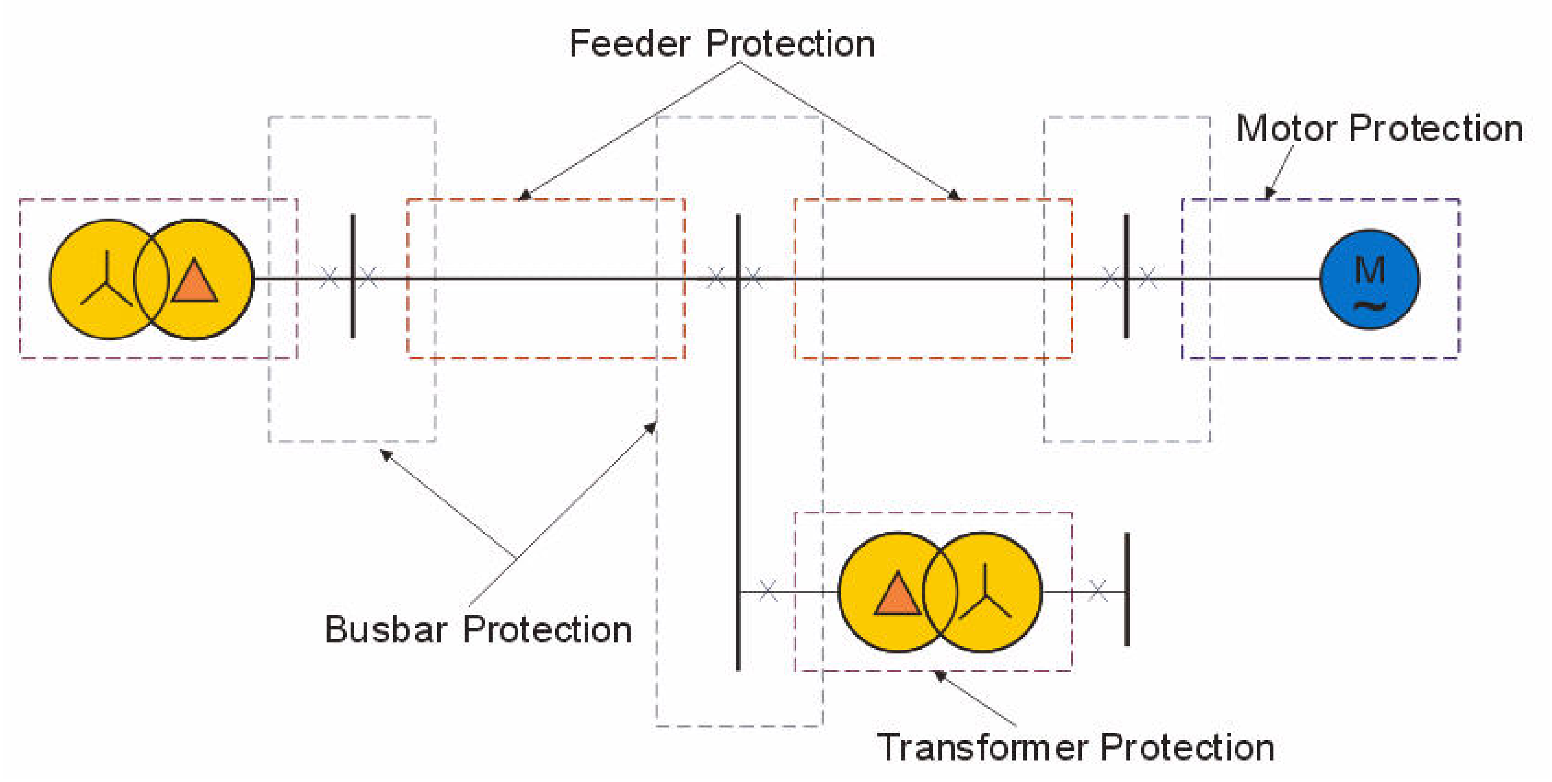
Topic 1: Power System Protection

1.1 Introduction



The Need for Protection

Protective relaying is the *Science* or *Art* of detecting faults on power systems and clearing those faults from the power system as quickly as possible.

Protective equipment or protective relay is used in a power network to detect, discriminate and isolate the faulty equipment in the network.

Basic Requirements of Power System Protection

1. to ensure continuity of supply.
2. to minimize damage and repair costs.
3. to ensure safety of personnel.

Effects of Short Circuits

If short circuits are allowed to persist on a power system for an extended period, the following effects are likely to occur:

- **Reduced stability** margins for the power system.
- **Damage** to the equipment that is in the vicinity of the fault due to heavy currents, or low voltages produced by the short circuit.
- **Explosions** in equipment containing insulating oil, cause fire.
- **Disruption** in the entire power system service area.

Attributes of Power System Protection

Basic Qualities

Protective Relays must have the following characteristics:

1. ***Selectivity:*** To detect and isolate the faulty item only.
2. ***Sensitivity:*** To detect even the smallest values of fault current or system abnormalities and operate correctly at its setting before the fault causes irreparable damage.
3. ***Speed:*** To operate speedily when it is called upon to do so, thereby minimizing damage to the surroundings and ensuring safety to personnel.
4. ***Stability:*** To leave all healthy circuits intact to ensure continuity or supply.

Electrical Fault Energy

Why Speed is Important?

Energy released into fault = $I^2 \times R \times t$

where I = Fault Current

R = Resistance of Fault Arc

t = Time in seconds when fault is ON.

So, the faster the fault clearing time, the lesser is the energy released.

The Need for Speed

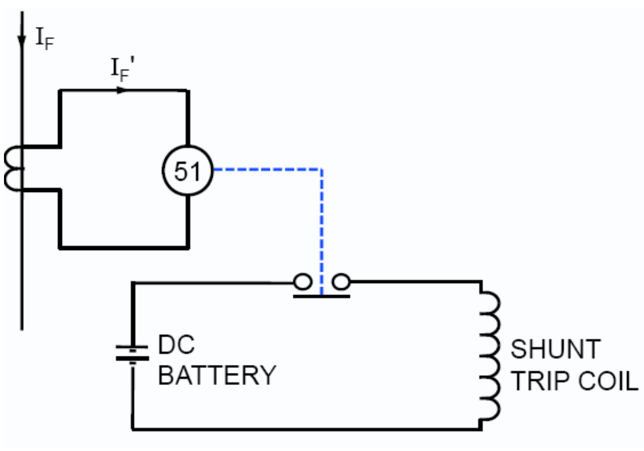
- Fault Current = 4000 Amps
- Clearance Time = 350 milliseconds (0.35 s)
- Assume ARC Resistance of 1 Ω
- Fault Energy = I^2 x R x t = 4000 x 4000 x 1 x 0.35

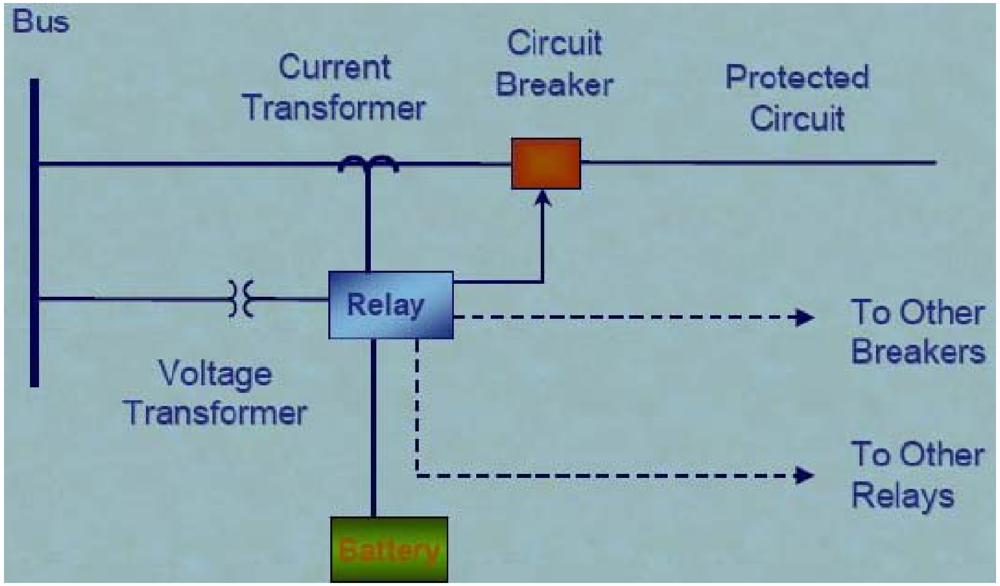
  = 5.6 Mega Joules
- If clearance time reduced to 100 milliseconds (0.1 s)
- Fault Energy = 4000 x 4000 x 1 x 0.1

  = 1.6 Mega Joules
- **HENCE A 70% REDUCTION !**
- If steps could be taken to also reduce level of fault current then major strides would be made.

Components of Protection Schemes

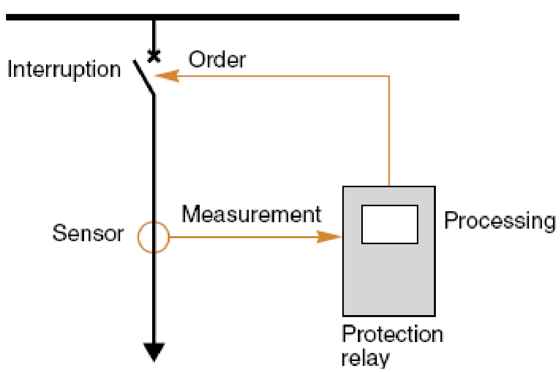
- Each power system protection scheme is made up from the following components:
  1. **Fault Detecting or Measuring Relays.**
  2. **Tripping and other Auxiliary Relays.**
  3. **Circuit Breakers.**
  4. **Current Transformers and Voltage transformers**
  5. **DC Batteries.**

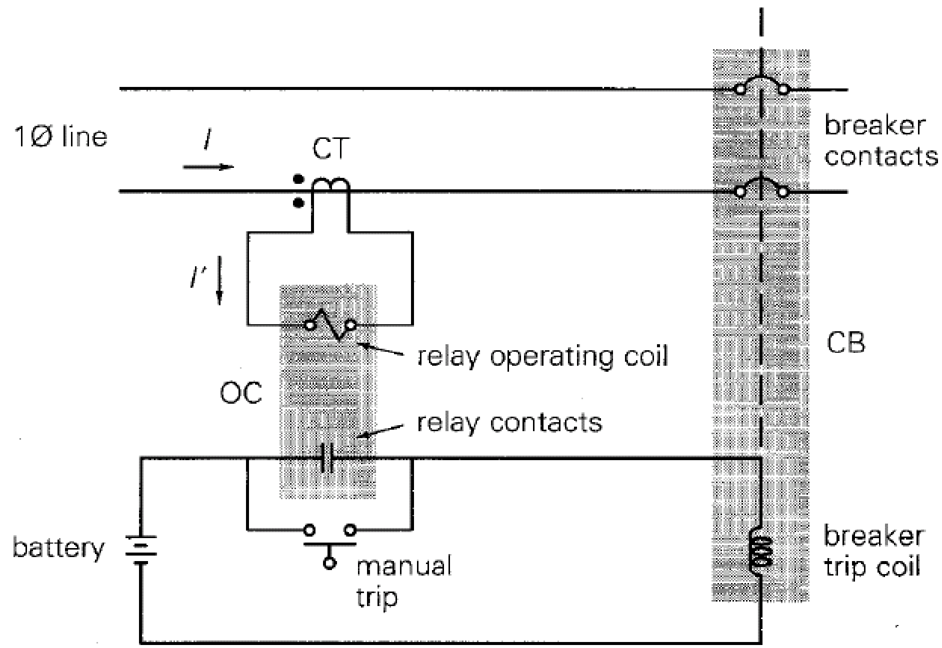




Components of Protection Schemes

- All power system elements are equipped with one or more protection schemes to detect faults on the system.
- When the protective relays have detected a fault, they send trip signals to the circuit breaker or breakers, which in turn clear the fault from the system.





Circuit Breaker and Relay Wiring Schematic
(1 of 3 phases)

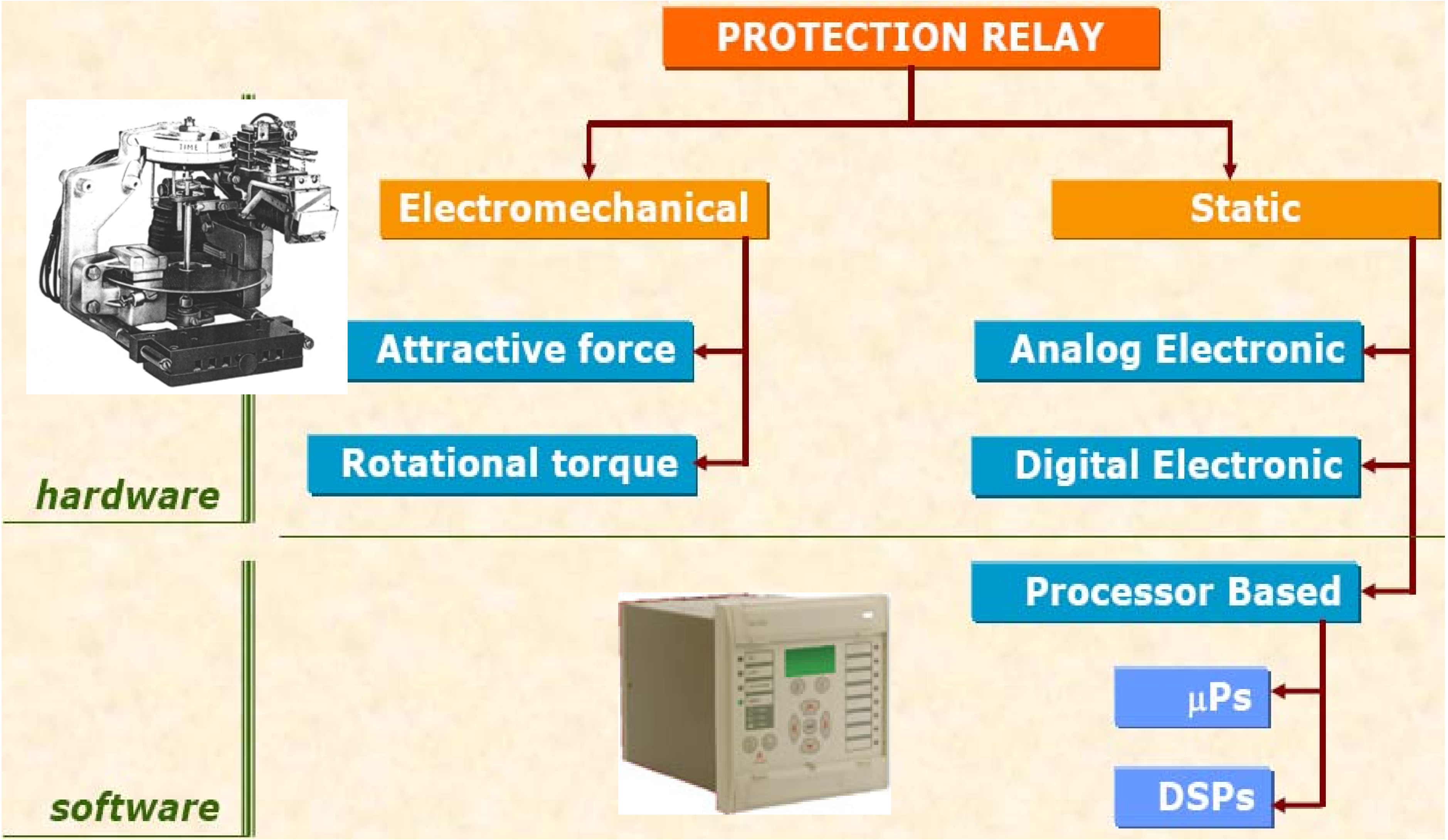
Relay Technology

Protection Relay Technology Evolution

Electromechanical: A protection relay design which uses magnetomotive force in its decision making stage and has moving parts in it.

Static: A protection relay design which does not have any moving part in the decision making stage.

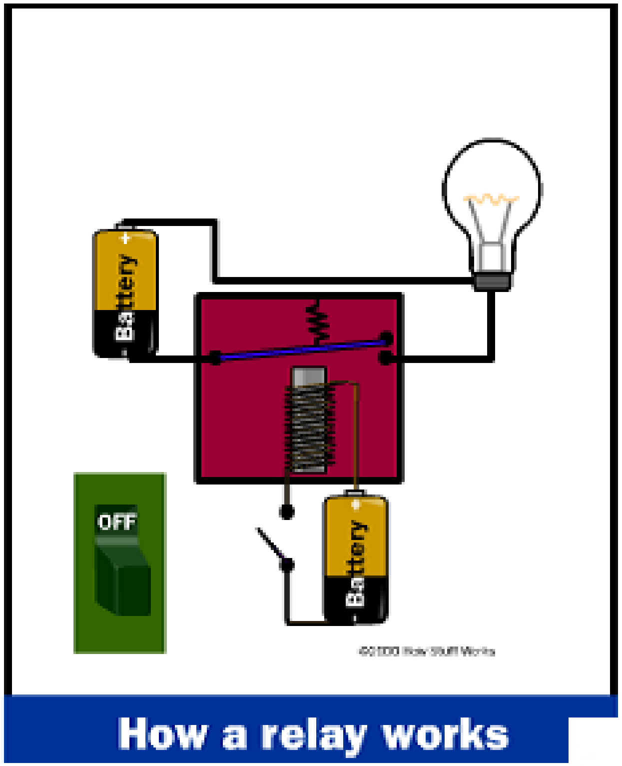
Protection Relay Technology Evolution

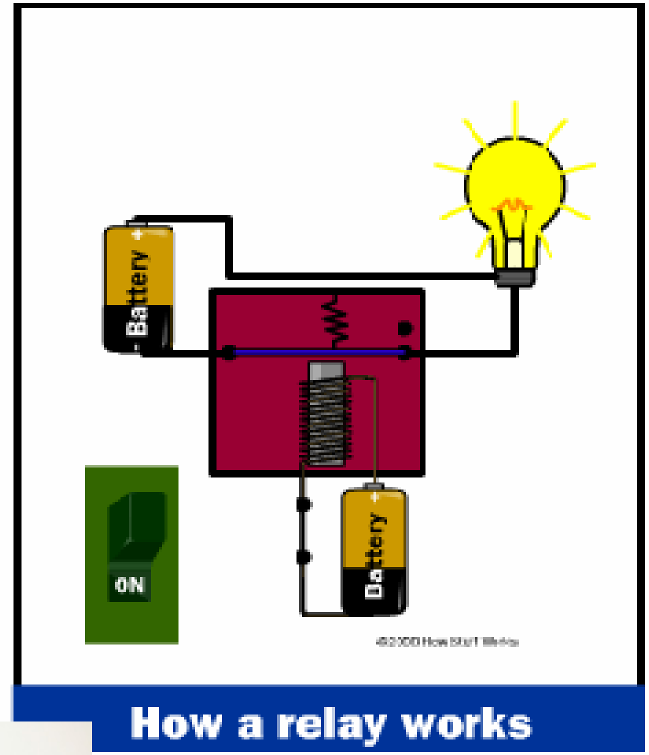


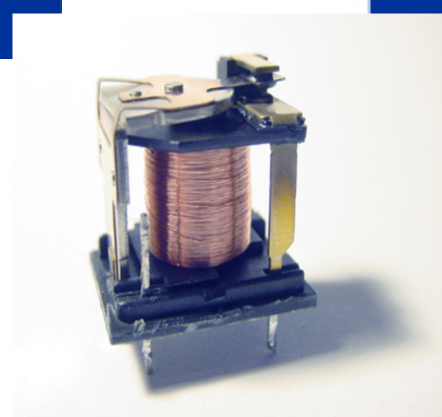
What are Relays?

- Relays are electrical switches that open or close another circuit under certain conditions.
- **Electromagnetic Relays (EMRs)**
  - EMRs consist of an input coil that's wound to accept a particular voltage signal, plus a set of one or more contacts that rely on an armature (or lever) activated by the energized coil to open or close an electrical circuit.
- **Solid-state Relays (SSRs)**
  - SSRs use semiconductor output instead of mechanical contacts to switch the circuit. The output device is optically-coupled to an LED light source inside the relay. The relay is turned on by energizing this LED, usually with low-voltage DC power.
- **Microprocessor Based Relays**
  - Use microprocessor for switching mechanism. Commonly used in power system monitoring and protection.

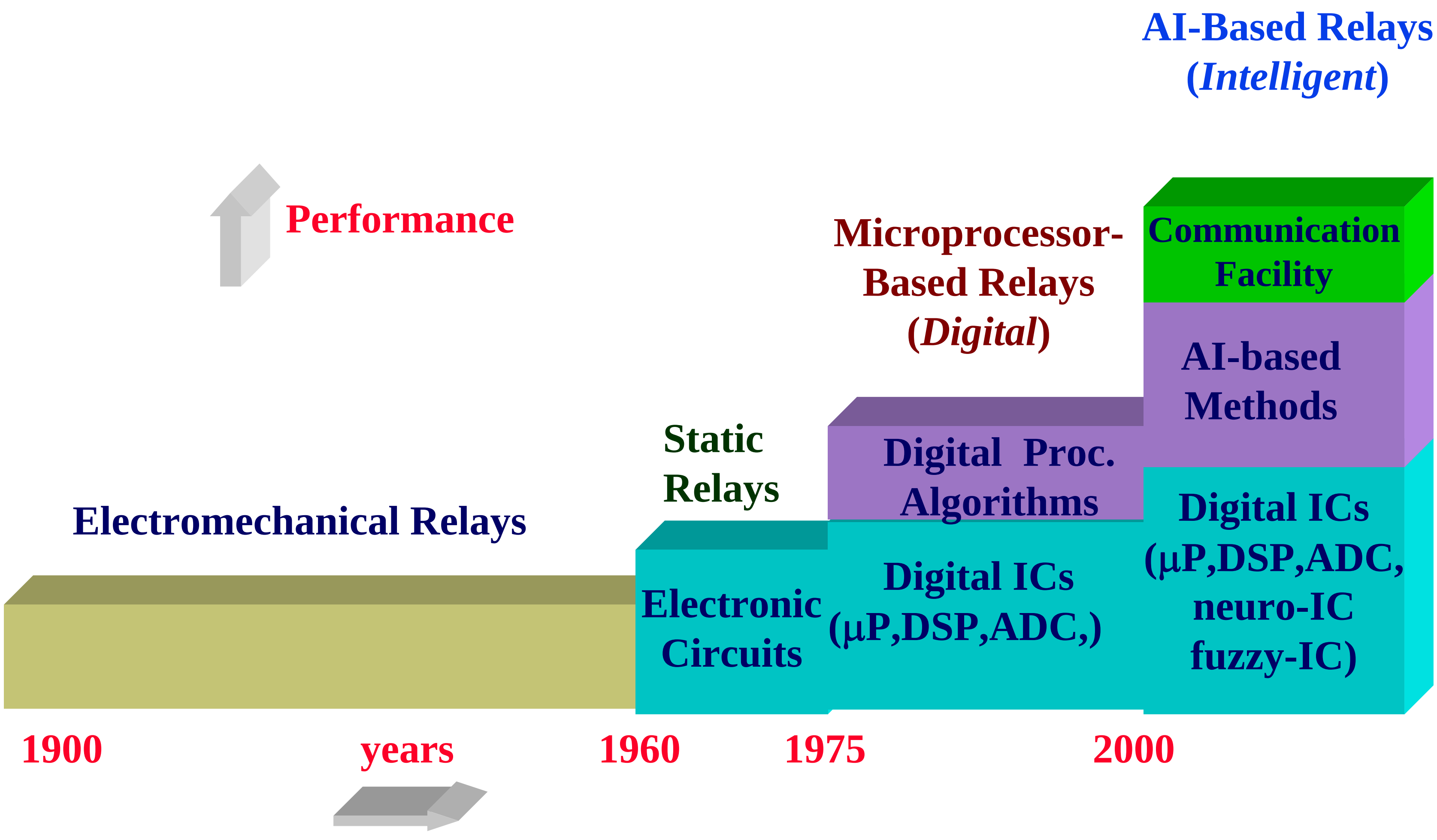
How a Relay Works?





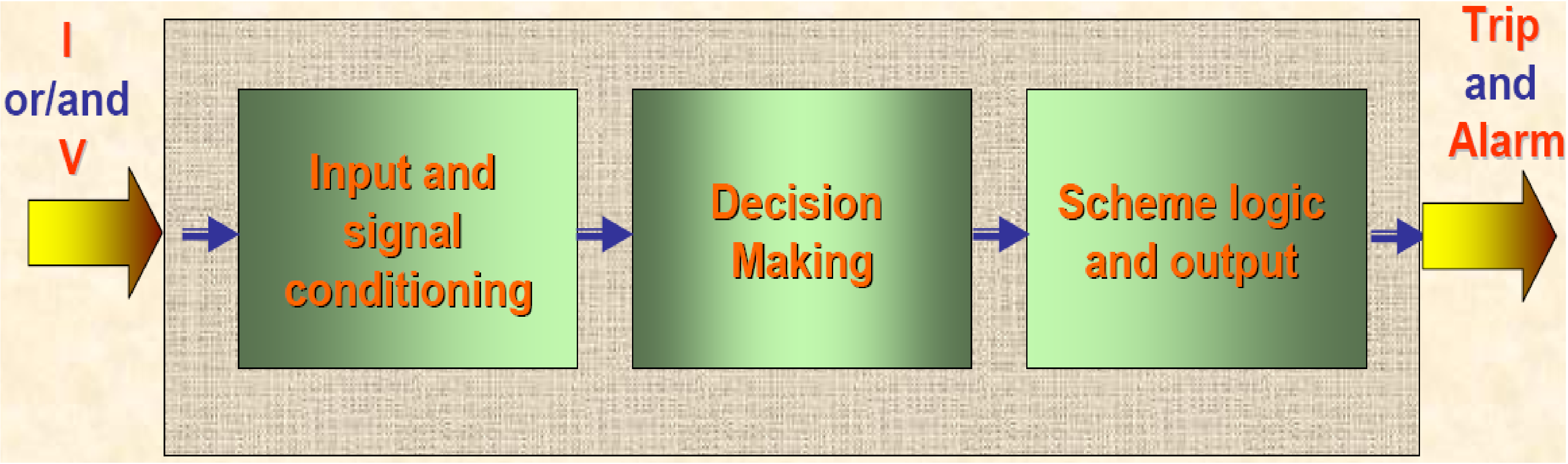


Development in Power System Relaying



Relay Technology

Protection Relay Functional Block Diagram

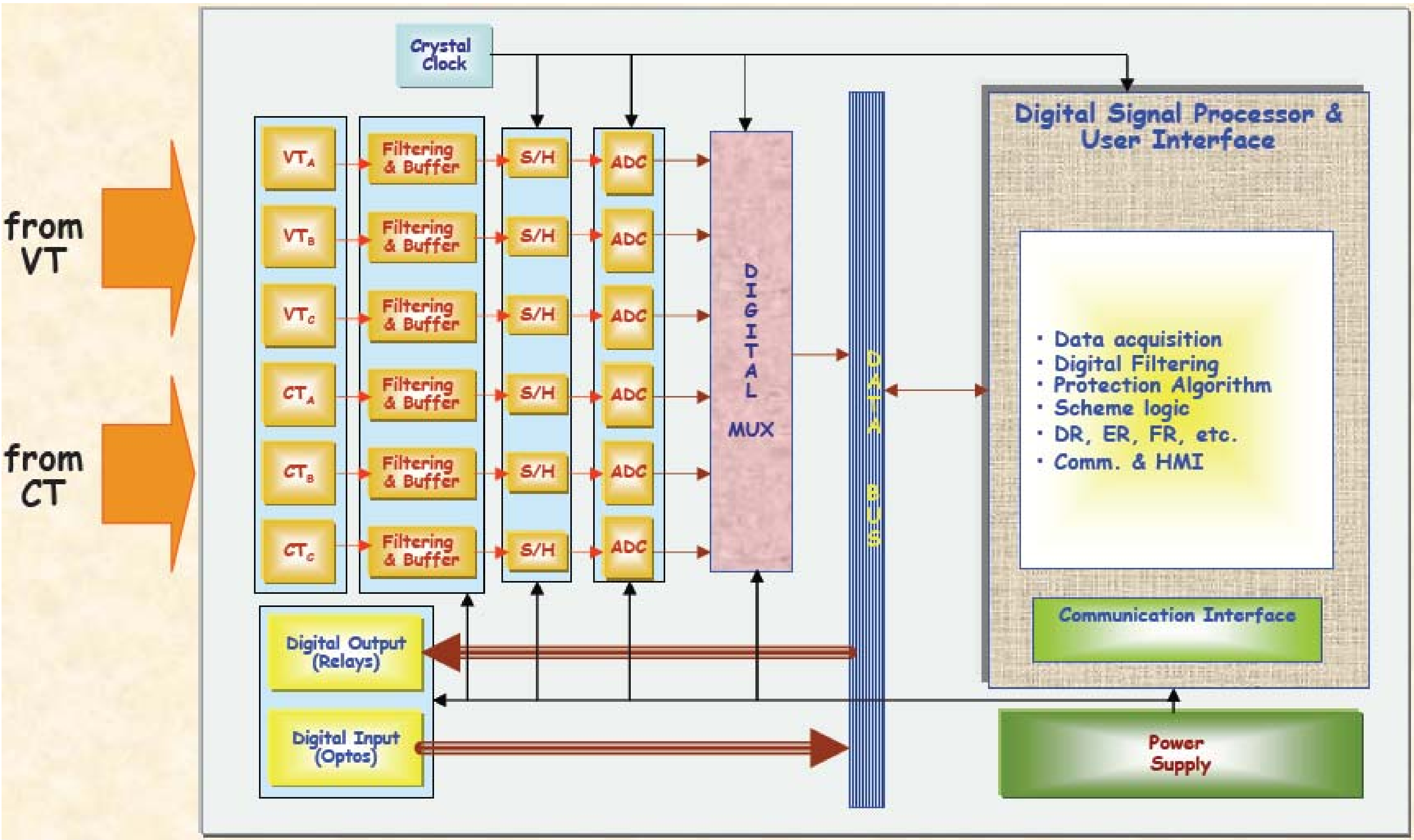


The voltage and/or current signal is first reduced to measurable quantities and necessary conditioning done .

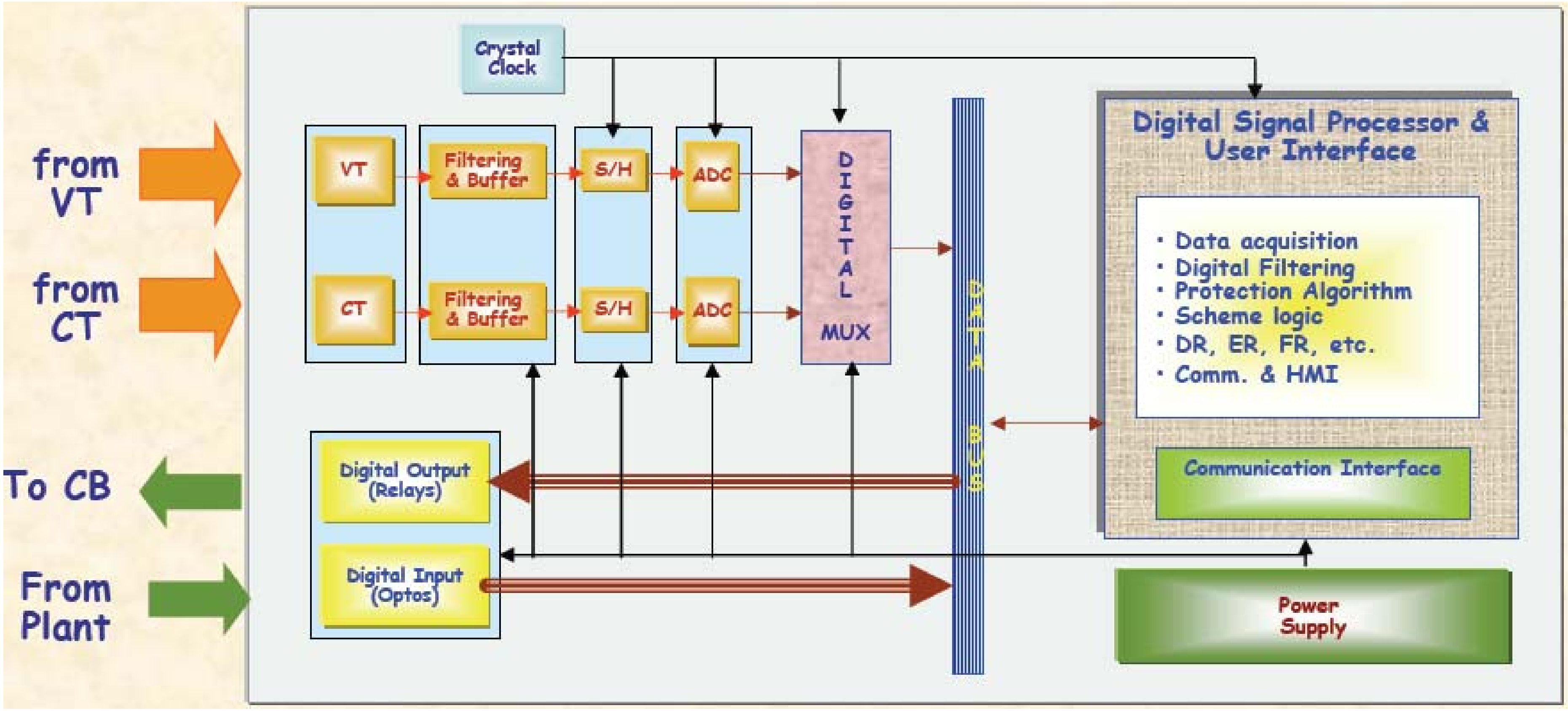
The decision making stage does the actual protection as per the set value.

The output stage implements the necessary logic before issuing trip and alarm commands.

Protection Relay Technology-Numerical



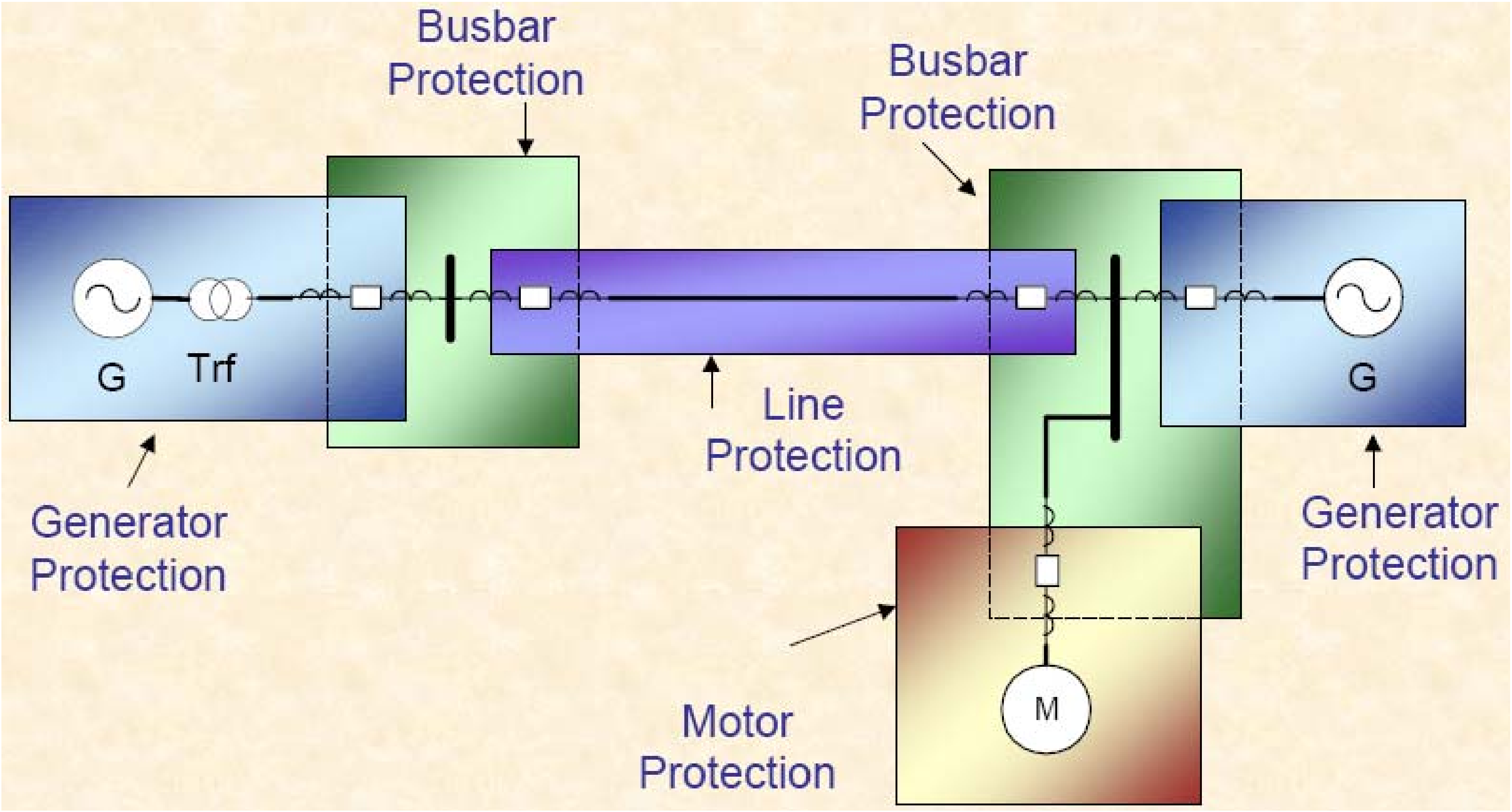
Single-Phase Impedance Relay-Numerical



Zones Of Protection

- For fault anyway within the zone, the protection system responsible to isolate everything within the zone from the rest of the system.
- Isolation done by CB
- Must isolate only the faulty equipment or section

Zones Of Protection

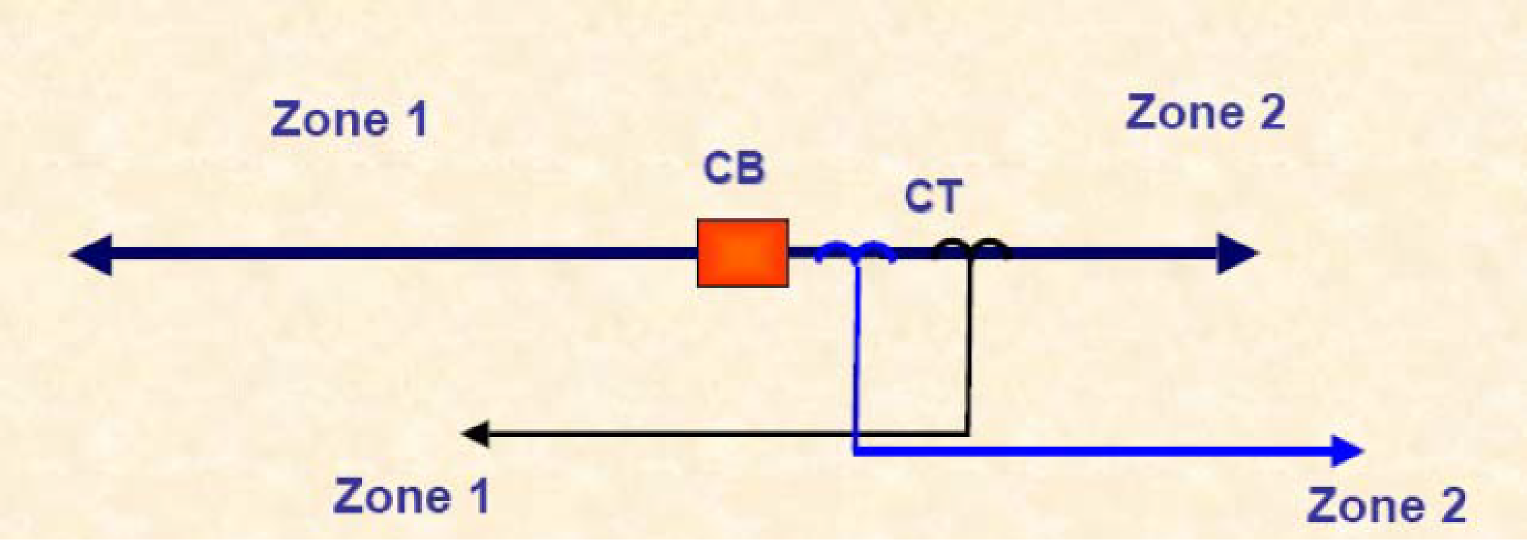


“Zones” of Protection

- Zones are defined for:
  - Generators
  - Transformers
  - Buses
  - Transmission and distribution lines
  - Motors
- Characteristics:
  - Zones are overlapped.
  - Circuit breakers are located in the overlap regions.
  - For a fault anywhere in a zone, all circuit breakers in that zone open to isolate the fault.

Overlapped Protection

- Overlap accomplish by having 2 sets of instrument transformers and relays for each CB.
- Achieved by the arrangement of CT and CB.



Types of Protection Schemes

A - Fuses

- For LV Systems, Distribution Feeders and Transformers, VT's, Auxiliary Supplies

B - Over current and earth fault

- Widely used in All Power Systems
  - Non-Directional
  - Directional

C - Differential

- For Distribution Feeders, Busbars, Transformers, Generators etc

Types of Protection Schemes

D - Distance

- For Transmission and Sub-transmission Lines and Distribution Feeders,
- Also used as back-up protection for transformers and generators without signaling with signaling to provide unit protection e.g.:
  - Time-stepped distance protection
  - Phase comparison for transmission lines
  - Directional comparison for transmission lines

Types of Protection Schemes

E - Miscellaneous:

- Under and over voltage
- Under and over frequency
- A special relay for generators, transformers, motors etc.
- Control relays: auto-reclose, tap change control, etc.
- Tripping and auxiliary relays

Which Relays are Used in What Applications?

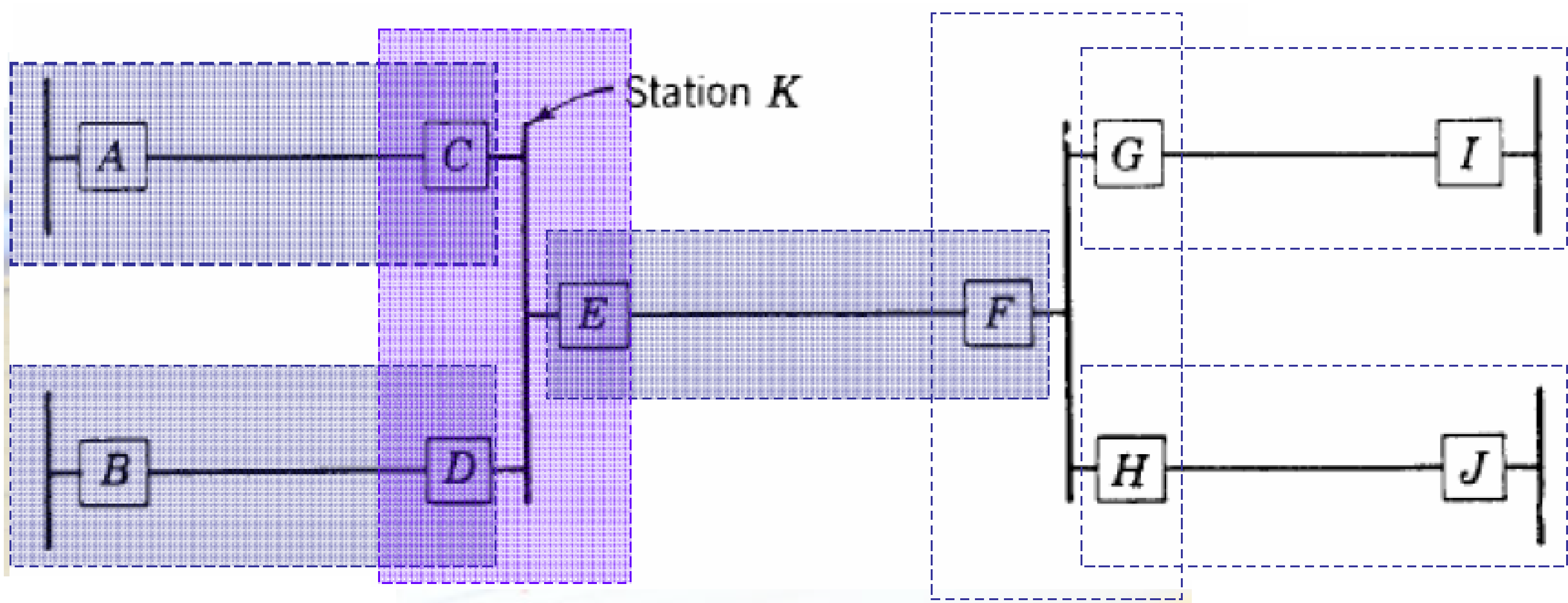
It depends on the importance of the power system element being protected.

Protection Type	Application Areas
Fuse	Local LV distributor
HRC Fuse	Major LV feeder, local HV spur line, HV side of distribution substation
Overcurrent and Earth Fault relay	Major HV distribution feeder, backup to transformer differential protection and feeder impedance protection on sub-transmission lines
Impedance relay	Primary protection on transmission and sub-transmission lines
Differential relay	Primary protection on large distribution and all sub-transmission and transmission level transformers; large generators
Thermal Overload relay	Transmission and sub-transmission level transformers, large motors, large generators
Oil Surge relay	Transmission and sub-transmission level transformers
Under and Over Volts relay	Large motors, large generators
Under and Over Frequency relay	Large generators
Negative Sequence relay	Large generators
Loss of Excitation relay	Large generators

Primary & Backup Protection Schemes

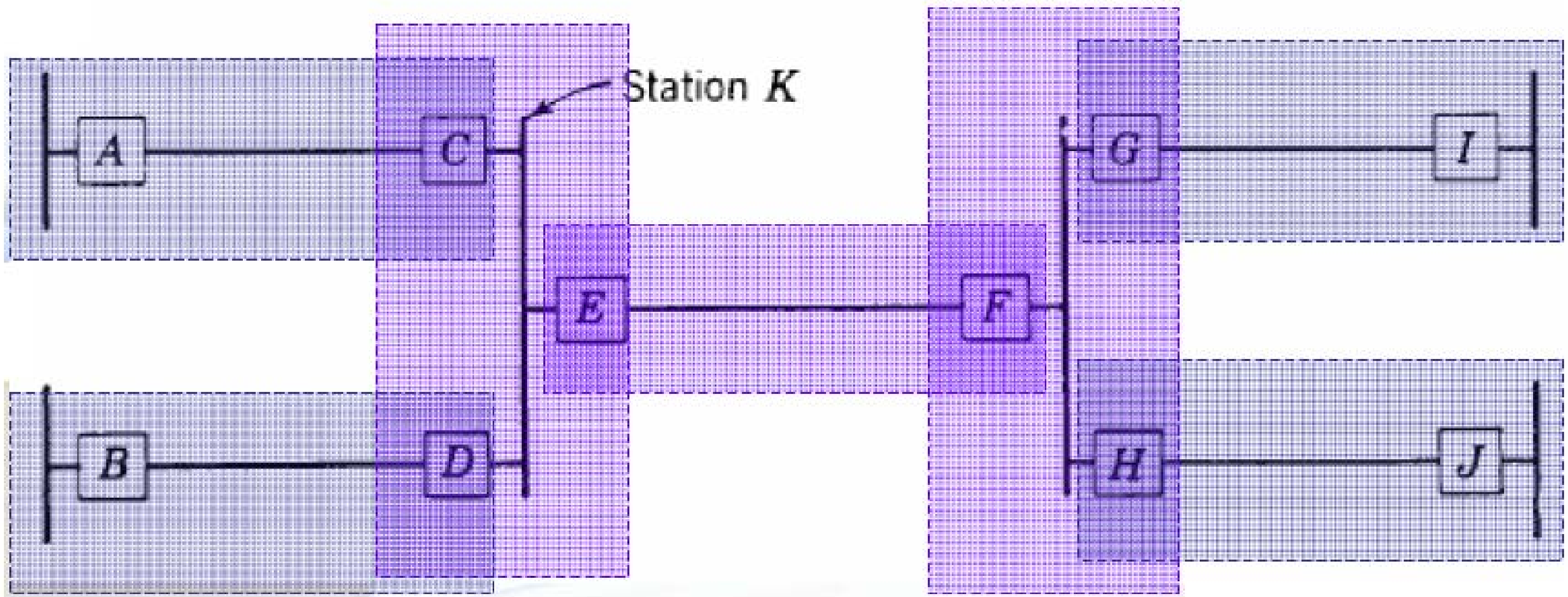
- Primary protection is the protection provided by each zone to its elements.
- However, some component of a zone protection scheme fail to operate.
- Back-up protection is provided which take over only in the event of primary protection failure.

Primary & Backup Protection - Example



Fault	Primary	Back-up
K	C, D, E	A, B, F

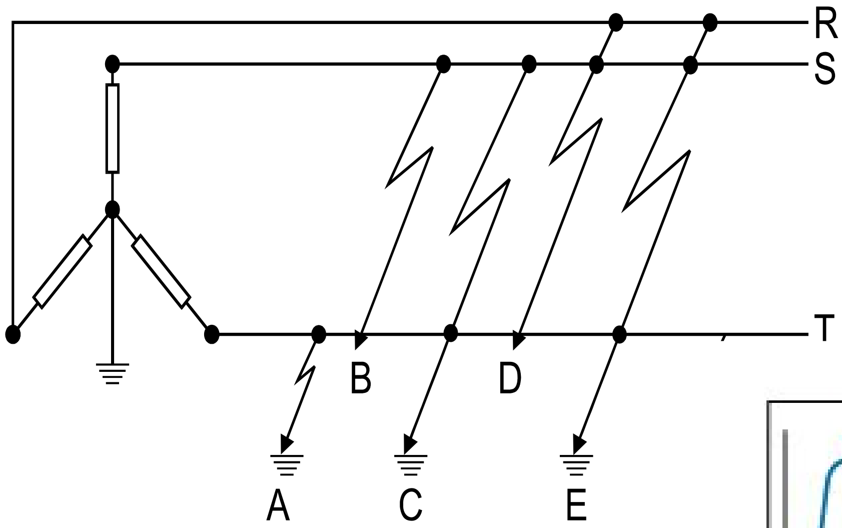
Primary & Backup Protection - Example

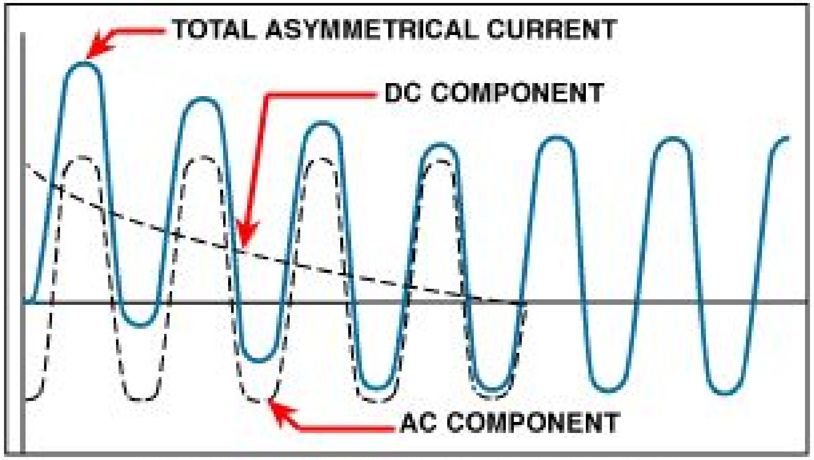


Fault	Primary	Back-up
Line E, F	C, D, E, F, G, H	A, B, I, J

Topic 1.2

Fault Types and Calculations





SHORT CIRCUIT CURRENT COMPONENTS

Definition

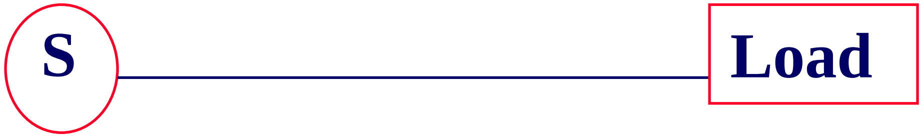
- Standard IEC 60038 defines voltage ratings as follows:
- ***Low voltage (LV):*** 100 V and 1,000 V, the standard ratings are: 400 V - 690 V - 1,000 V (at 50 Hz).
- ***Medium voltage (MV):*** between 1,000 V and 35 kV, the standard ratings are: 3.3 kV - 6.6 kV - 11 kV - 22 kV - 33 kV.
- ***High voltage (HV):*** between 35 kV and 230 kV, the standard ratings are: 45 kV - 66 kV - 110 kV - 132 kV - 150 kV - 220 kV.

In this Topic we will look at:

- types of HV and MV consumer substations;
- structure of MV networks inside a site;

Simple Distribution Systems

- **Radial**



- **Advantages**

Simple (lowest capital cost)

Easy and simple to protect

- **Disadvantages**

Little security of supply for customer - a single fault will cause loss of supply

Simple Distribution Systems

- **Parallel**

  - **Advantages**

    Better power availability for customer

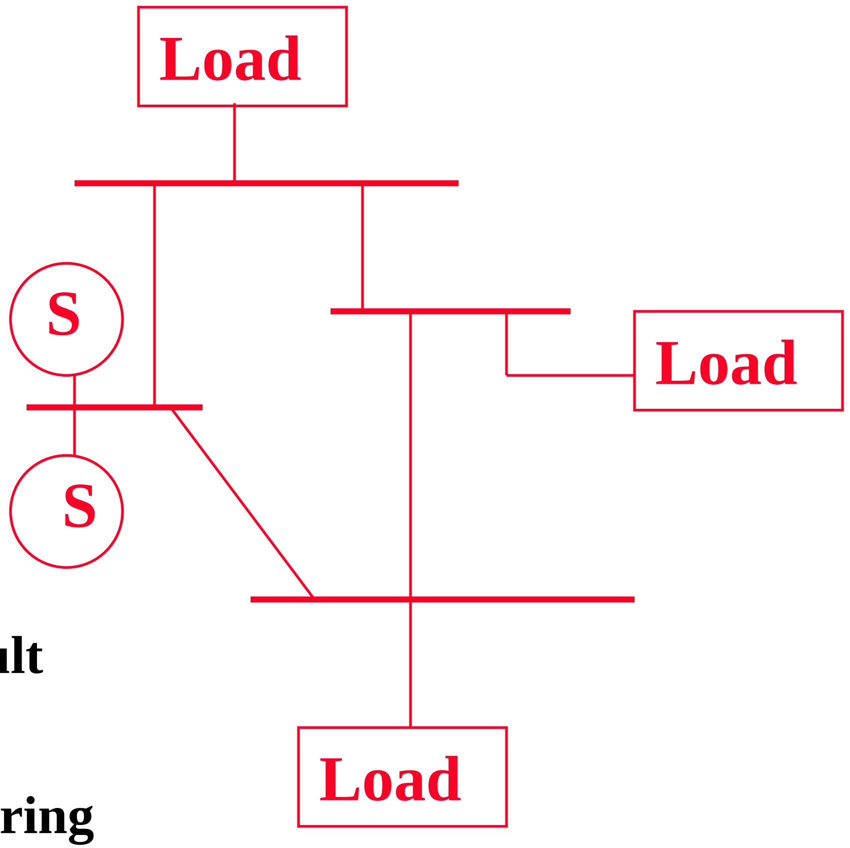
  - **Disadvantages**

    More expensive

    Increased fault currents

Simple Distribution Systems

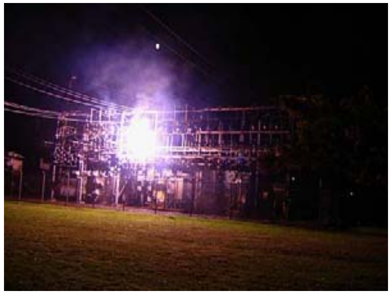
- **Ring Main**
  - **Advantages**
    - **Maintains continuity even if one source fails.**
    - **Savings in Copper compared to parallel type.**
  - **Disadvantages**
    - **Lower impedance and Higher Fault current as in feed from two points**
    - **Requires better discrimination during faults due to alternate paths**



What is a Power System Fault?

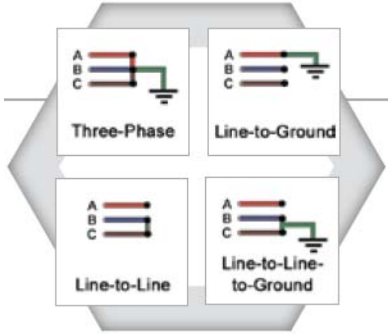
- A power system fault is the breakdown of insulation (between conductors, or between a phase conductor and ground) which results in excess current flow.

Possible Faults



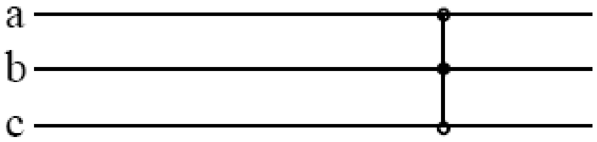
- Cable Faults
- Transformer faults
- Busbar Faults

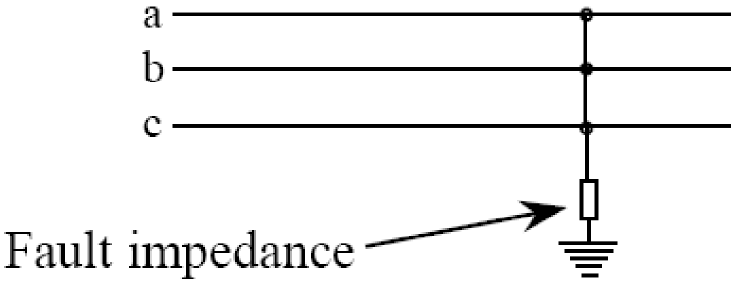
Types of Faults



- **Balanced Faults (Symmetrical Faults)**

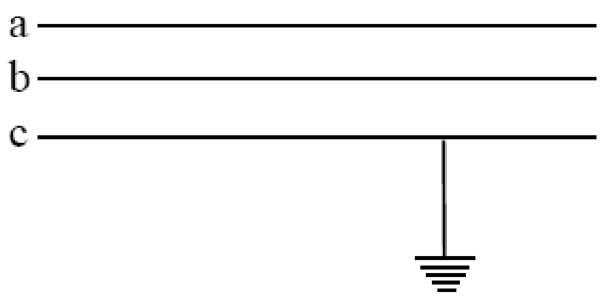
  - **3-Phase Fault (with or without ground) → (5%)**

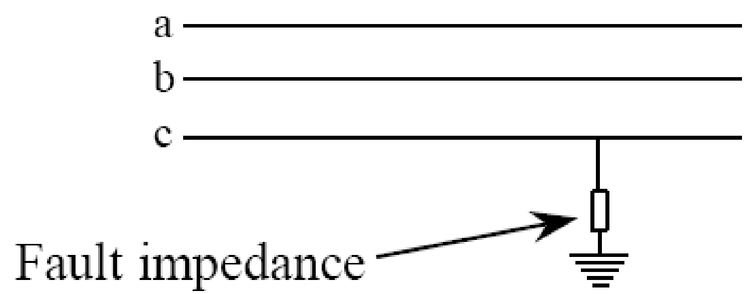


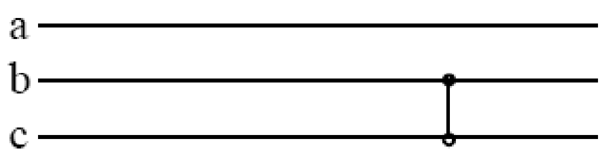


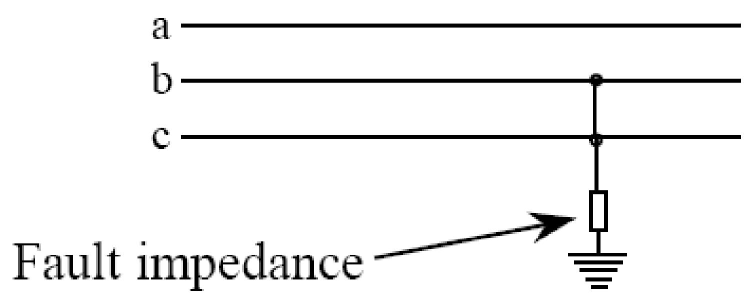
Types of Faults

- **Unbalanced Faults (Unsymmetrical Faults)**
  - **Single phase (Phase-Ground) → (70%)**
  - **Two phase to ground (Phase-Phase-Ground) → (15%)**
  - **Two phase (Phase-Phase) → (10%)**





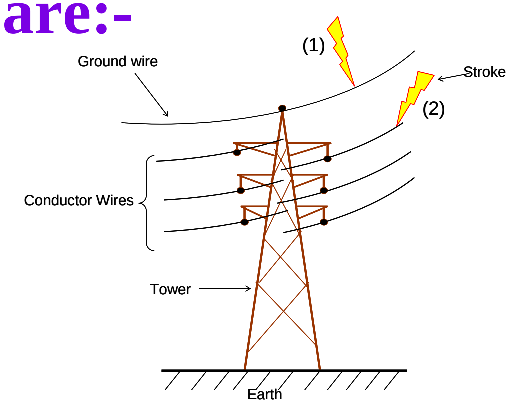


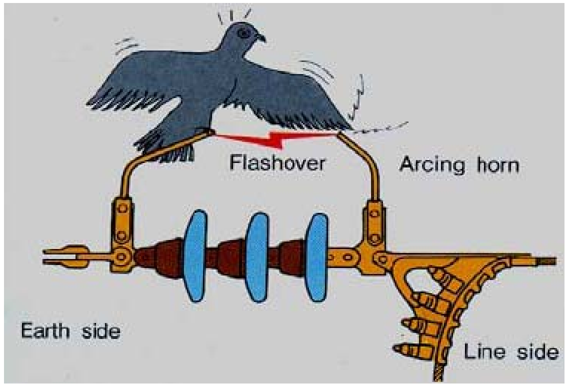


Causes of Faults on Power System

➔ **The most common causes of faults on OHL are:-**

- **Lightning**
- **Contaminated Insulators**
- **Punctured or broken insulators**
- **Birds and animals**
- **Cars hitting lines and structures**
- **Ice and snow loading**
- **Wind**



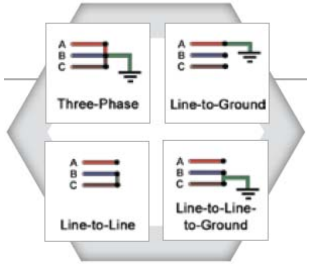


Causes of Faults on Power System

➔ **In electrical machines, cables and transformers, faults are caused by:**

- **Failure of insulation because of moisture**
- **Mechanical damage**
- **Flashover caused by overvoltage or abnormal loading.**

Simple Calculation of Short- Circuit Currents



- Calculation Methods
  - Ohmic Method - Where all impedances are expressed in Ohms
  - Per Unit Method - Similar to % impedance method removing the % factor

Standardized I_{sc} calculations

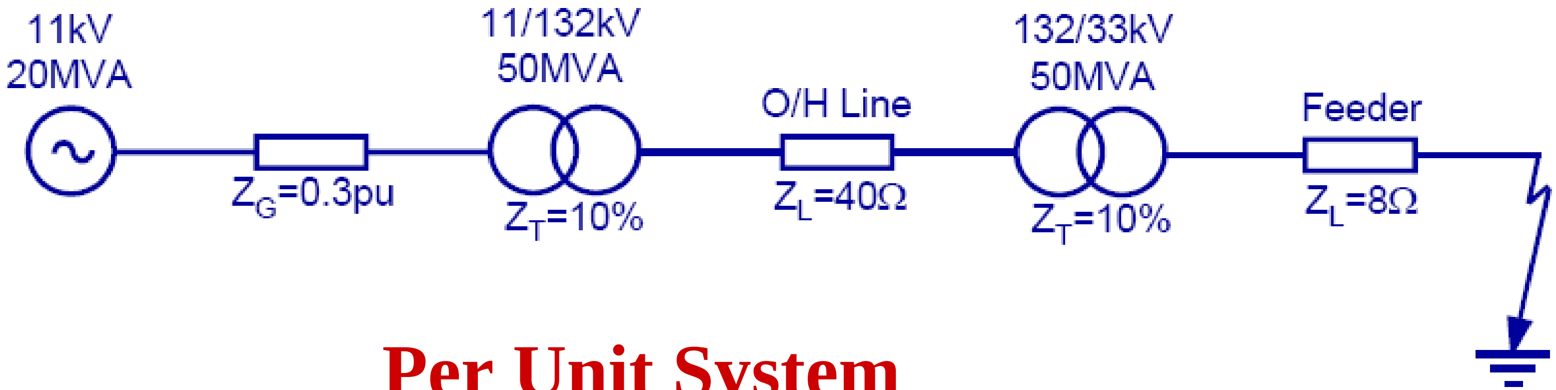
The Impedance Method-Classical

- Used to calculate fault currents at any point in a network with a high degree of accuracy.
- The impedance method, reserved primarily for LV networks.
- This method involves adding the various resistances and reactances of the fault loop separately, from the source to the given point, and then calculating the corresponding impedance.
- The I_{sc} value is obtained by applying Ohm's law:

$$I_{sc} = \frac{V_{LL}}{\sqrt{3}\sum Z}$$

Different Voltages

How do we Analyze?



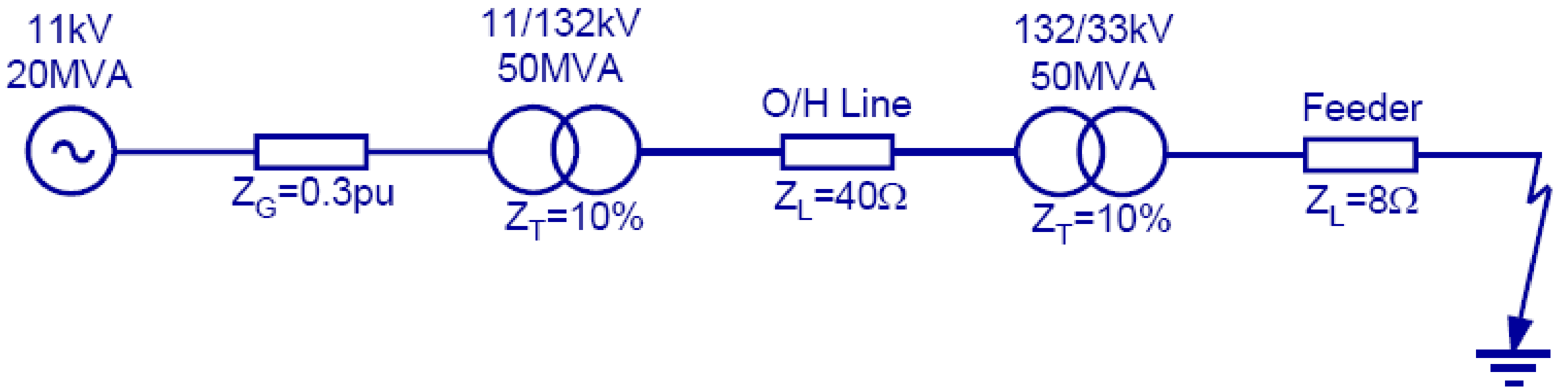
Per Unit System

- **Used to simplify calculations on systems with more than 2 voltages.**

Definition:

$$\text{Per Unit Value of a Quamtity} = \frac{\text{Actual Value}}{\text{Base value of the Same units}}$$

Base Quantities and Per Unit Values



- **Particularly useful when analyzing large systems with several voltage levels**
- **All system parameters referred to common base quantities**
- **Base quantities fixed in one part of system**
- **Base quantities at other parts at different voltage levels depend on ratio of intervening transformers**

Base Quantities and Per Unit Values

Base quantities normally used :-

- **Base MVA**

$\mathbf{MVA_{BASE} = MVA_b = MVA_{\ 3Ph}}$

- **Constant at all voltage levels**
- **Value ~ MVA rating of largest item of plant or 100MVA**

- **BASE VOLTAGE**

$\mathbf{kVA_{BASE} = kV_b = LL\ Voltage\ in\ kV}$

- **Fixed in one part of system**
- **This value is referred through transformers to obtain base voltages on other parts of system.**
- **Base voltages on each side of transformer are in same ratio as voltage ratio.**

Base Quantities and Per Unit Values

Other Base quantities:-

BASE Impedance = $$Z_b = \frac{(kV_b)^2}{MVA_b}$$ **in Ohms**

BASE Current = $$I_b = \frac{MVA_b}{\sqrt{3} \times kV_b}$$ **in kA**

Base Quantities and Per Unit Values

$$\text{Per Unit Value} = \frac{\text{Actual Value}}{\text{Base value of the Same units}}$$

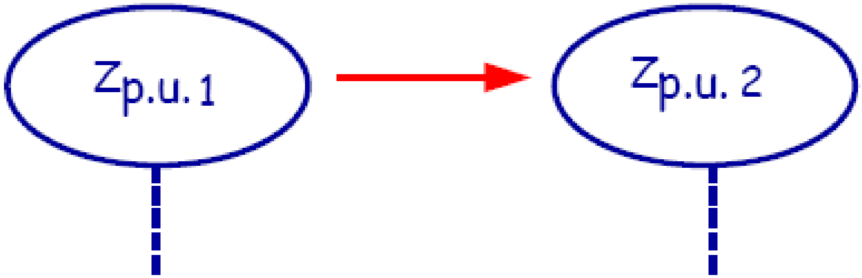
$$\text{Per Unit MVA} = MVA_{p.u.} = \frac{MVA_a}{MVA_b}$$

$$\text{Per Unit Voltage} = kV_{p.u.} = \frac{KV_a}{KV_b}$$

$$\text{Per Unit Impedance} = Z_{p.u.} = \frac{Z_a}{Z_b} = Z_a \cdot \frac{MVA_b}{(kV_b)^2}$$

$$\text{Per Unit Current} = I_{p.u.} = \frac{I_a}{I_b}$$

Conversion of Per Unit Values from One Set of Quantities to Another



Z_{b1} $\quad$ Z_{b2}

MVA_{b1} $\quad$ MVA_{b2}

kV_{b1} $\quad$ kV_{b2}

Actual $Z = Z_a$

$$Z_{p.u.1} = \frac{Z_a}{Z_{b1}}$$

$$Z_{p.u.2} = \frac{Z_a}{Z_{b2}} = Z_{p.u.1} \times \frac{Z_{b1}}{Z_{b2}}$$

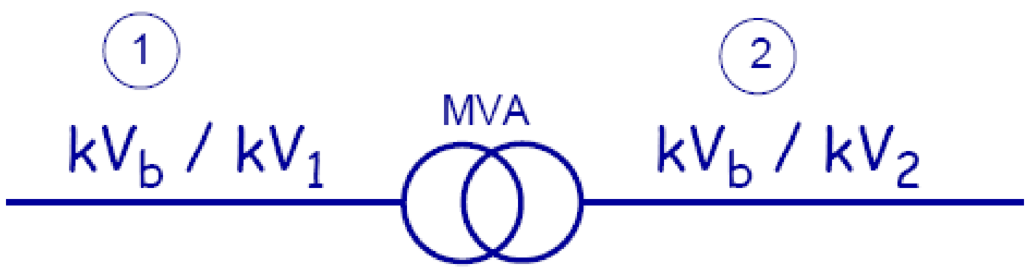
$$= Z_{p.u.1} \times \frac{(kV_{b1})^2}{MVA_{b1}} \times \frac{MVA_{b2}}{(kV_{b2})^2}$$

$$= Z_{p.u.1} \times \frac{MVA_{b2}}{MVA_{b1}} \times \frac{(kV_{b1})^2}{(kV_{b2})^2}$$

Transformer Percentage Impedance

Per unit impedance of transformer is same on each side of the transformer.

Consider transformer of ratio kV_1 / kV_2



Actual impedance of transformer viewed from side 1 = Z_{a1}

Actual impedance of transformer viewed from side 2 = Z_{a2}

Transformer Percentage Impedance

$$Z_{p.u.1} = \frac{Z_{a1}}{Z_{b1}} = Z_{a1} \times \frac{MVA}{kV_1^2}$$

$$Z_{p.u.2} = \frac{Z_{a2}}{Z_{b2}} = Z_{a2} \times \frac{MVA}{kV_2^2}$$

but $$Z_{a2} = Z_{a1} \times \frac{kV_2^2}{kV_1^2}$$

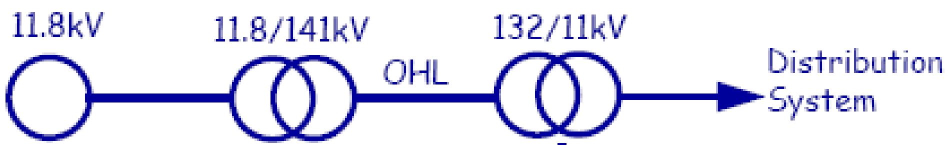
$$\therefore \quad Z_{p.u.2} = Z_{a1} \times \frac{kV_2^2}{kV_1^2} \times \frac{MVA}{kV_2^2}$$

$$= Z_{a1} \times \frac{MVA}{kV_1^2}$$

$$= Z_{p.u.1}$$

Transformer – Base Voltage Selection

Base voltage on each side of a transformer must be in the same ratio as voltage ratio of transformer.



Incorrect selection of kV_b	11.8kV	132kV	11kV
Correct selection of kV_b	$\frac{132\times11.8}{141}$ = 11.05kV	132kV	11kV
Alternative correct selection of kV_b	11.8kV	141kV	$\frac{141\times11}{132}$ = 11.75kV

Procedure For Calculating Maximum Fault Current

1. **Draw a single-line diagram of the power system.**
2. **Collect detailed impedance data for all of the components of the power system. i.e *Resistance R* and *Reactance X***
3. **Although fault current can be calculated using the *Ohmic* method, it is usually simpler to use the *Per-Unit Method* where all of the impedances are referred to an arbitrarily chosen common *BASE MVA*.**
4. **Convert all of the various impedances to *per-unit* values with a common base MVA.**
5. **Find the total *Resistance R*, and *Reactance X*, from the source to the fault.**
6. **Calculate the total Impedance:**

$$Z = \sqrt{R^2 + X^2}$$

Power System Fault Analysis

Balanced 3-Phase Faults

➔ **RARE:- Majority of faults are unbalanced**

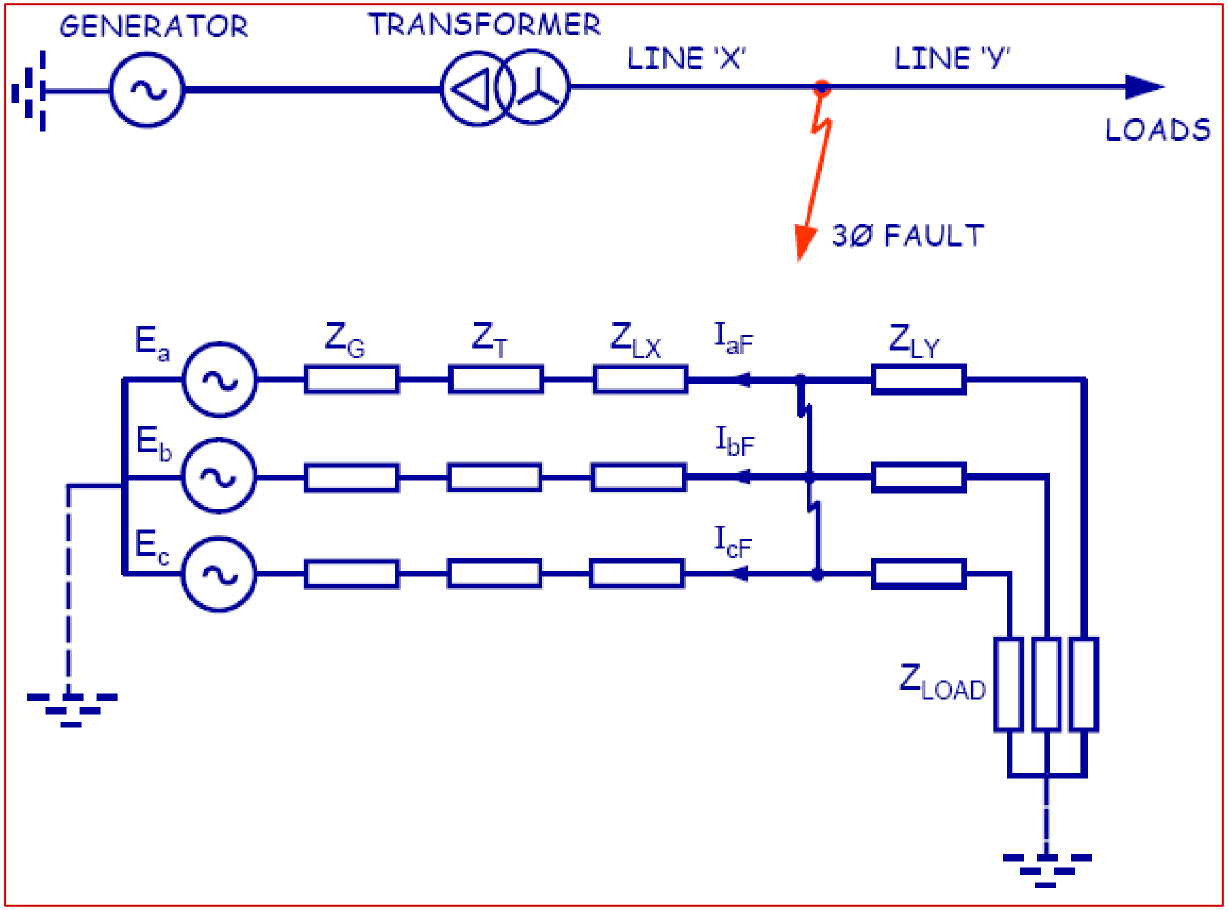
➔ **CAUSES:-**

- **System energization with maintenance earthing clamps still connected.**
- **1-Phase faults developing into 3-Phase faults**

➔ **3-Phase faults may be represented by 1-phase circuit**

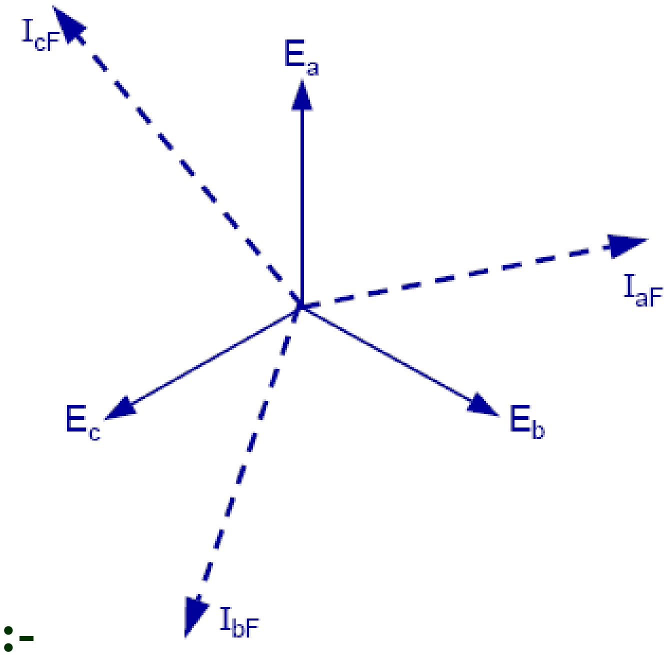
Power System Fault Analysis

Balanced 3-Phase Faults

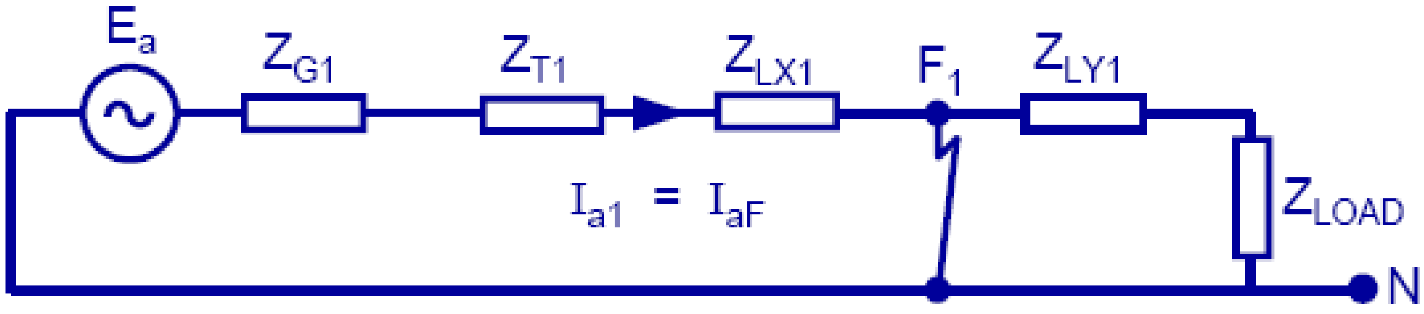


Power System Fault Analysis

Balanced 3-Phase Faults

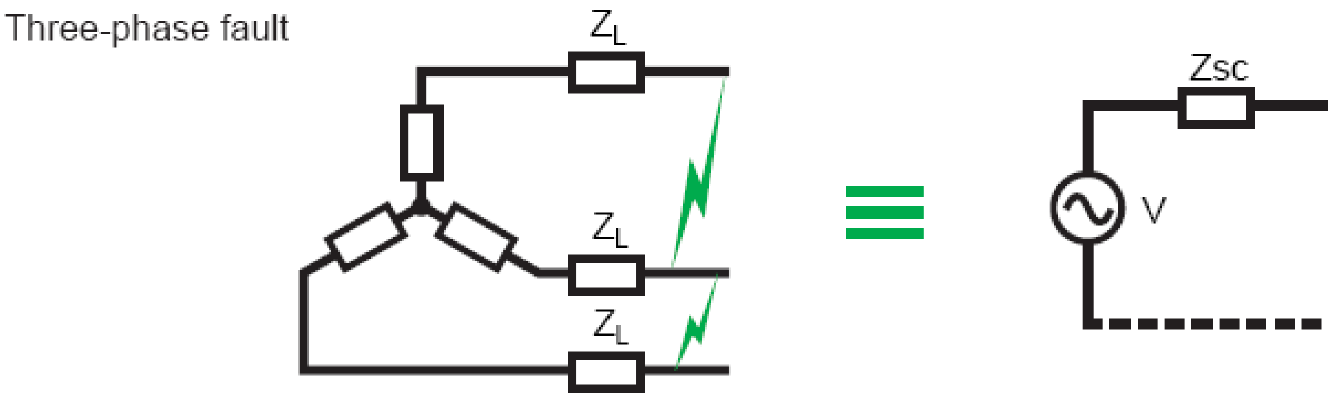


Positive Sequence (Single Phase) Circuit :-



Procedure For Calculating Maximum Fault Current
Using the Classical Method

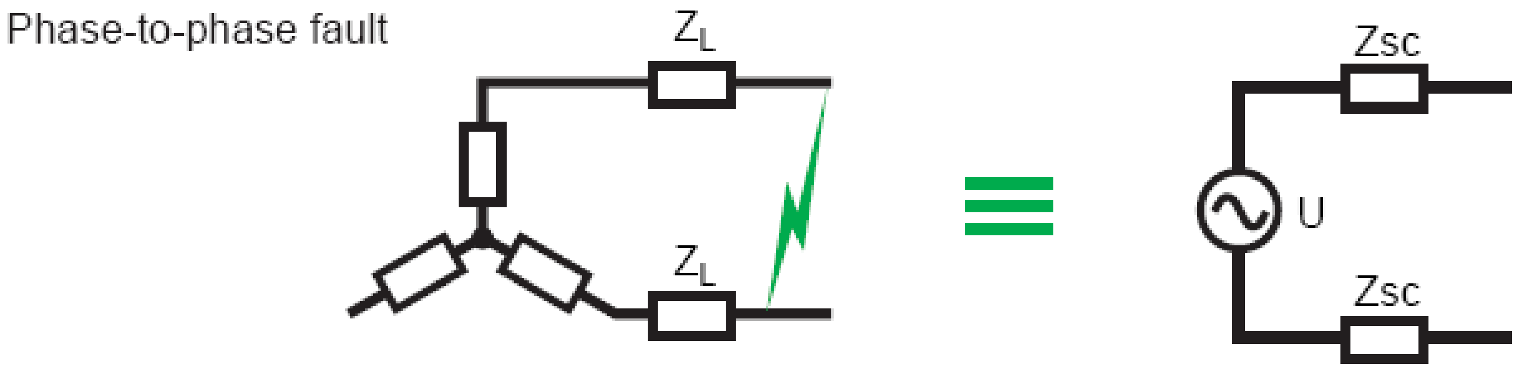
7. **Calculate the 3-PHASE FAULT CURRENT:**



$$I_{sc\text{-}3ph} = \frac{V_{ph}}{Z_{sc}}$$

Procedure For Calculating Maximum Fault Current
Using the Classical Method

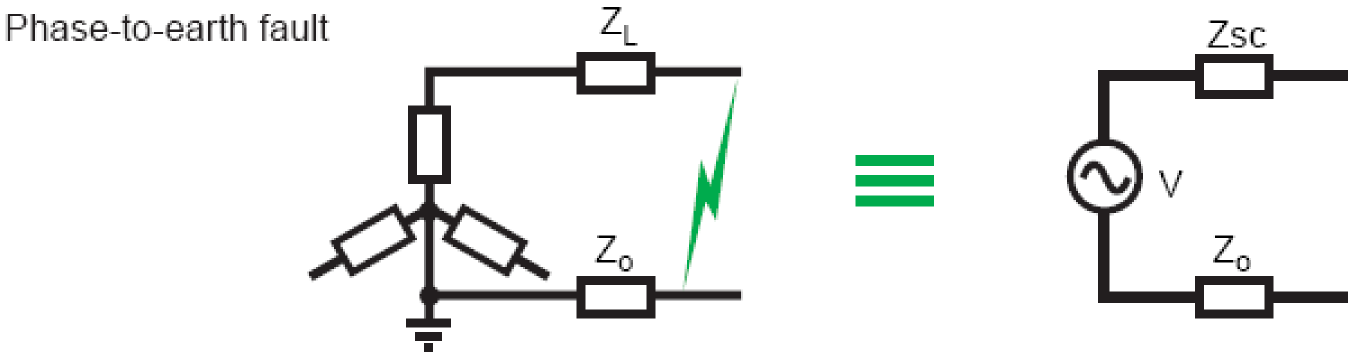
Calculate the PHASE-TO PHASE FAULT CURRENT:



$$I_{sc\text{-}2ph} = \frac{V_{ph\text{-}ph}}{2Z_{sc}} = \frac{\sqrt{3}}{2} I_{sc\text{-}3ph} = 0.866 I_{sc\text{-}3ph}$$

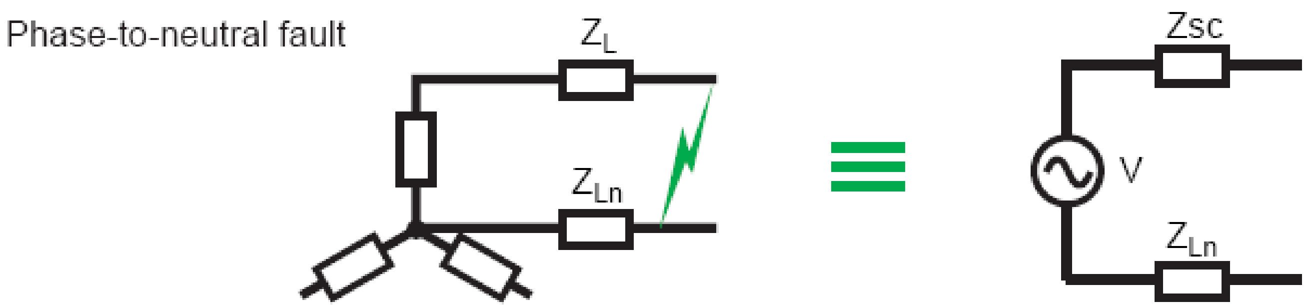
Procedure For Calculating Maximum Fault Current
Using the Classical Method

Calculate the PHASE-TO-GROUND FAULT CURRENT:



$$I_{sco} = \frac{V_{ph}}{Z_{sc} + Z_o}$$

Calculate the PHASE-TO-NEUTRAL FAULT CURRENT:



$$I_{sc1\text{-}ph} = \frac{V_{ph}}{Z_{sc} + Z_{LN}}$$

Procedure For Calculating Maximum Fault Current
Using the Classical Method

When using the PER-UNIT METHOD to calculate fault levels the following formulae are used to convert all impedances to per-unit values.

$$\mathrm{SOURCE\ P.U.\ IMPEDANCE} = \frac{\mathrm{Base\ MVA}}{\mathrm{SOURCE\ S.C.\ MVA}} = \frac{V_s^2}{MVA_{SC}}$$

$$\mathrm{TRANSFORMER\ P.U.\ IMPEDANCE\ Z_{PU}} = \frac{\mathrm{Z_T\ \%}}{100} \times \frac{\mathrm{Base\ MVA}}{\mathrm{TRANSFORMER\ MVA}}$$

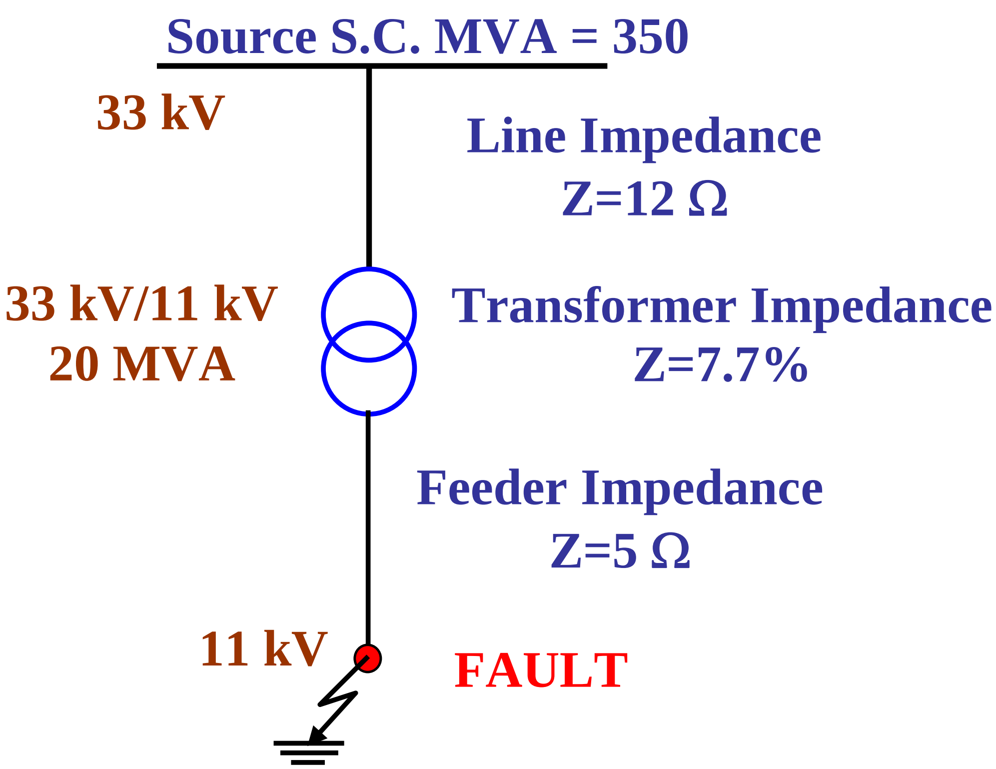
$$\mathrm{FEEDER\ P.U.\ IMPEDANCE\ Z_{PU}} = \frac{\mathrm{Z_{OHMS}}}{\mathrm{Z_{Base}}} = \frac{\mathrm{Z_{OHMS}}}{\dfrac{\mathrm{kV_{Base}^2}}{\mathrm{Base\ MVA}}} = \mathrm{Z_{OHMS}} \times \frac{\mathrm{Base\ MVA}}{\mathrm{kV_{Base}^2}}$$

$$\mathrm{3-Phase\ S.C.\ MVA\ at\ FAULT} = \frac{1}{\mathrm{TOTAL\ Z_{PU}}} \times \mathrm{Base\ MVA}$$

$$\mathrm{RMS\ SYMM.\ S.C.\ CURRENT\ at\ FAULT} = \frac{1}{\mathrm{Z_{PU}}} \times \frac{\mathrm{Base\ MVA}}{\sqrt{3} \times \mathrm{kV}}$$

$$= \frac{\mathrm{S.C.\ MVA}}{\sqrt{3} \times \mathrm{kV}}$$

Example of Maximum Fault Current



Base. MVA = 100

Assume HIGH *X*/*R* resistances are ignored

$$I_{sym} = 889.6 \text{ A}$$

Example of Maximum Fault Current

$$\text{SOURCE P.U. Z} = \frac{100\ \text{MVA}}{350.\ \text{MVA}} = 0.286\ \text{pu}$$

$$33\ \text{kV Line}\ \ Z_{PU} = \frac{12 \times 100\ \text{MVA}}{(33\ \text{kV})^2} = 1.1\ \text{pu}$$

$$\text{TRANSFORMER P.U. IMPEDANCE}\ Z_{PU} = \frac{7.7\%}{100} \times \frac{100\ \text{MVA}}{20\ \text{MVA}} = 0.385\ \text{pu}$$

$$11\ \text{kV FEEDER P.U. Z} = 5 \times \frac{100\ \text{MVA}}{(11\ \text{kV})^2} = 4.13\ \text{pu}$$

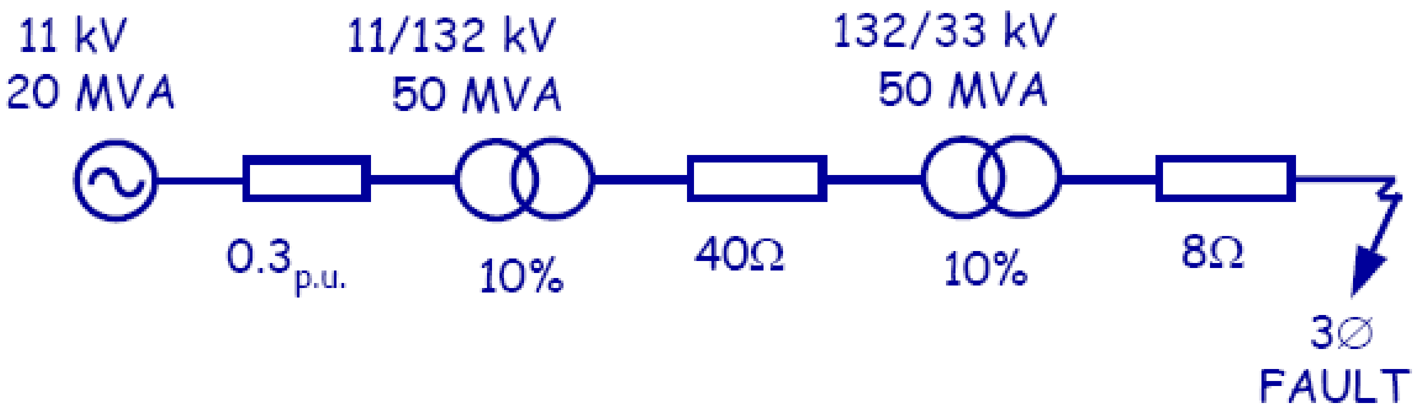
$$\text{Total Impedance from Source to FAULT} = \qquad = 5.90\ \text{pu}$$

$$3 - \text{Phase S.C. MVA at FAULT} = \frac{100\ \text{MVA}}{5.90\ \text{pu}} = 16.95\ \text{MVA}$$

$$\text{RMS SYMM. S.C. CURRENT at FAULT} == \frac{16.95\ \text{MVA}}{\sqrt{3} \times 11\ \text{kV}} = 889.6\ \text{A}$$

Example

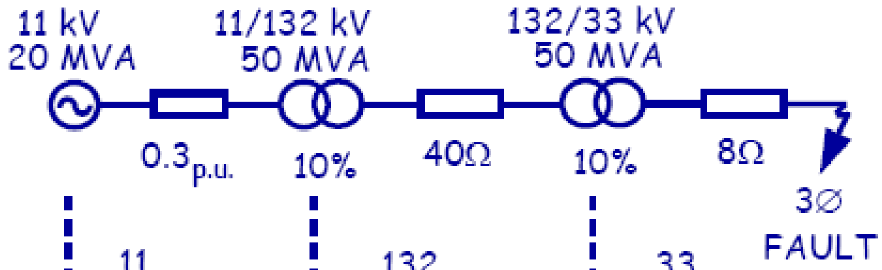
Calculate the fault currents in 11kV, 132kV and 33kV system for the three phase fault shown.

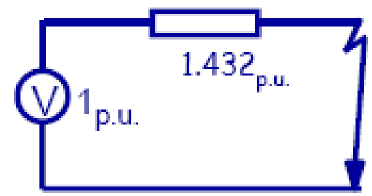


$I_{sym} = 611\ A$

Example

Calculate the fault currents in 11kV, 132kV and 33kV system for the three phase fault shown.





kV_b	11	132	33
MVA_b	50	50	50
$Z_b = \frac{kV_b^2}{MVA_b}$	2.42 Ω	348.5 Ω	21.8 Ω
$I_b = \frac{MVA_b}{\sqrt{3}kV_b}$	2624 A	219 A	875 A

$Z_{p.u.}$	$0.3 \times \frac{50}{20} = 0.75_{p.u.}$	$0.1_{p.u.}$	$\frac{40}{348.5} = 0.115_{p.u.}$	$0.1_{p.u.}$	$\frac{8}{21.8} = 0.367_{p.u.}$

$$I_F = \frac{1}{1.432} = 0.698\text{p.u.}$$

$$I_{11\,kV} = 0.698 \times I_b = 0.698 \times 2624 = 1832A$$

$$I_{132\,kV} = 0.698 \times 219 = 153A$$

$$I_{33\,kV} = 0.698 \times 875 = 611A$$

Unbalanced 3-Phase System

$$V_A = V_{A1} + V_{A2} + V_{A0}$$
$$V_B = V_{B1} + V_{B2} + V_{B0}$$
$$V_C = V_{C1} + V_{C2} + V_{C0}$$

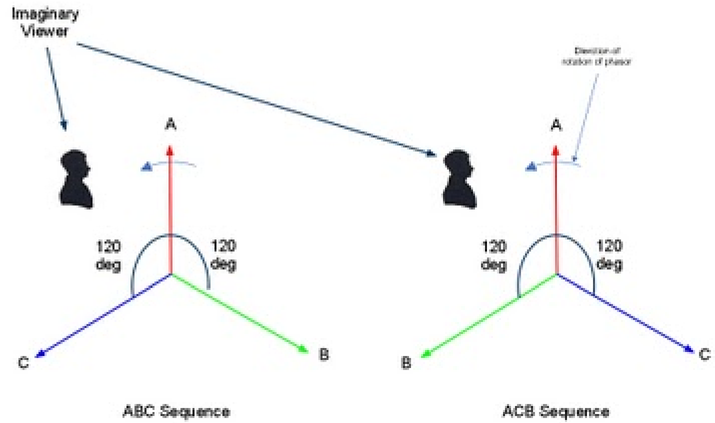
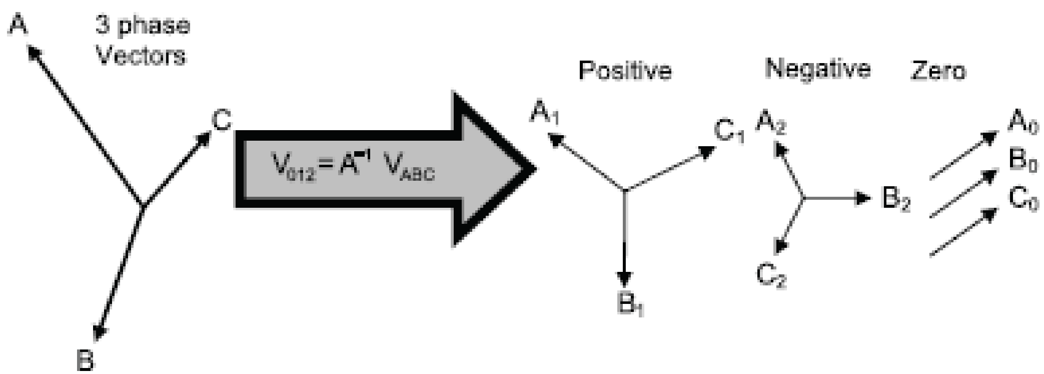
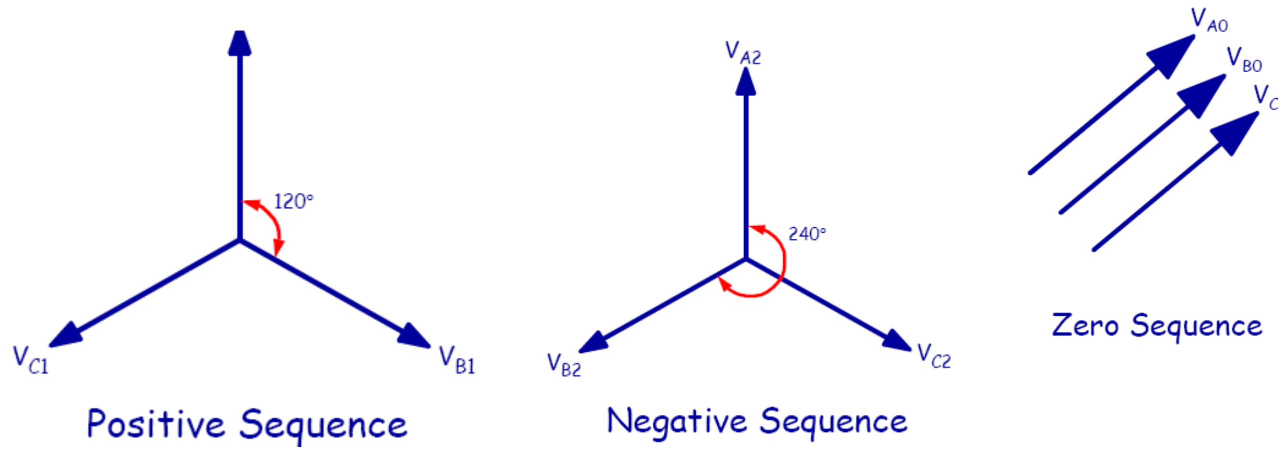


Fig A: ABC and ACB phase sequence,
Anti clock wise Rotation

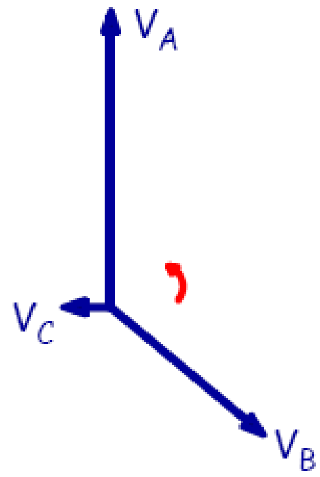


Symmetrical Component Conversion



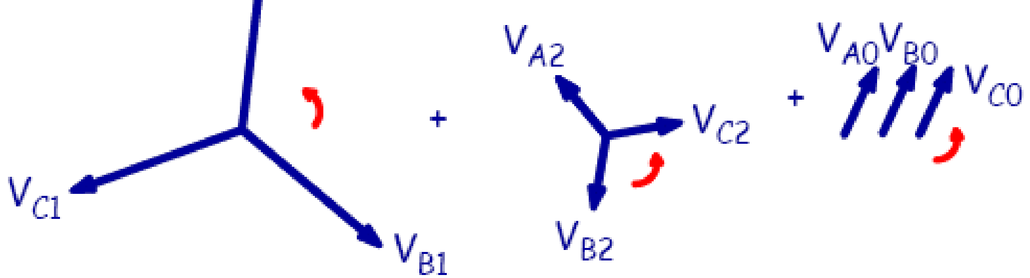
Symmetrical Components

Phase ≡ Positive + Negative + Zero



$$V_A = V_{A1} + V_{A2} + V_{A0}$$
$$V_B = V_{B1} + V_{B2} + V_{B0}$$
$$V_C = V_{C1} + V_{C2} + V_{C0}$$

***a*=1∠120º**



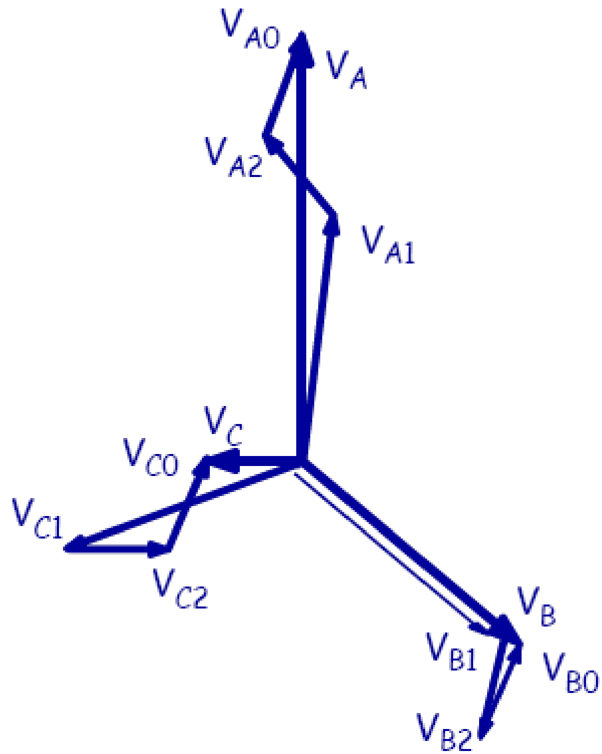
$$V_{B1} = a^2 V_{A1} \qquad V_{B2} = a\,V_{A2} \qquad V_{B0} = V_{A0}$$
$$V_{C1} = a\,V_{A1} \qquad V_{C2} = a^2 V_{A2} \qquad V_{C0} = V_{A0}$$

Converting from Sequence Components to Phase Values

$$V_A = V_{A1} + V_{A2} + V_{A0}$$

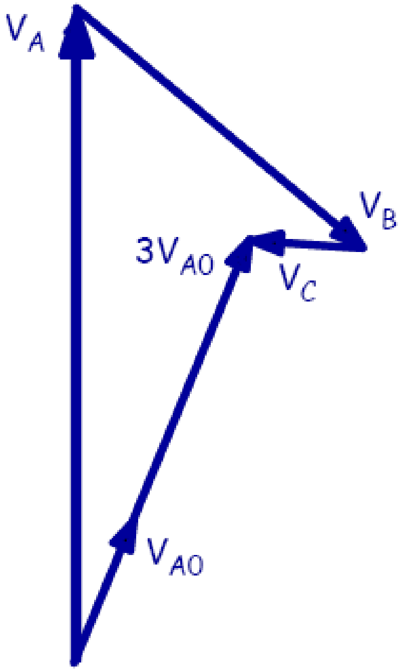
$$V_B = V_{B1} + V_{B2} + V_{B0} = a^2 V_{A1} + a V_{A2} + V_{A0}$$

$$V_C = V_{C1} + V_{C2} + V_{C0} = a V_{A1} + a^2 V_{A2} + V_{A0}$$



Converting from Phase Values to Sequence Components

$$V_{A1} = 1/3 \{V_A + a\,V_B + a^2 V_C\}$$
$$V_{A2} = 1/3 \{V_A + a^2 V_B + a\,V_C\}$$
$$V_{A0} = 1/3 \{V_A + V_B + V_C\}$$



Sequence Networks

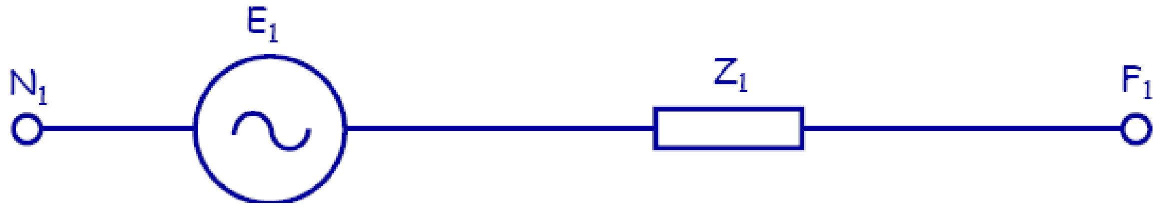
- **It can be shown that provided the system impedances are balanced from the points of generation right up to the fault, each sequence current causes voltage drop of its own sequence only**
- **+ve, -ve and zero sequence networks are drawn for a 'reference' phase. This is usually taken as the 'A' phase.**
- **Faults are selected to be 'balanced' relative to the reference 'A' phase.**

e.g. For Ø/E faults consider an A-E fault

For Ø/Ø faults consider a B-C fault

Positive Sequence Diagram

1. **Start with neutral point N_1**
   - ❖ **All generator and load neutrals are connected to N_1**
2. **Include all source voltages**
   - ❖ **Phase-neutral voltage**
3. **Impedance network**
   - ❖ **Positive sequence impedance per phase**
4. **Diagram finishes at fault point F_1**

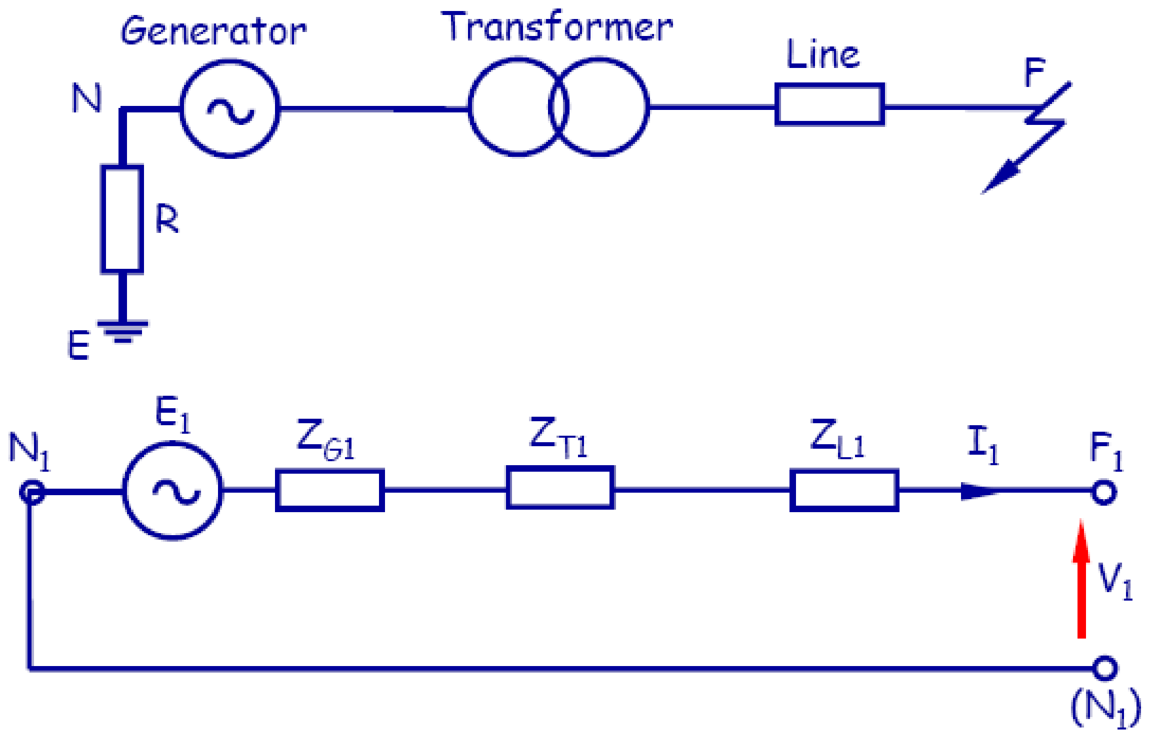


Positive Sequence Diagram

V_1 = Positive sequence Ph-N voltage at fault point

I_1 = Positive sequence phase current flowing into F_1

$$V_1 = E_1 - I_1 (Z_{G1} + Z_{T1} + Z_{L1})$$



Negative Sequence Diagram

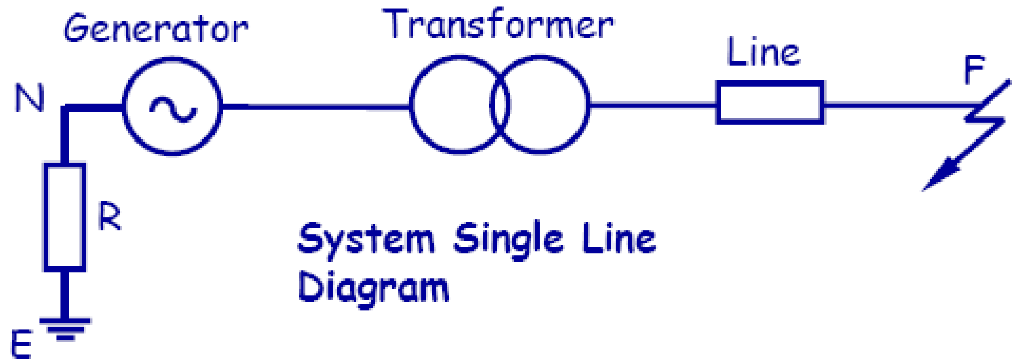
1. **Start with neutral point N_2**
   - **All generator and load neutrals are connected to N_2**
2. **No Voltages included**
   - **No negative sequence voltage is generated!**
3. **Impedance network**
   - **Negative sequence impedance per phase**
4. **Diagram finishes at fault point F_2**

Negative Sequence Diagram

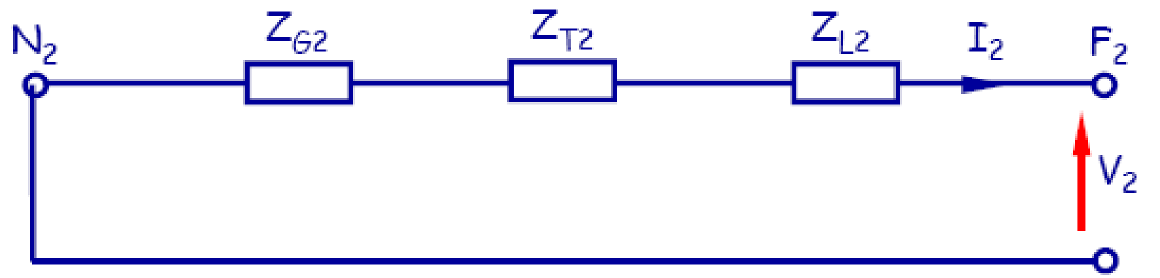
V_2 = Negative sequence Ph-N voltage at fault point

I_2 = Negative sequence phase current flowing into F_2

$V_2 = -I_2\,(Z_{G2} + Z_{T2} + Z_{L2})$



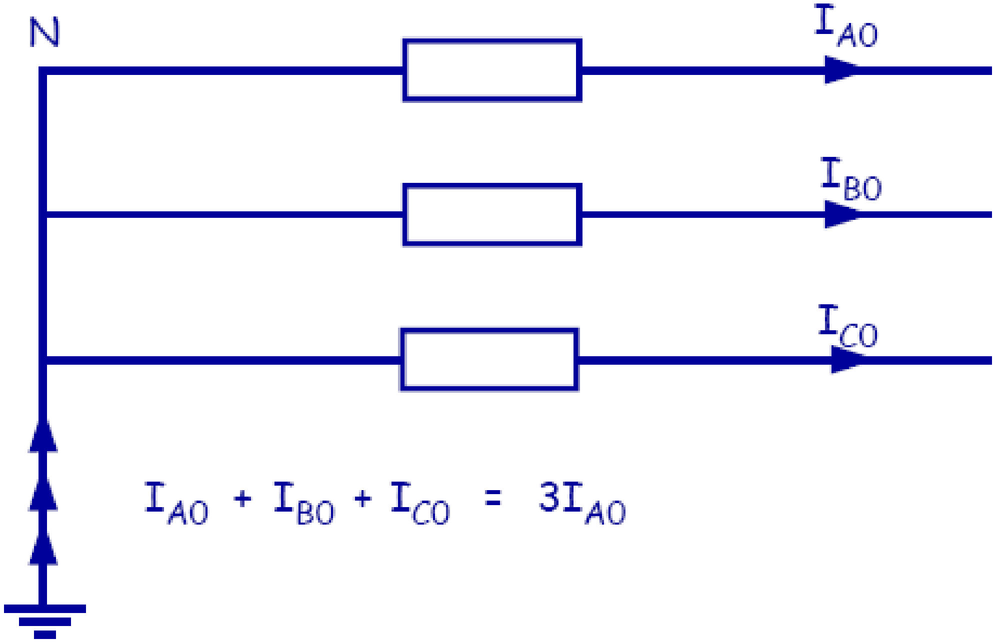
System Single Line Diagram

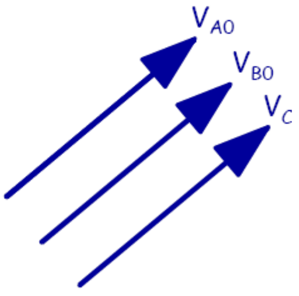


Negative Sequence Diagram

Zero Sequence Diagram

❖ **For “In Phase” (Zero Phase Sequence) currents to flow in each phase of the system, there must be a fourth connection (this is typically the neutral or earth connection).**

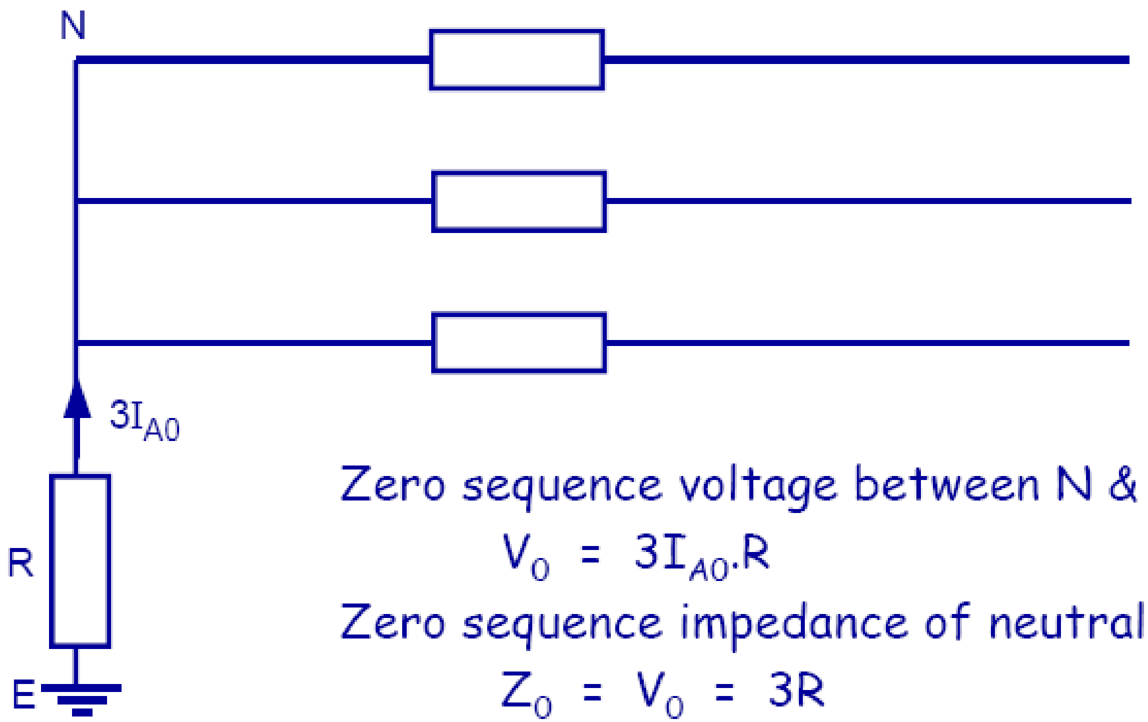




Zero Sequence

Zero Sequence Diagram

❖ **Resistance Earthed System :-**



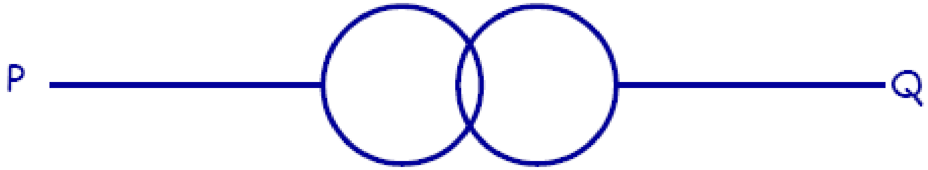
Zero sequence voltage between N & E given by

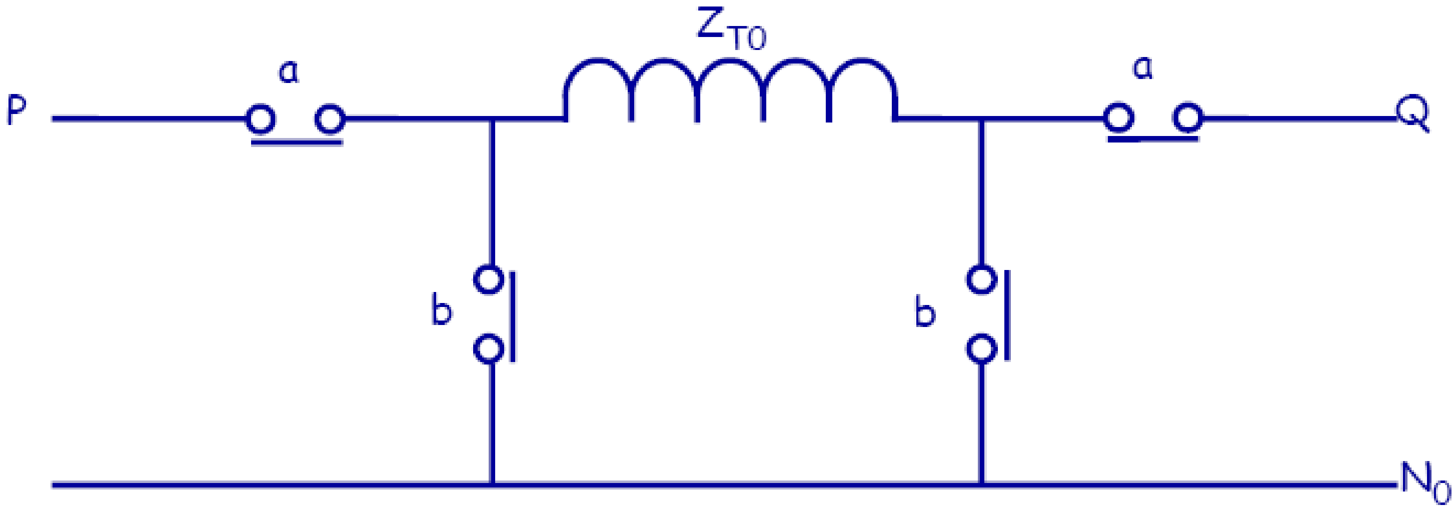
$$V_0 = 3I_{A0}.R$$

Zero sequence impedance of neutral to earth path

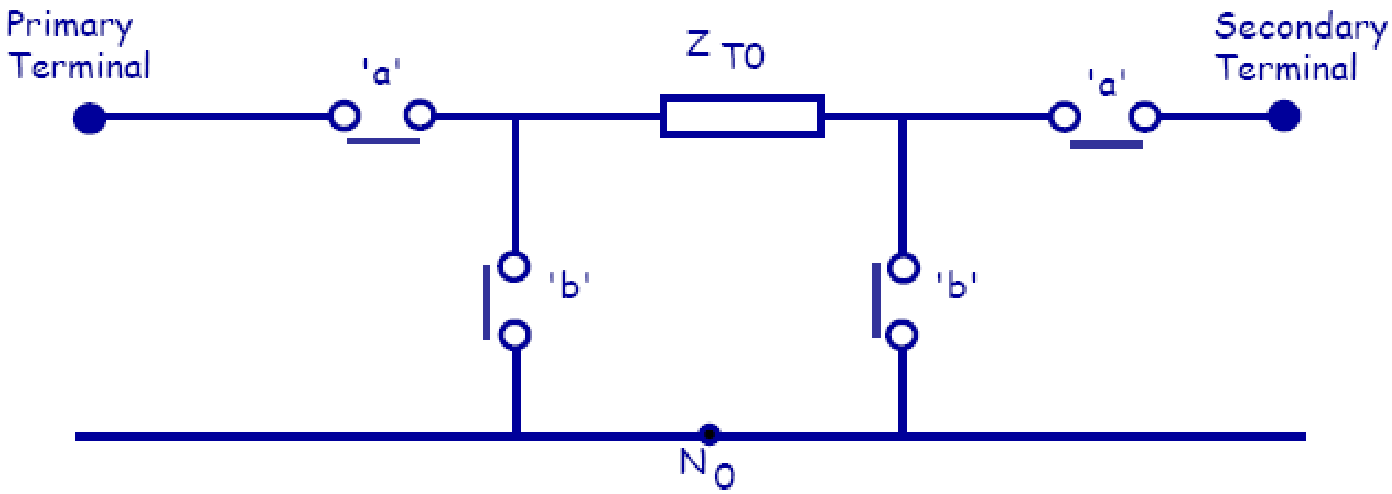
$$Z_0 = \frac{V_0}{I_{A0}} = 3R$$

Transformer Zero-Sequence Diagram





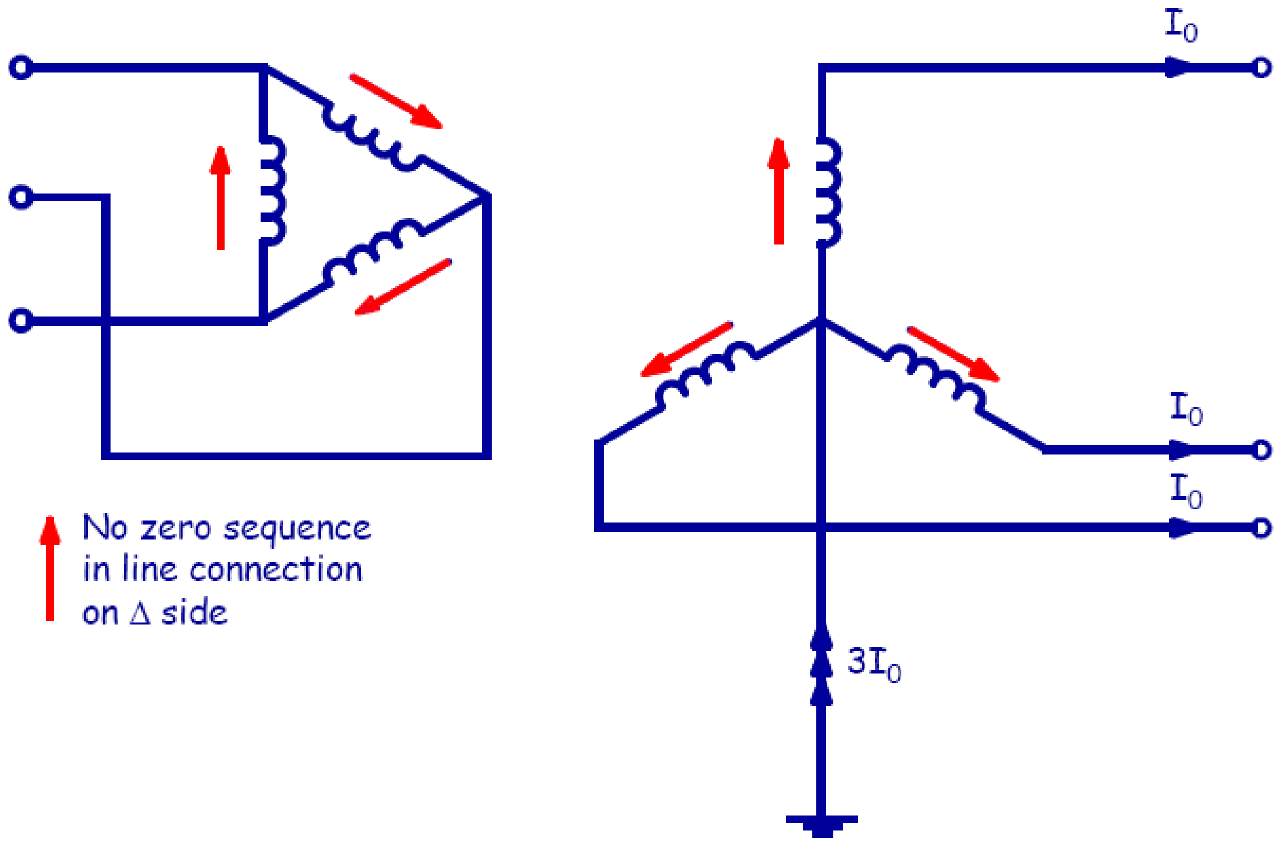
General Zero-Sequence Equivalent Circuit for Two Winding Transformer



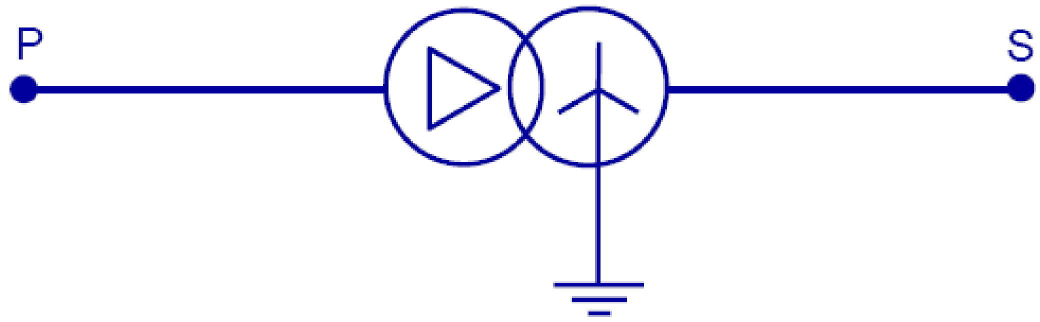
On appropriate side of transformer :

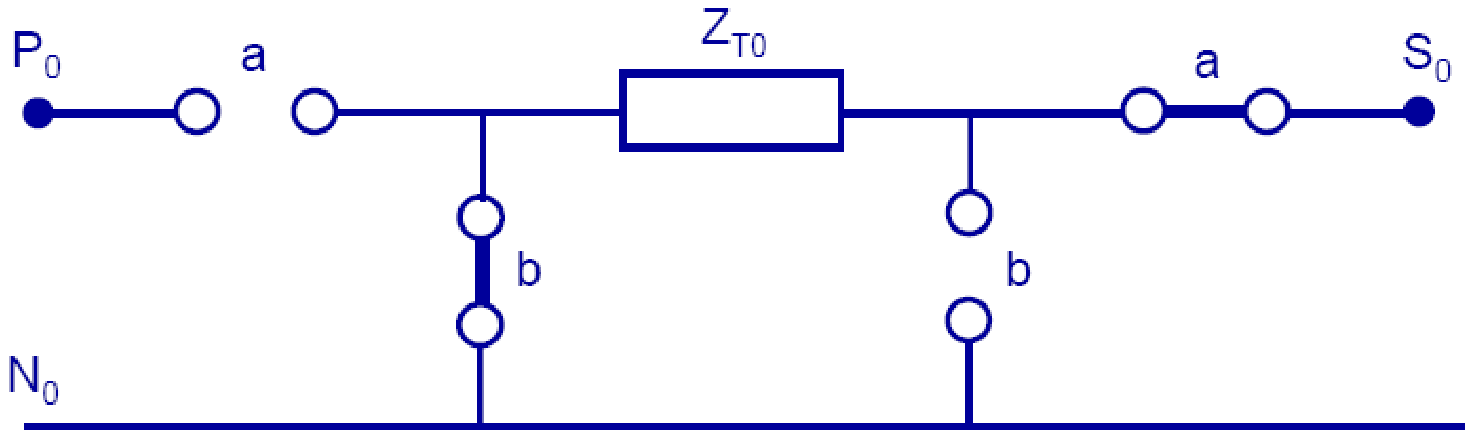
Earthed Star Winding	-	Close link 'a' Open link 'b'
Delta Winding	-	Open link 'a' Close link 'b'
Unearthed Star Winding	-	Both links open

Zero-Sequence Equivalent Circuit for "Dyn" Transformer



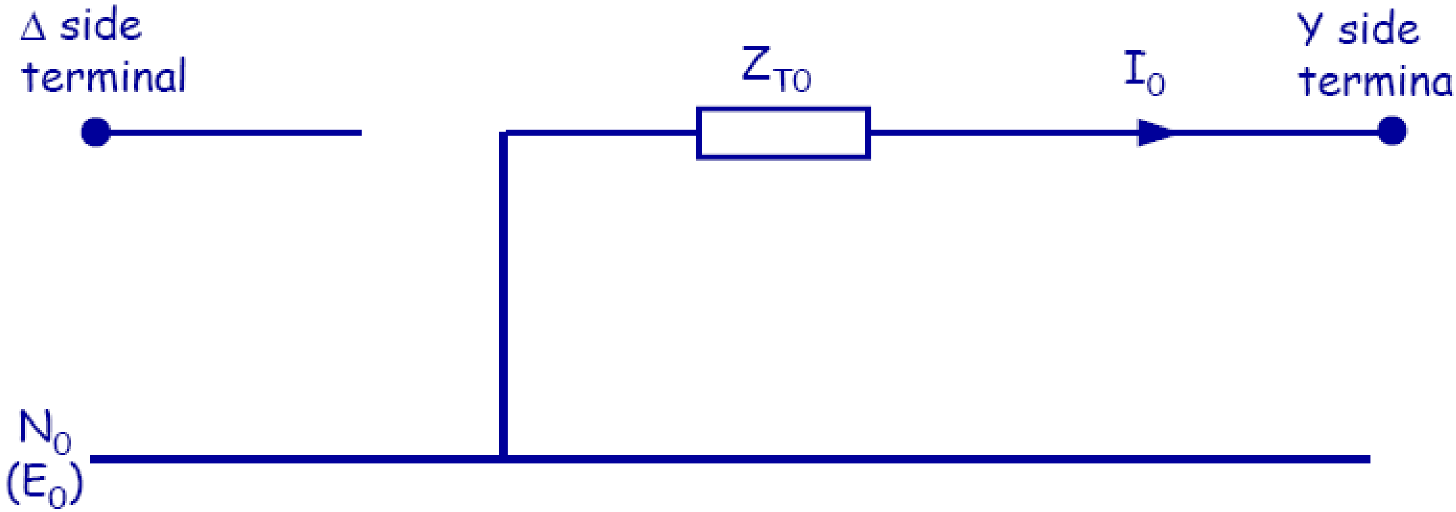
Zero-Sequence Equivalent Circuit for "Dyn" Transformer



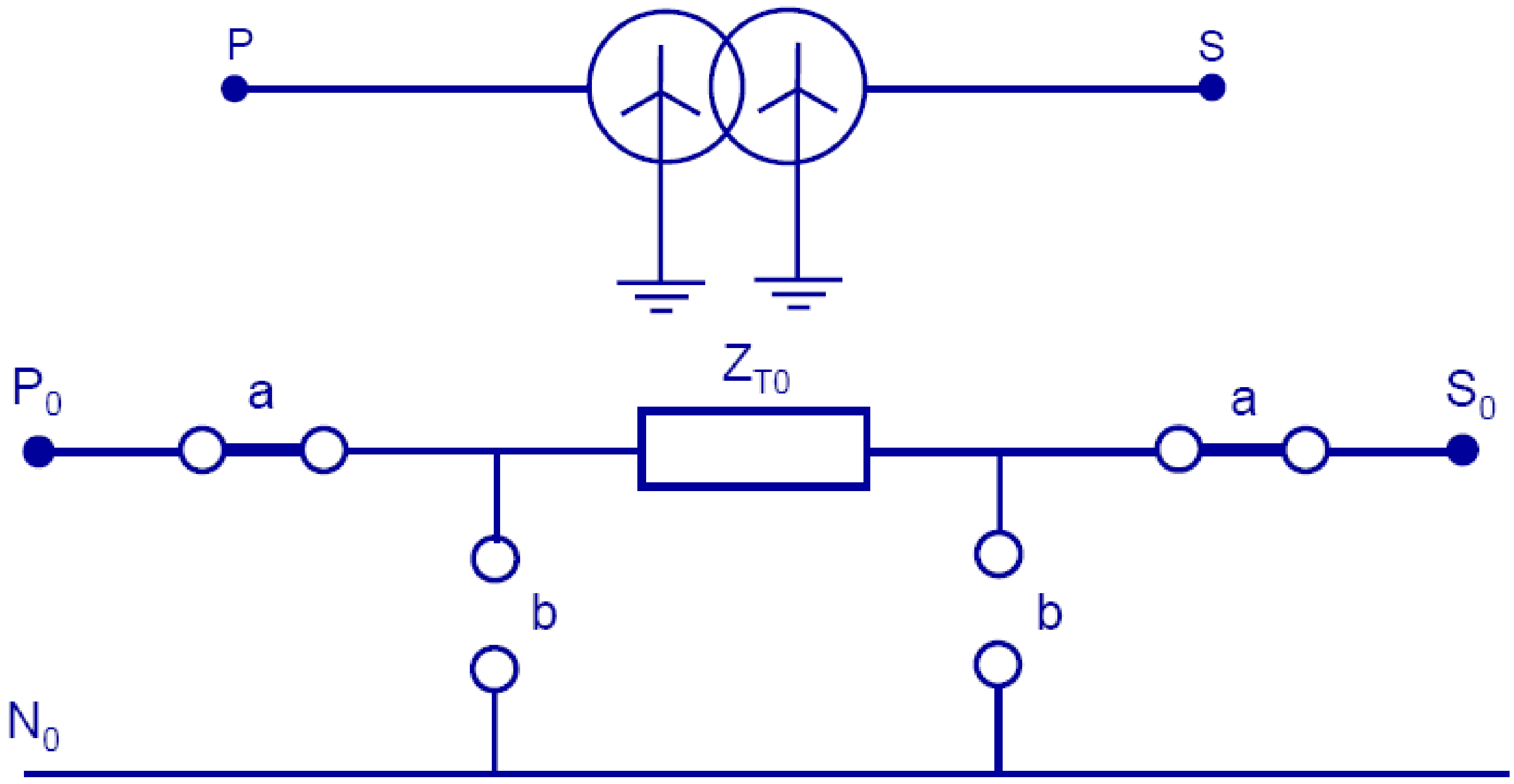


Zero-Sequence Equivalent Circuit for "Dy" Transformer

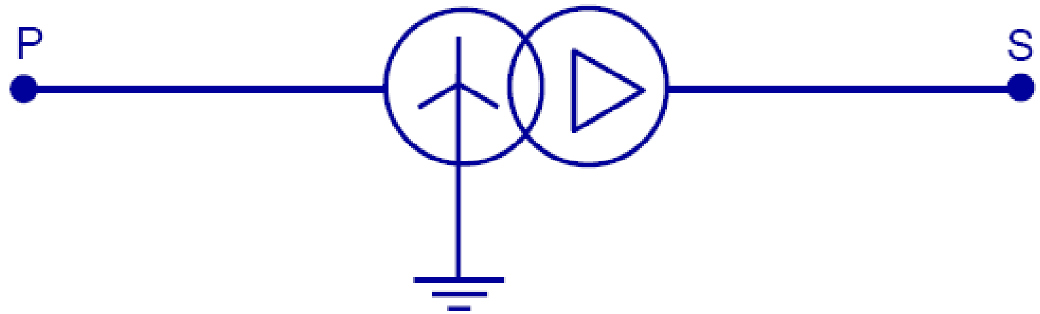
Thus, equivalent single phase zero sequence diagram :-

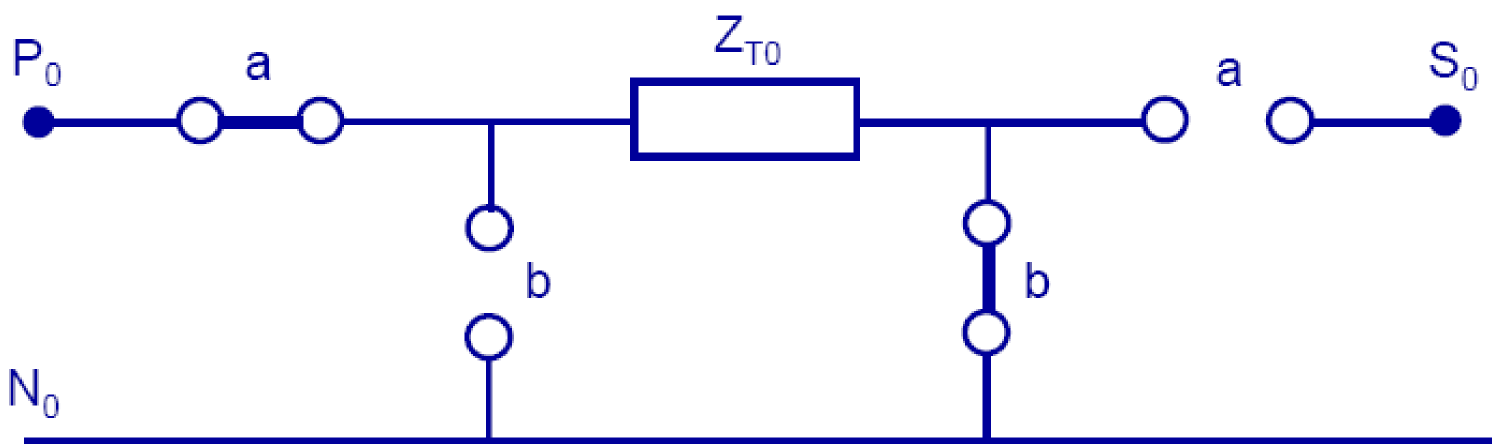


Zero-Sequence Equivalent Circuit for "YNyn" Transformer

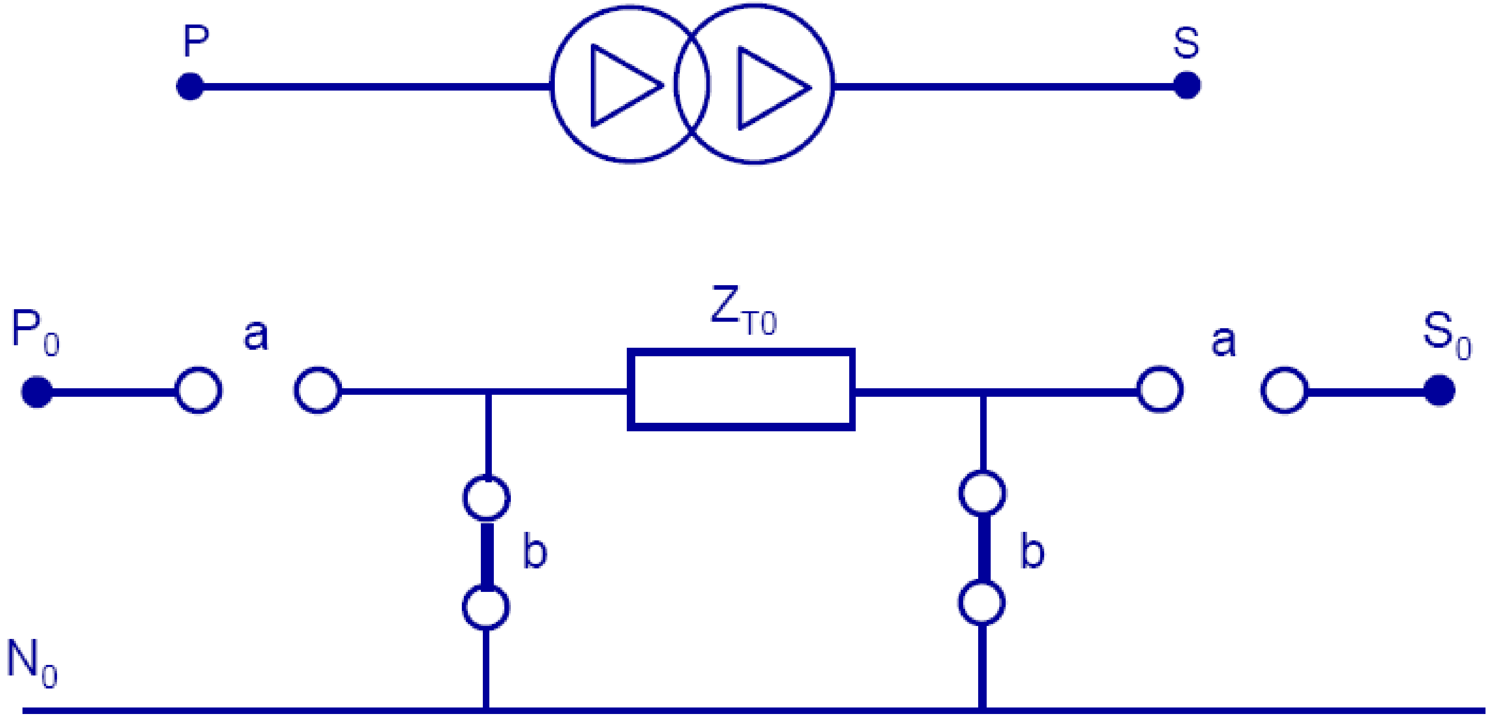


Zero-Sequence Equivalent Circuit for "YNd" Transformer

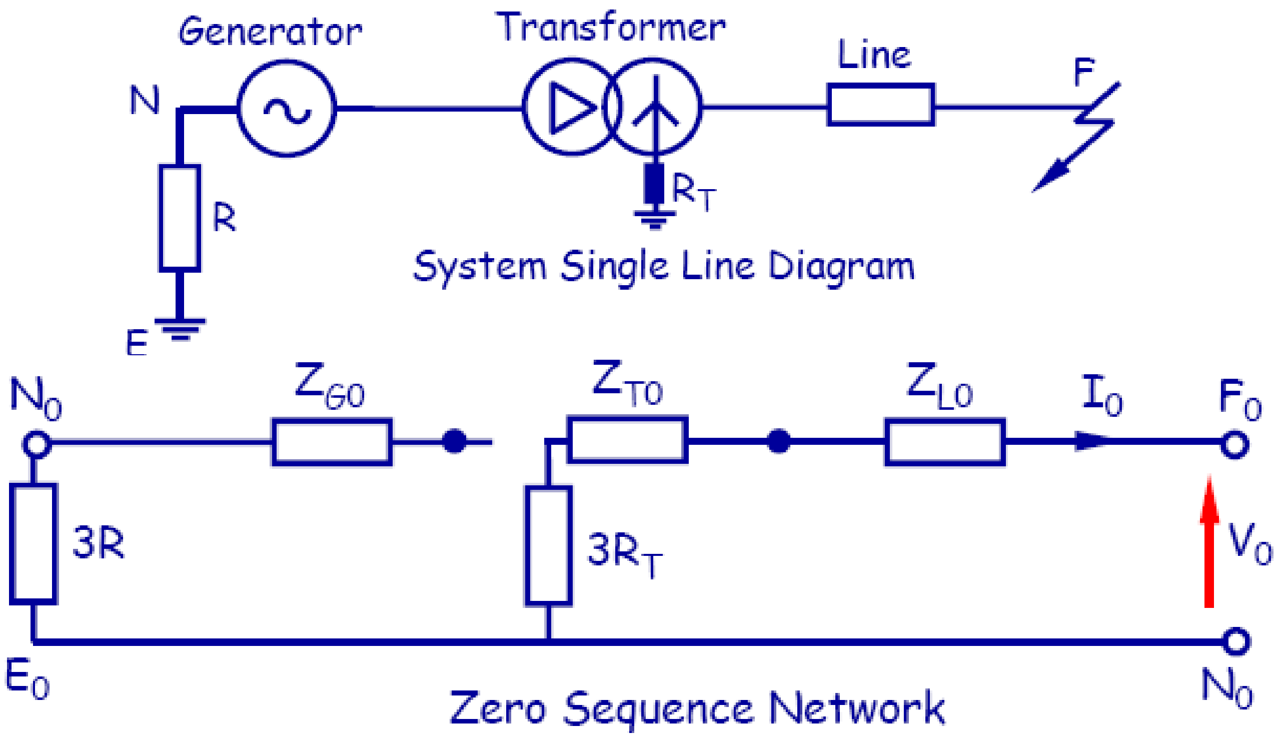




Zero-Sequence Equivalent Circuit for "Dd" Transformer



Zero-Sequence Equivalent Circuit
Example

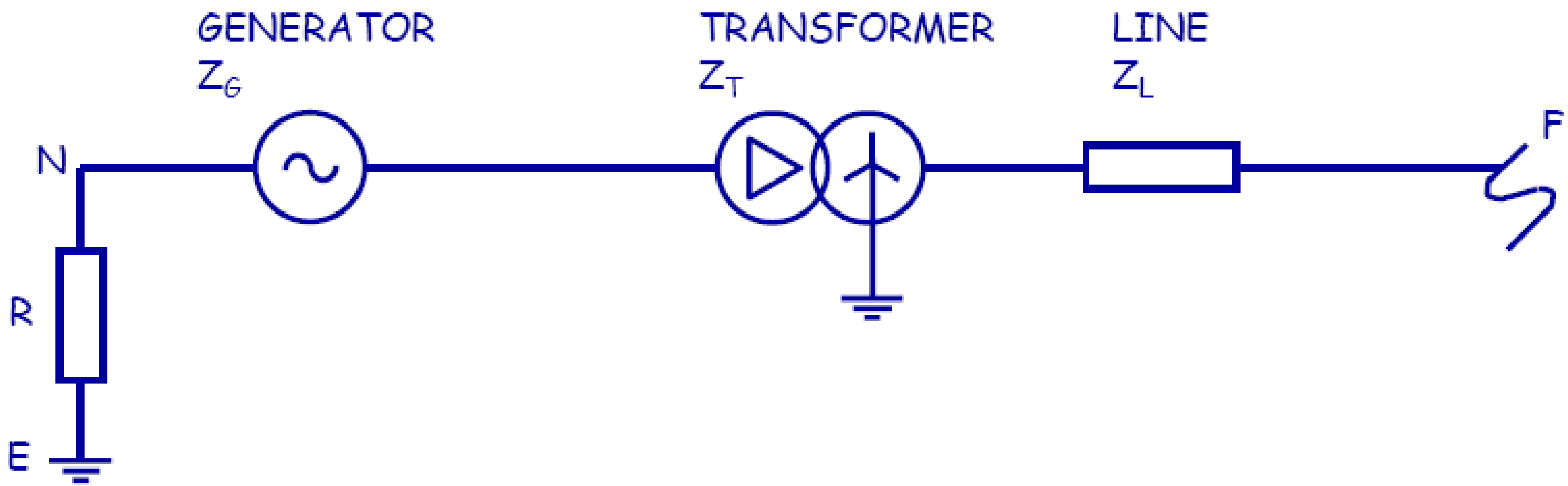


V_0 = Zero-sequence Ph-E voltage at fault point

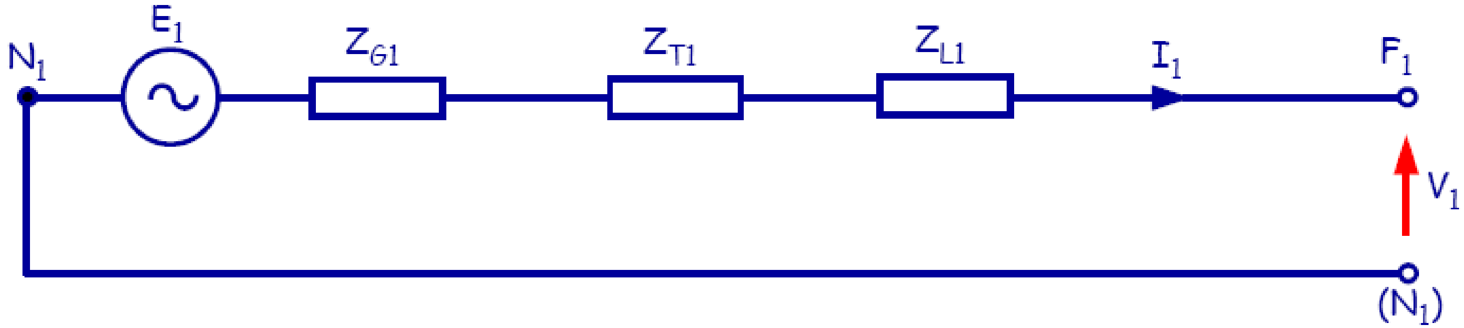
I_0 = Zer0-sequence current flowing into F_0

$V_0 = -I_0 (Z_{T0} + Z_{L0})$

Summary of Sequence Diagrams

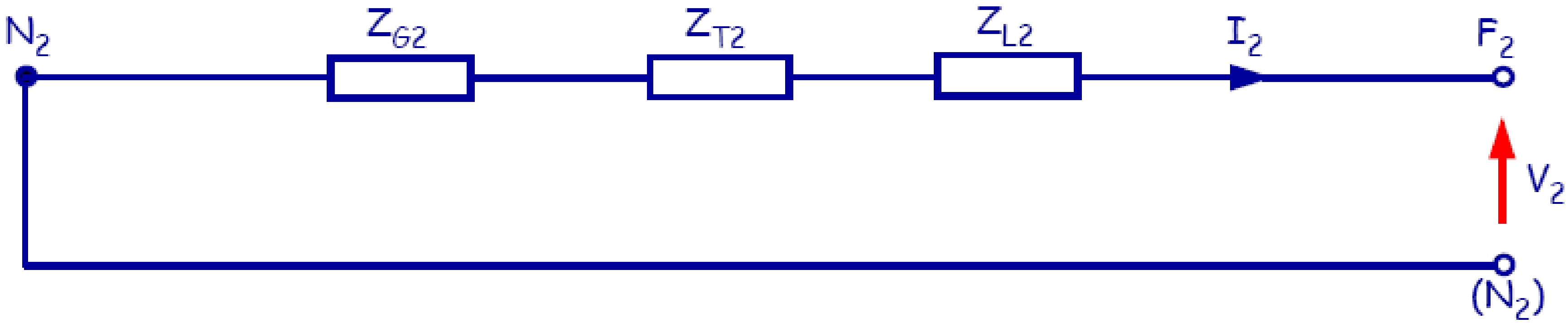


System Single Line Diagram

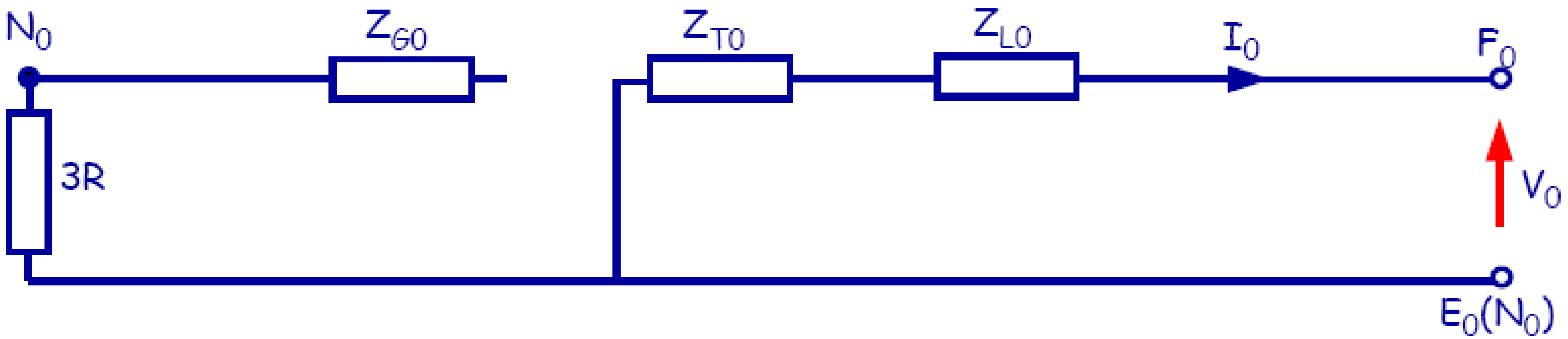


Positive-Sequence Diagram

Summary of Sequence Diagrams

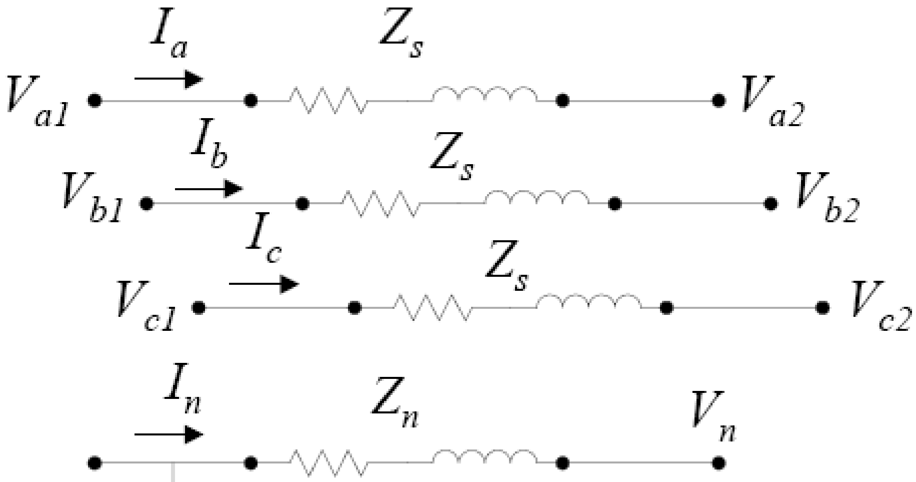


Negative-Sequence Diagram



Zero-Sequence Diagram

Sequence Impedances-Transmission Lines



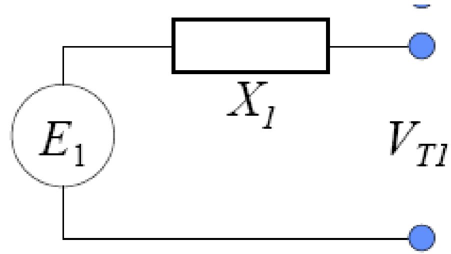
$$\mathbf{Z}_{012} = \mathbf{A}^{-1}\mathbf{Z}_{abc}\mathbf{A}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} Z_S + Z_n & Z_n & Z_n \\ Z_n & Z_S + Z_n & Z_n \\ Z_n & Z_n & Z_S + Z_n \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$= \begin{bmatrix} Z_S + 3Z_n & 0 & 0 \\ 0 & Z_S & 0 \\ 0 & 0 & Z_S \end{bmatrix}$$

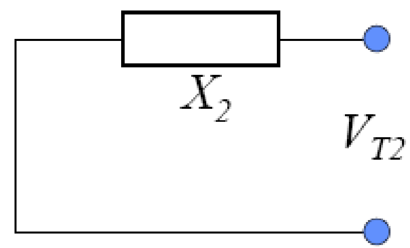
Sequence Impedances-Generators

Positive Sequence



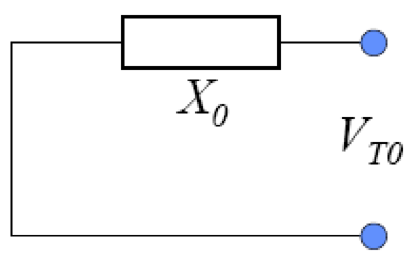
X_d, X_d', and X_d''

Negative Sequence



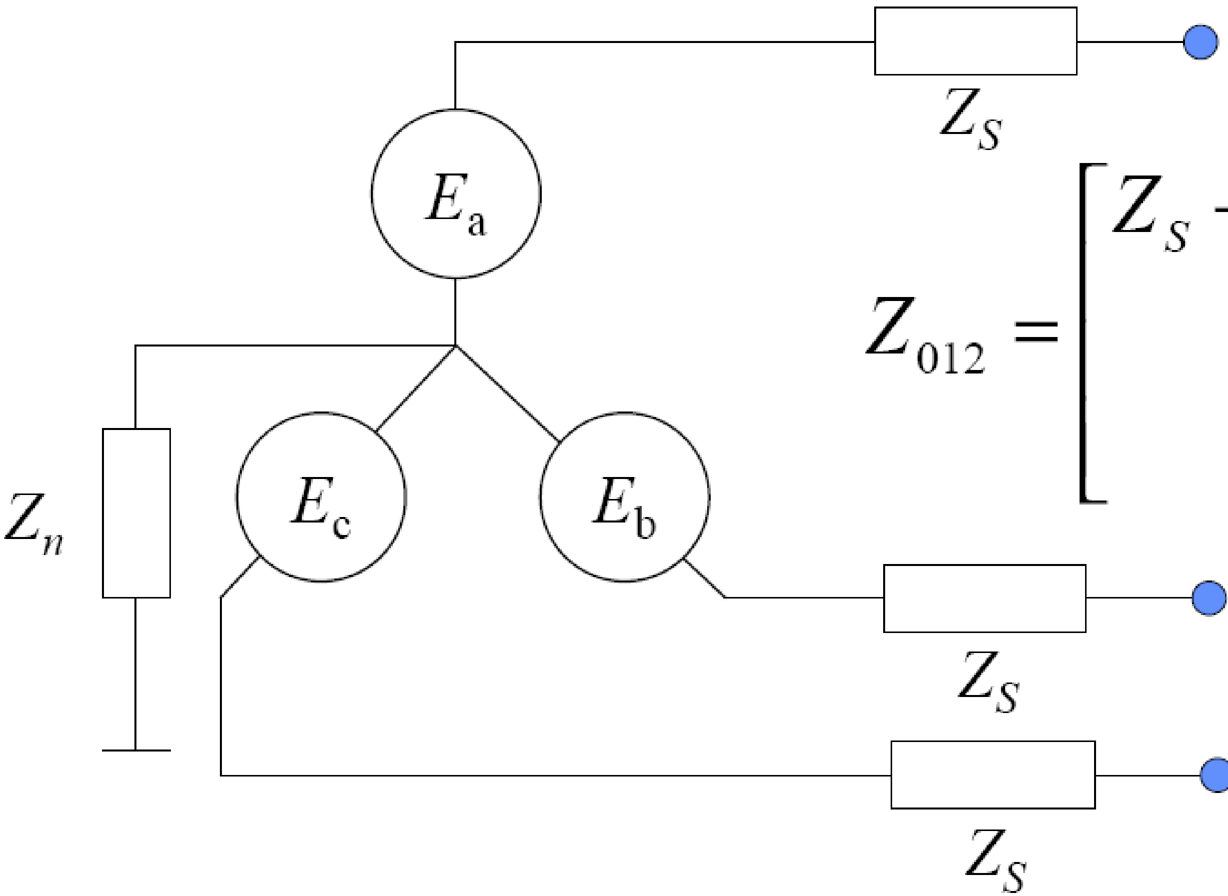
$X_2 \sim X_d''$

Zero Sequence



$X_0 \sim X_L$

Sequence Impedances-Grounded Generator



$$Z_{012} = \begin{bmatrix} Z_S + 3Z_n & 0 & 0 \\ 0 & Z_S & 0 \\ 0 & 0 & Z_S \end{bmatrix}$$

Sequence Impedances-Transformer

$$Z_0 = Z_1 = Z_2 = Z_\ell$$

- **Wye-delta transformers create a phase shifting pattern for the various sequences**
  - the positive sequence quantities rotate by +30 degrees
  - the negative sequence quantities rotate by -30 degrees
  - the zero sequence quantities can not pass through the transformer
- **USA standard**
  - independent of the winding order (Δ-Y or Y- Δ)
  - the positive sequence line voltage on the HV side leads the corresponding line voltage on the LV side by 30°
  - consequently, for the negative sequence voltages the corresponding phase shift is -30°

Common Unbalanced Network Faults

Single Line to Ground Fault

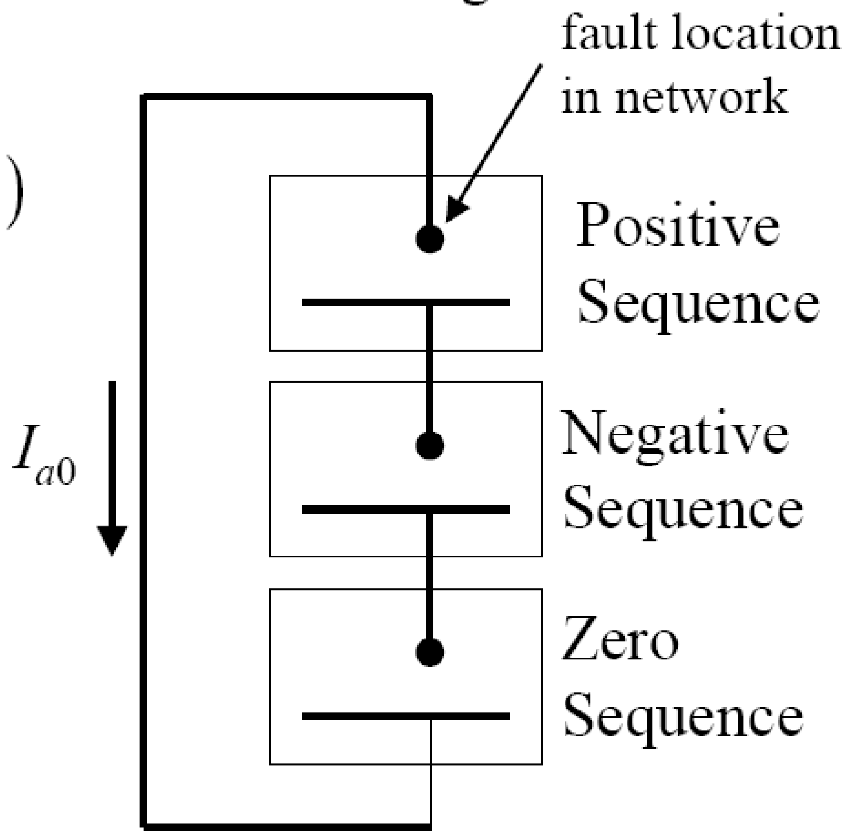
$$
\begin{aligned}
V_a &= E_a - Z_a I_a = 0 \\
&= E_a - A\left(Z_{012} I_{012}\right) \\
&= E_a - \left(Z_{a0} I_{a0} + Z_{a1} I_{a1} + Z_{a2} I_{a2}\right) \\
&= E_a - \left(Z_{a0} + Z_{a1} + Z_{a2}\right) I_{a0}
\end{aligned}
$$

$$I_{a0} = \frac{E_a}{\left(Z_{a0} + Z_{a1} + Z_{a2}\right)}$$

$$I_{a0} = I_{a1} = I_{a2}$$

$$I_f = 3\, I_{a0}$$

Network Diagram



Common Unbalanced Network Faults
Double Line to Ground Fault

$$I_{a1} = \left(1 + \frac{Z_1}{Z_0} + \frac{Z_1}{Z_2}\right) = V_{a1} \frac{Z_2 + Z_0}{Z_2 Z_0}$$

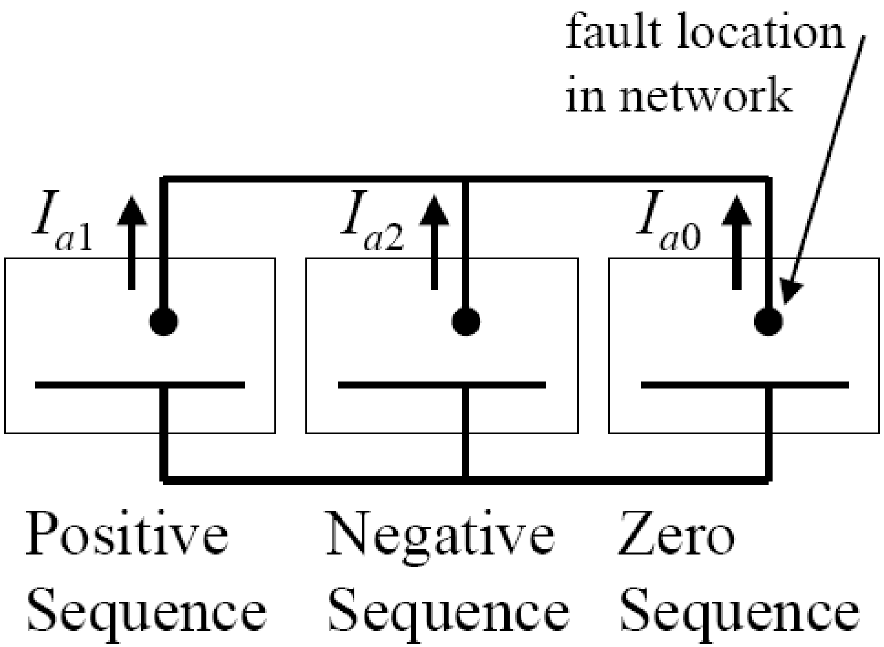
$$I_{a1} = \frac{V_{a1}(Z_0 + Z_2)}{(Z_1 Z_0 + Z_2 Z_0 + Z_1 Z_2)}$$

$$I_{a0} = \frac{V_{a1}}{\left(Z_1 + \frac{Z_0 Z_2}{Z_0 + Z_2}\right)}$$

$$I_{a0} = -\frac{V_{a0}}{Z_0}, \quad I_{a2} = -\frac{V_{a2}}{Z_2}$$

$$I_f = 2\, I_{a0} - (I_{a1} + I_{a2})$$

Network Diagram



Common Unbalanced Network Faults

Line-Line Fault

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

$$I_{a0} = 0$$

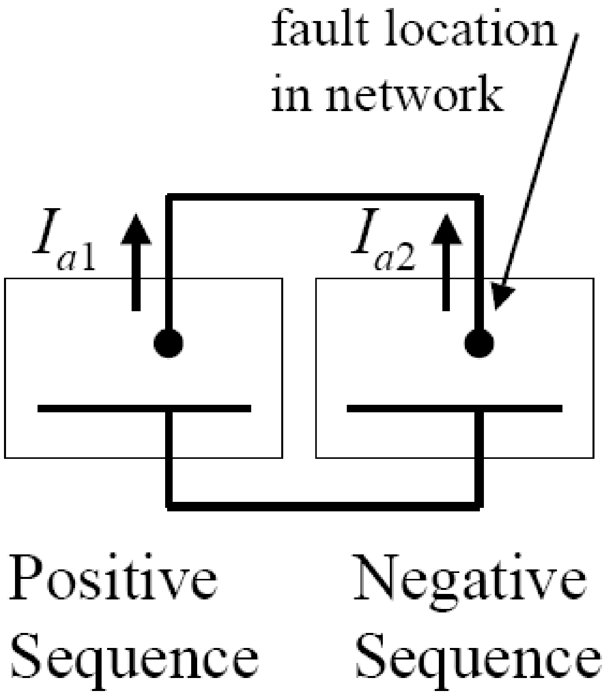
$$I_{a1} = \tfrac{1}{3}\left(a - a^2\right) I_b$$

$$I_{a2} = \tfrac{1}{3}\left(a^2 - a\right) I_b$$

$$I_{a1} = -I_{a2}$$

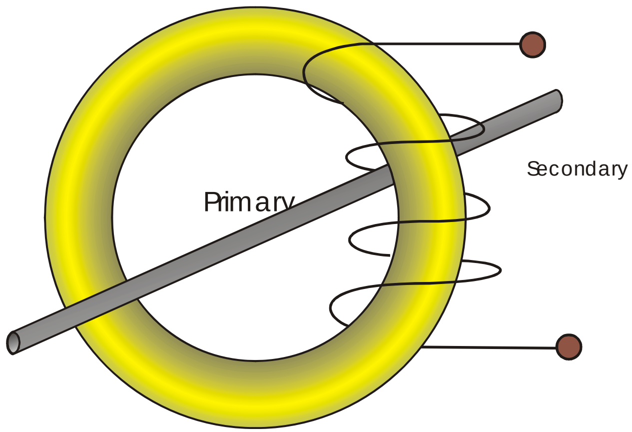
$$I_k^{a1} = \frac{V_{k,pre-f}^{a}}{Z_k^1 + Z_k^2}$$

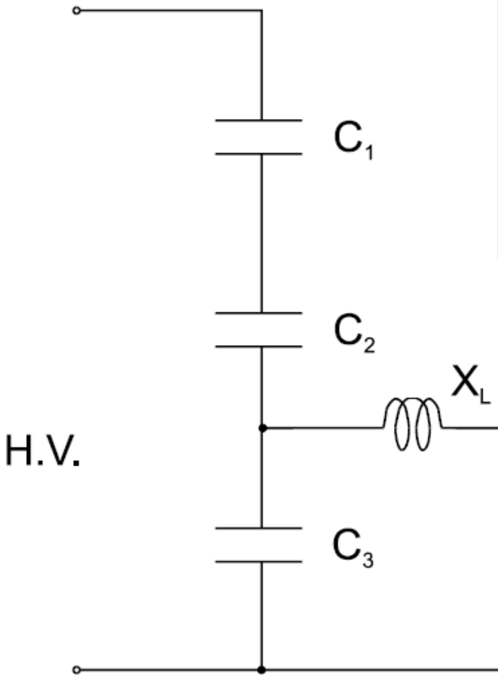
Network Diagram

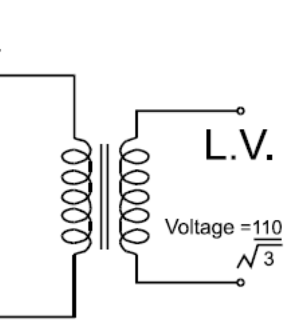


Topic 1: Power System Protection

1.3: Instrument Transformers







Instrument Transformer (VT)

The main tasks of **instrument transformers** are:

- To transform ***currents*** *or* ***voltages*** from a usually high value to a value appropriate for relays and instruments (1 or 5 Amps)
- To insulate the relays, metering and instruments from the primary high voltage system.
- To provide possibilities of standardizing the relays and instruments etc. to a few rated currents and voltages.

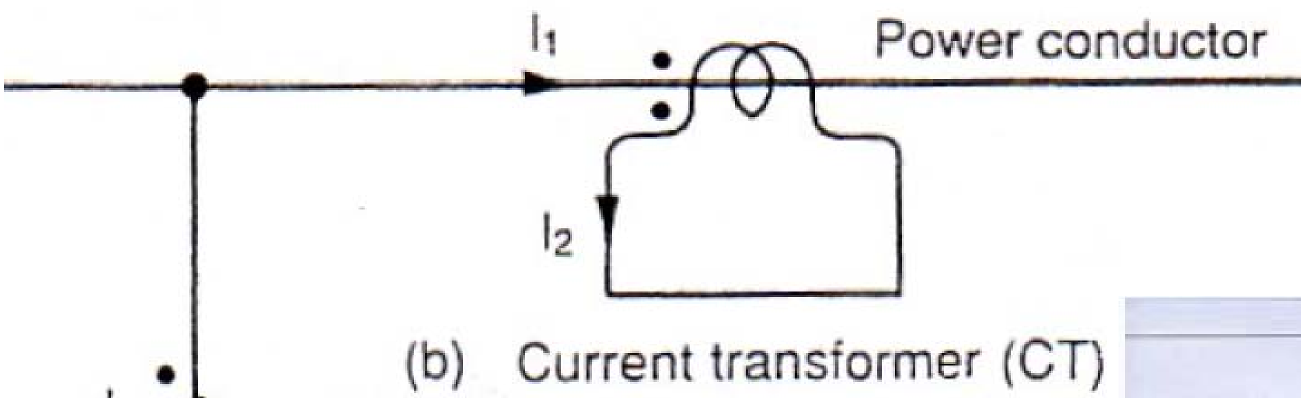
Theory of Operation

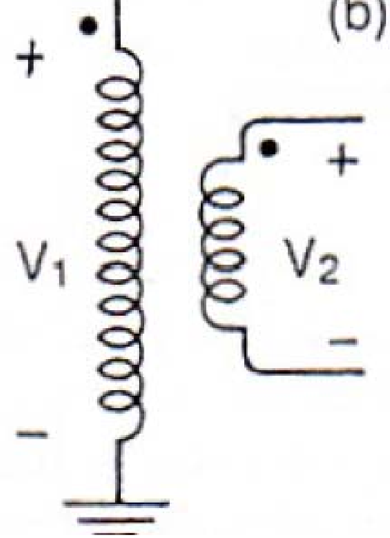
- Follows the basic Transformer principle to convert voltage on primary to an appropriate value on secondary through a common magnetic core.

- **Voltage Transformers** - Connected across the open circuit ends of the point of measurement

- **Current Transformers** - Connected in Series to carry the full rated / short circuit current of the circuit under measurement

CT and VT Schematic

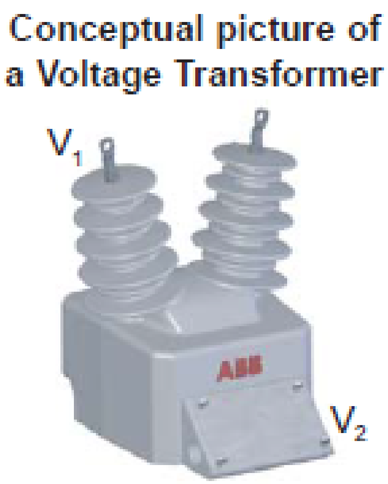


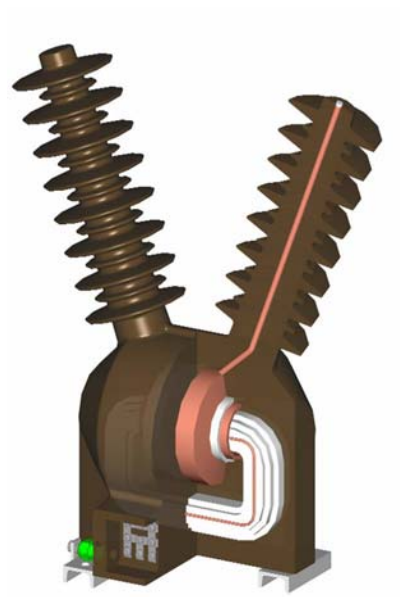


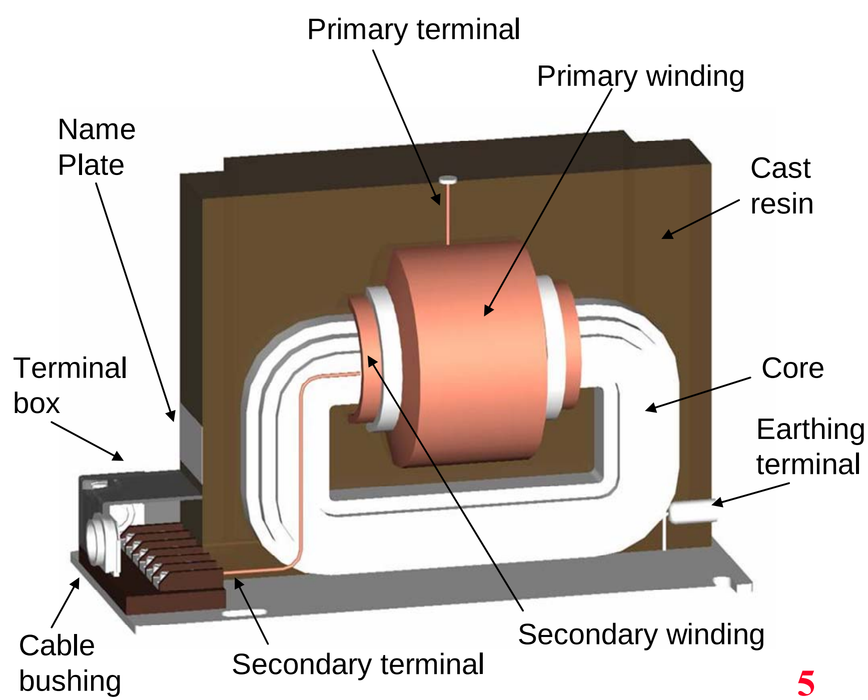
1. Voltage Transformers

Voltage Transformers (**VT**) are used to step the power system primary voltage from, say 132 kV or 33 kV to 120 volts phase-to-phase, or 69 volts phase-to-ground.

It is this secondary voltage that is applied to the fault detecting relays, and meters.



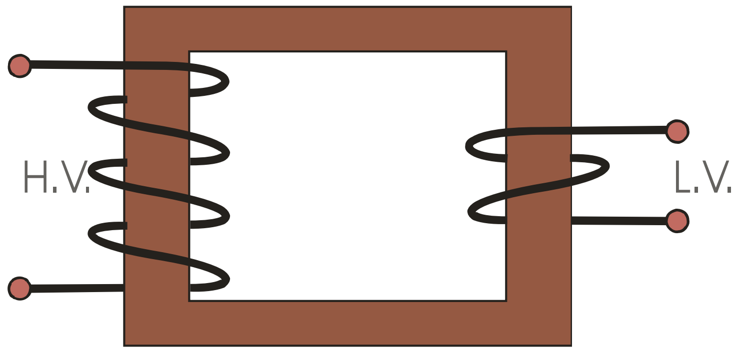


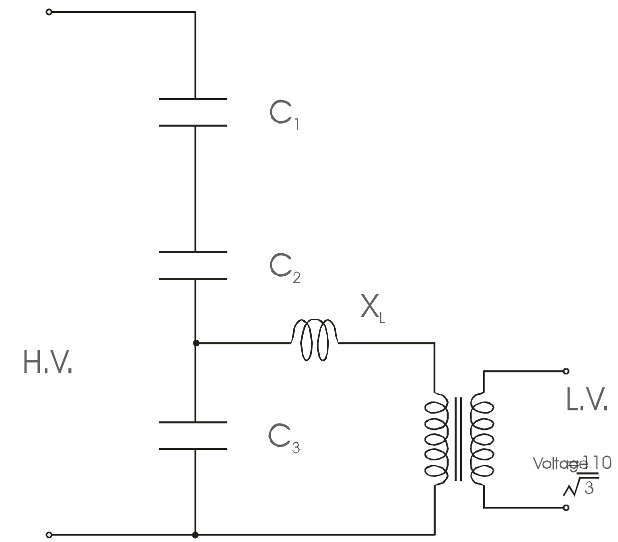


1. Voltage Transformers

The voltage transformers at these primary voltages of 50 kV and 33 kV are normally of the **WOUND** type. That is, a two winding transformer in an oil filled steel tank, with a turns ratio of 416.6:1 or 275:1.

On higher voltage systems, such as 132kV, 220kV and 400kV, **CAPACITOR VOLTAGE TRANSFORMERS** (**CVT's**) are normally used.

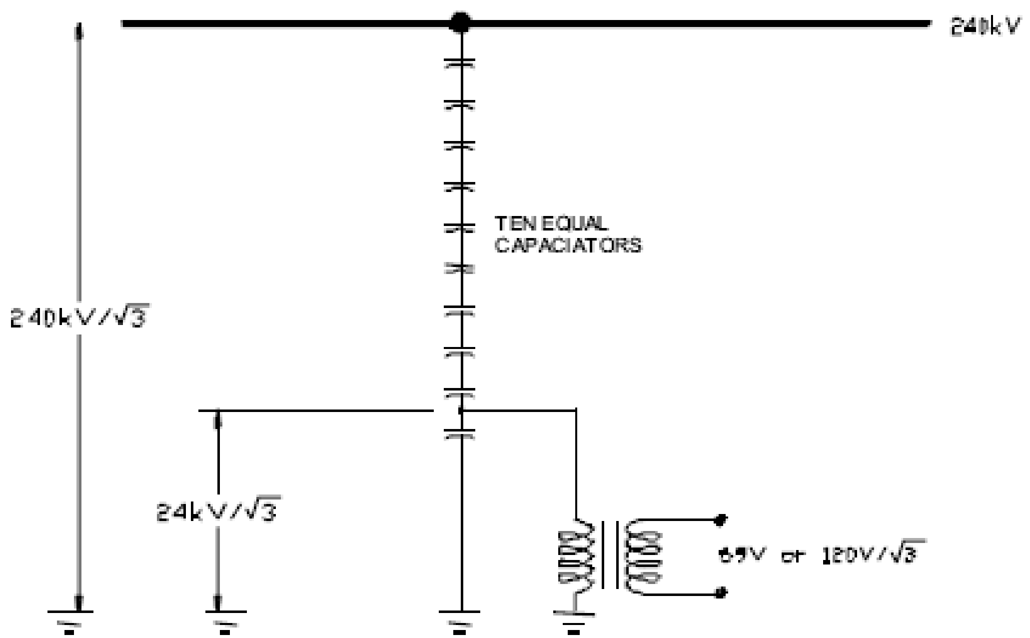




1. Voltage Transformers

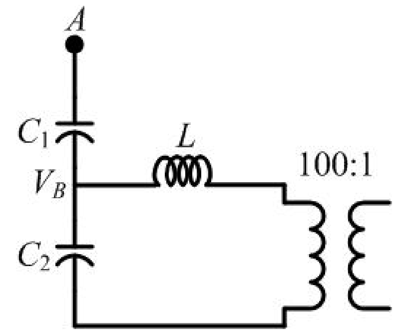
A **CVT** is comprised of a capacitor divider made up from typically 10 equal capacitors, connected in series from the phase conductor to ground, with a voltage transformer connected across the bottom capacitor.

This V.T. actually measures one-tenth of the line voltage, as illustrated in the diagram beside:



Electromagnetic Voltage Transformers

Values of C_1 and C_2 such that there is no phase displacement between the line voltage and the output of the CVT



Consider the circuit of CVT. The open-circuit voltage across C_2 is given by

$$V_B = \frac{V_A\left(1/j\omega C_2\right)}{1/j\omega C_1 + 1/j\omega C_2} = V_A \frac{C_1}{C_1 + C_2}$$

Also the short circuit current is

$$I_{sc} = \frac{V_A}{1/j\omega C_1} = j\omega C_1 V_A$$

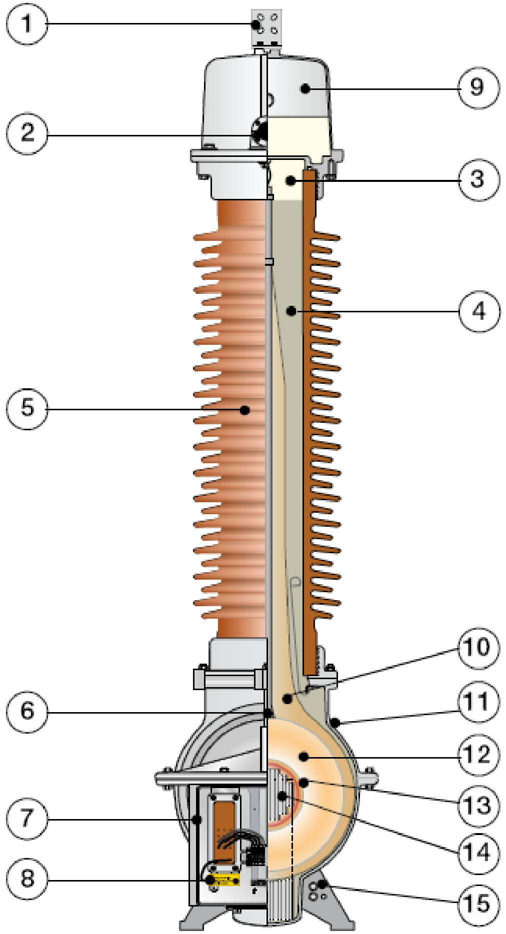
Thevenin impedance is given by

$$Z_{TH} = \frac{V_B}{I_{sc}} = \frac{1}{j\omega\left(C_1 + C_2\right)}$$

Let us assume L to be leakage impedance of the transformer. Let us now choose C_1 and C_2 such that

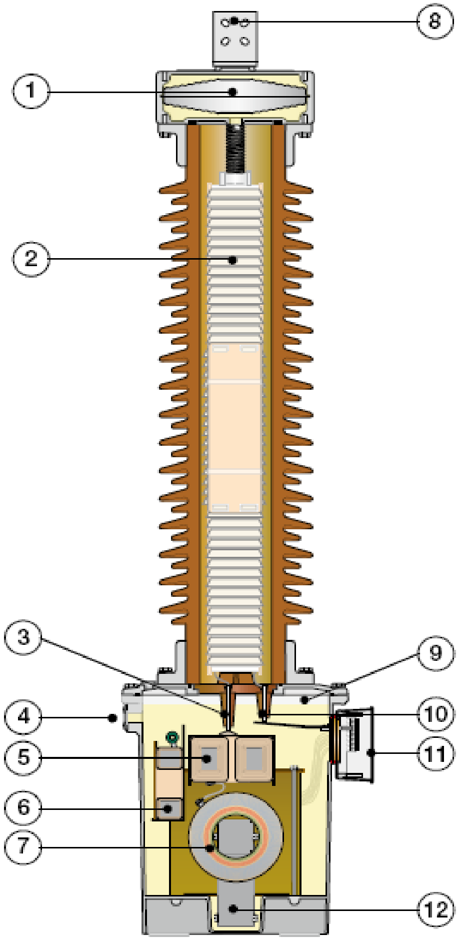
$$-\frac{1}{j\omega\left(C_1 + C_2\right)} = j\omega L \Rightarrow L = \frac{1}{\omega^2\left(C_1 + C_2\right)}$$

Electromagnetic Voltage Transformers



1. **Primary terminal**
2. Oil level sight glass
3. **Oil**
4. Quartz filling
5. **Insulator**
6. Lifting lug
7. Secondary terminal box
8. Neutral end terminal
9. Expansion system
10. Paper insulation
11. Tank
12. **Primary winding**
13. **Secondary windings**
14. Core

Capacitive Voltage Transformers



Capacitor Voltage Divider

1. Expansion system
2. **Capacitor elements**
3. Intermediate voltage bushing
8. **Primary terminal, flat 4-hole Al-pad**
10. **LV terminal (for carrier frequency use)**

Electromagnetic unit

4. Oil level glass
5. Compensating reactor
6. Ferroresonance damping circuit
7. **Primary and secondary windings**
9. Gas cushion
11. Terminal box
12. **Core**

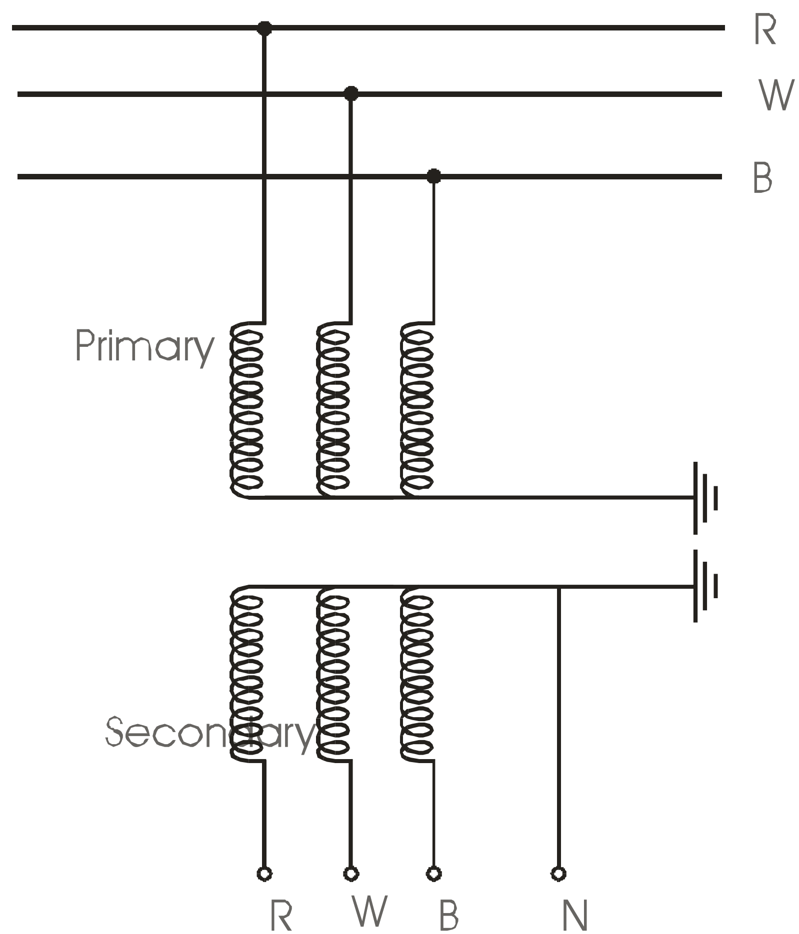
Voltage Transformers

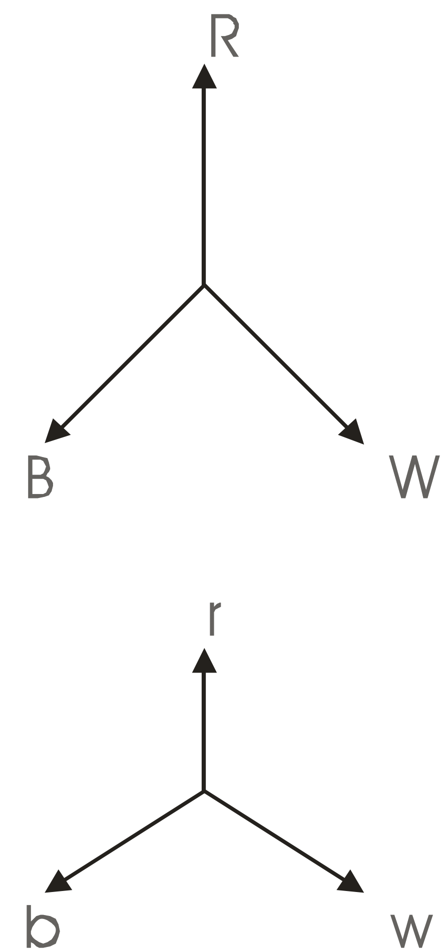
- VT ratios:
  - ratio of the high voltage/secondary voltage

1:1	2:1	2.5:1	4:1
5:1	20:1	40:1	60:1
80:1	100:1	200:1	300:1
400:1	600:1	800:1	1000:1
2000:1	3000:1	4500:1	

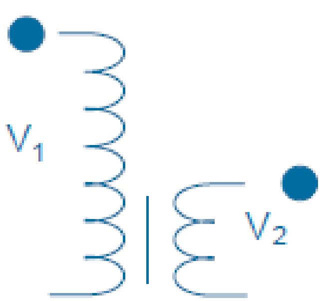
Connection of VT's

- VT's can be connected between phases or between phase and neutral (CVT's only phase - earth)



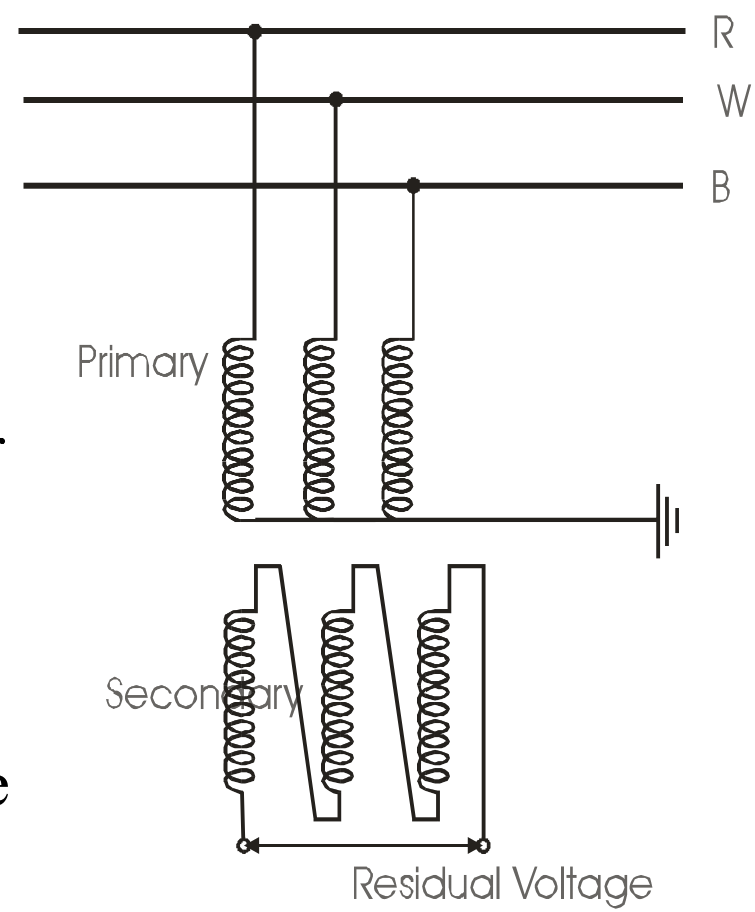


Symbol of a Voltage Transformer

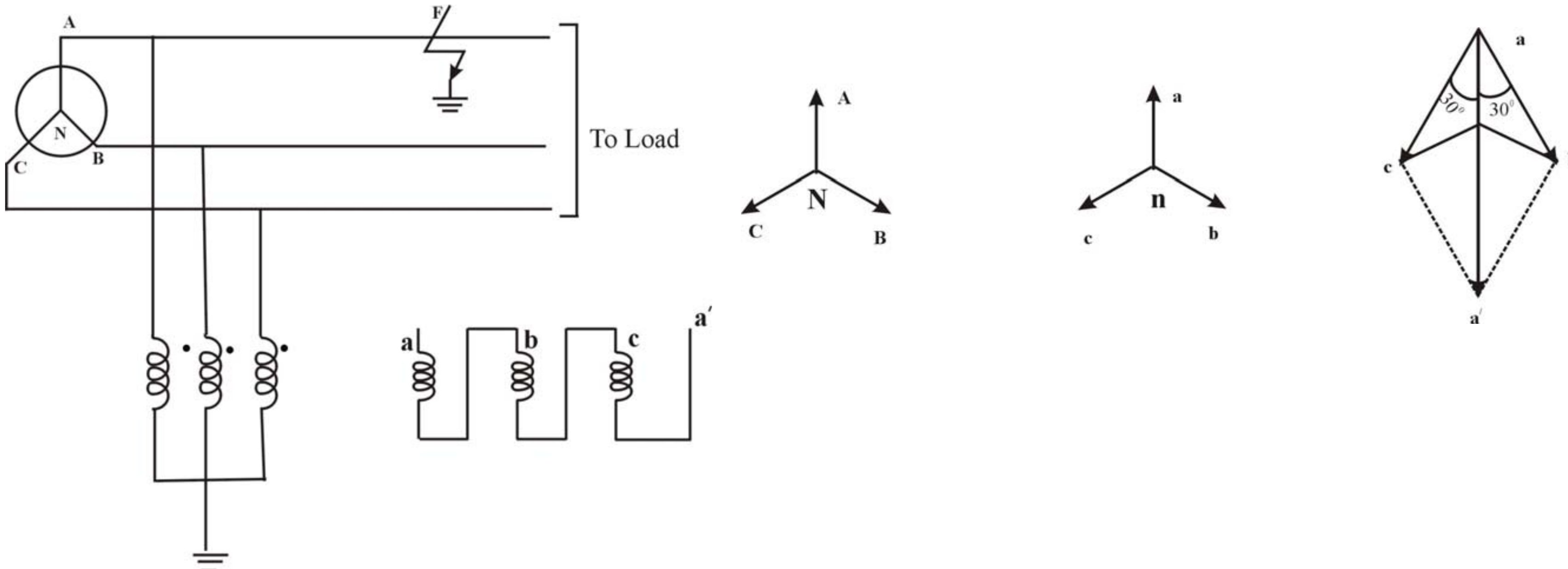


Residual voltage connection

- Normally used for earth fault detection
- VT's are connected in open-delta on primary and the vectorial sum of the 3 phase voltages will appear across the secondary output
- when there is an earth fault present the residual voltage will be non-zero



Residual voltage connection



When there is an Earth fault in line A it assumes Earth Potential.

Therefore Voltage across PT primary windings become

$V_A = 0$, $V_B = V_{BA}$, , $V_c = V_{CA}$

Thus Secondary Vectors are

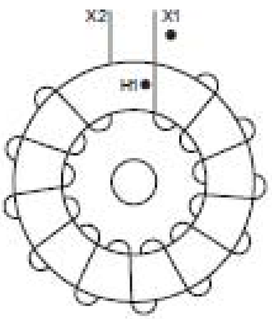
$V_a = 0$, $V_b = V_{ba}$, $V_c = V_{ca}$

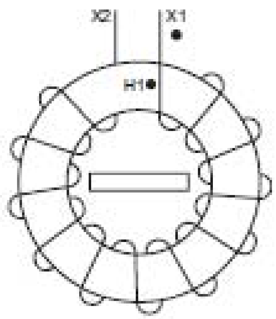
2. Current Transformers (CT)

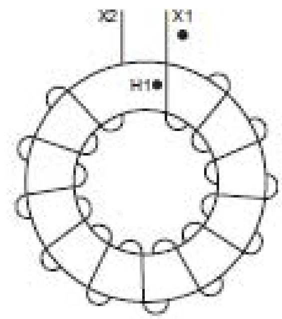
Window-type

Bar-type

Wound

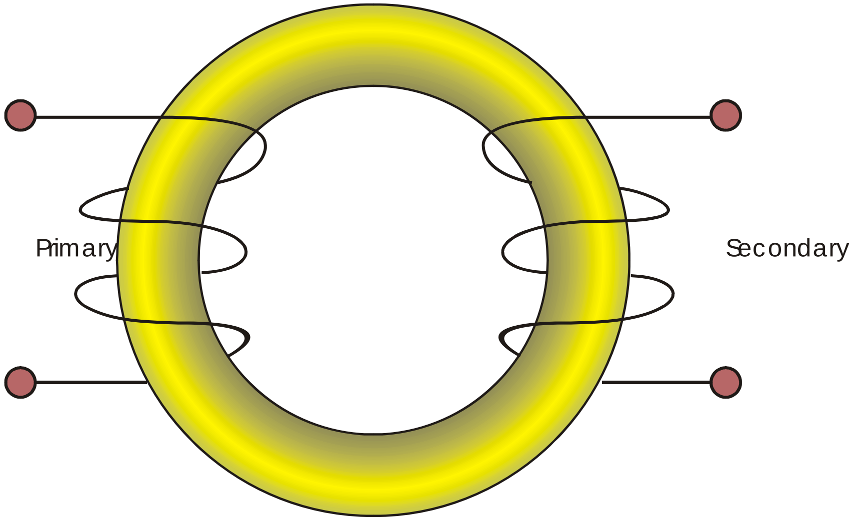




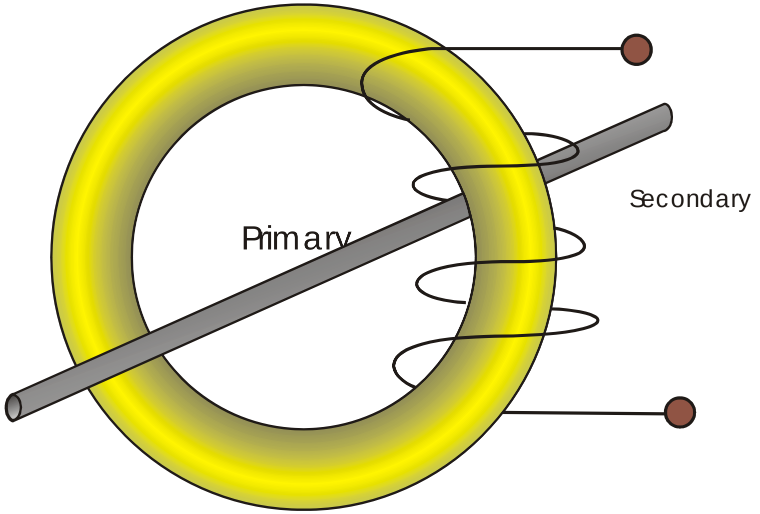


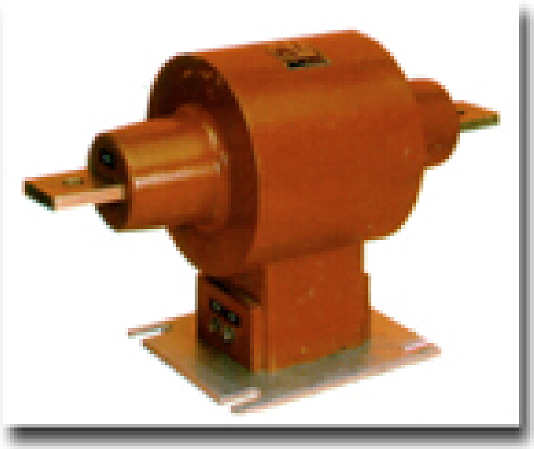
Current Transformer Types

1. Wound Primary



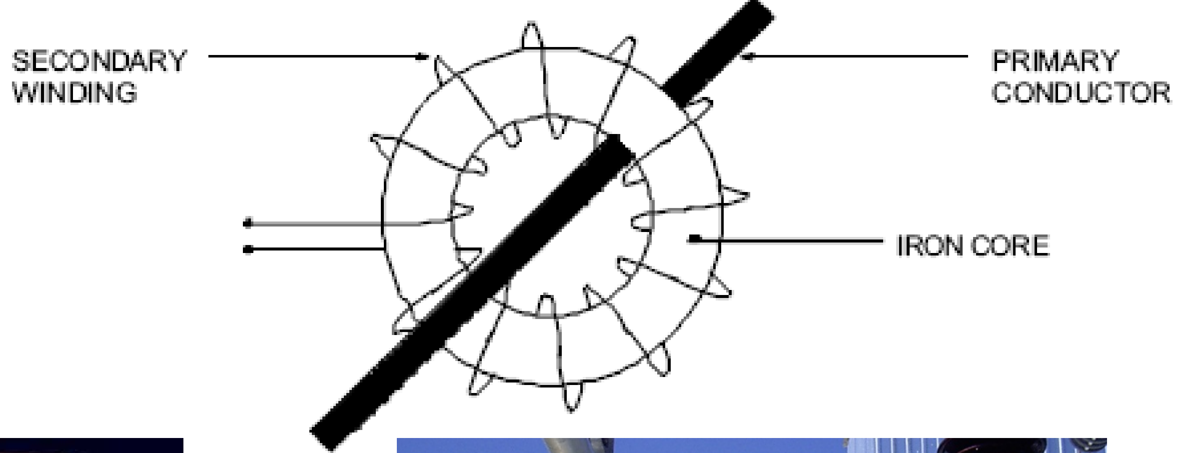
2. Bar Primary

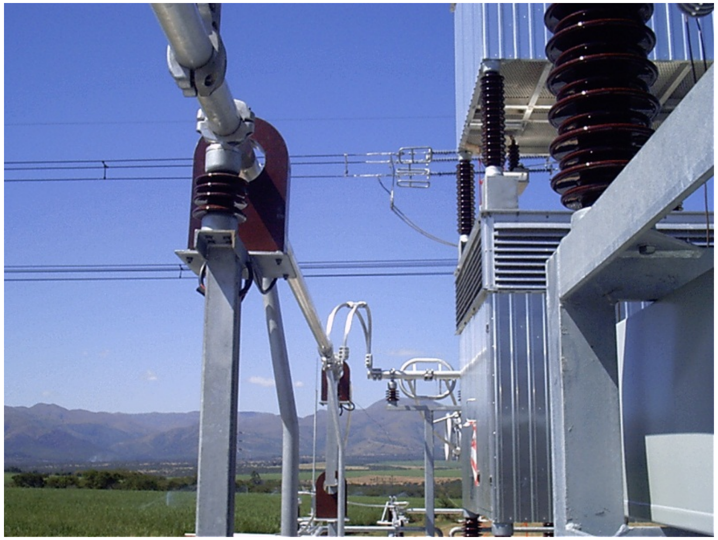




Doughnut CT

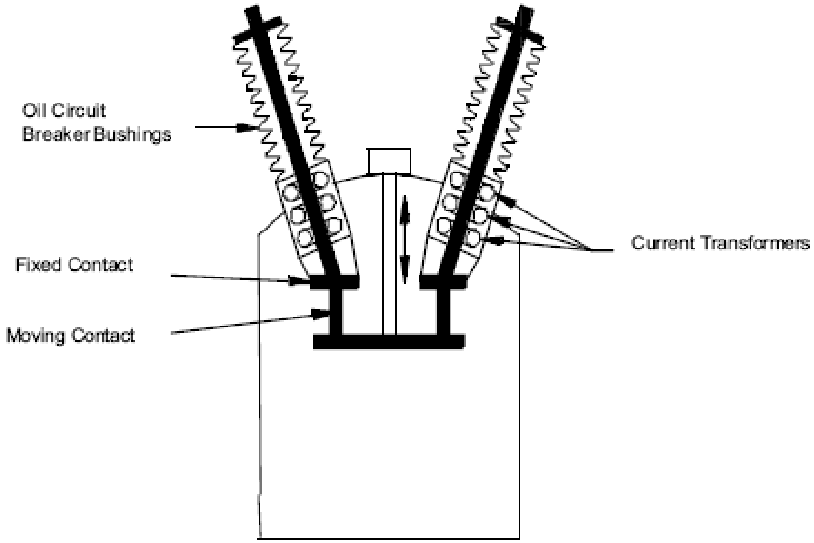
The most common type of C.T. construction is the `**DOUGHNUT**' type. It is constructed of an iron toroid, which forms the core of the transformer, and is wound with secondary turns.





CT-Instrument Transformers

This type of `*doughnut*' C.T. is most commonly used in circuit breakers and transformers. The C.T. fits into the bushing `*turret*', and the porcelain bushing fits through the centre of the `*doughnu*t'.

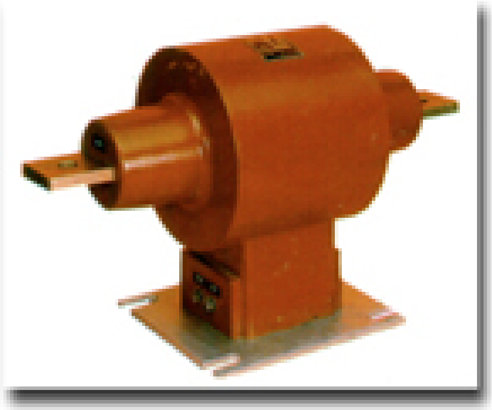


Bushing-type CTs installed on the bushings of 66kV Dead Tank Breaker

CT-Instrument Transformers

The **toroid**, wound with secondary turns, is located in the **live tank** at the **top** of the C.T. High voltage insulation must, of course, be provided, between the H.V. primary conductor, and the secondary winding, which operates at essentially ground potential.

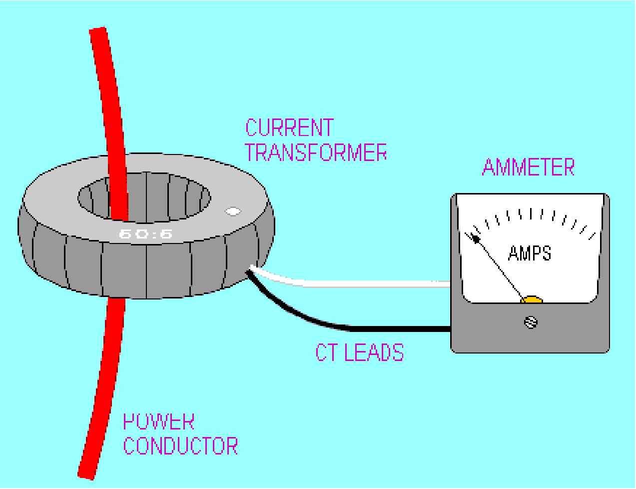
Current transformers of this type are often used at voltage levels of 44 kV, 33kV, and 13.8 kV.

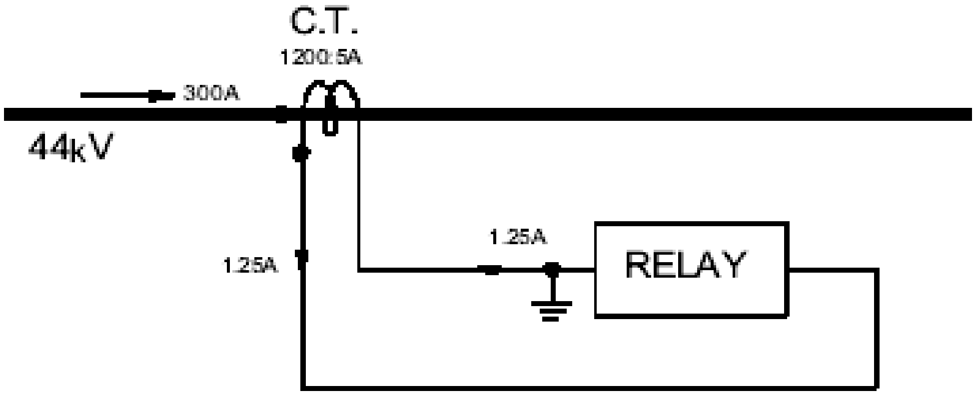


CT-Instrument Transformer

The `*doughnut*' fits over the primary conductor, which constitutes one primary turn. If the toroid is wound with 240 secondary turns, then the ratio of the C.T. is 240:1 or 1200:5A

The continuous rating of the secondary winding is normally **5 AMPS** in North America, and **1 AMP** or **0.5 AMP** in many other parts of the world.





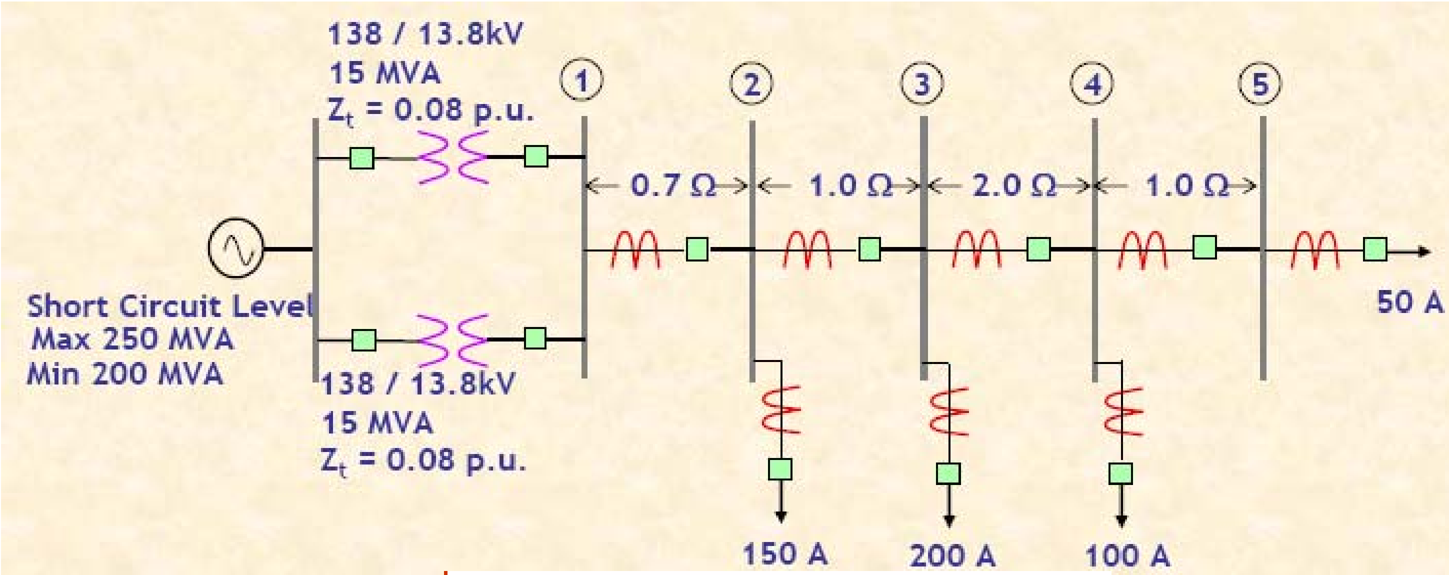
Current Transformers

- CTs ratio(secondary current rating is 5A)

50:5	100:5	150:5	200:5
250:5	300:5	400:5	450:5
500:5	600:5	800:5	900:5
1000:5	1200:5		

- CTs also available with the secondary rating of 1A

Example 1: Selection of CT Ratio

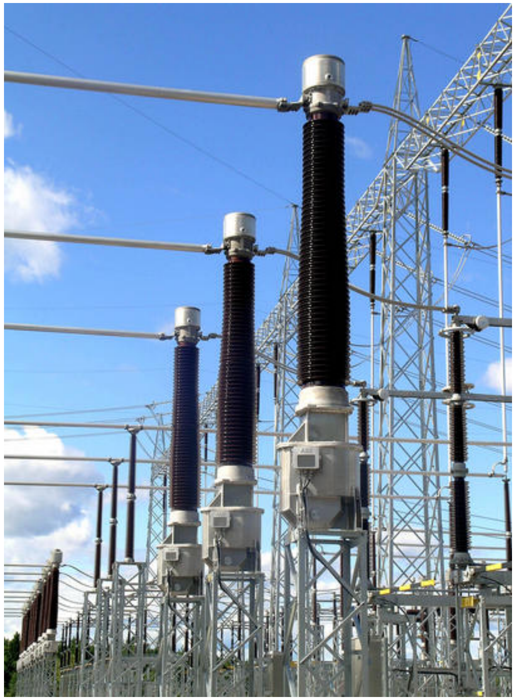


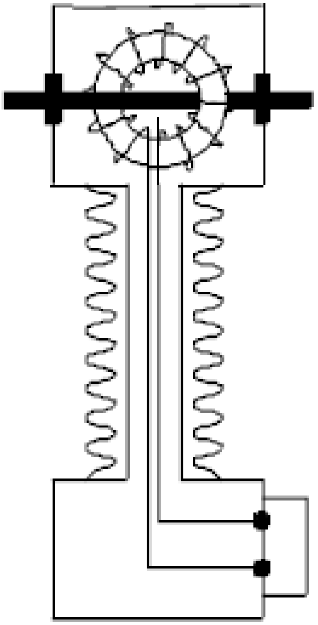
$$I_{CT}=1.25\times I_L$$

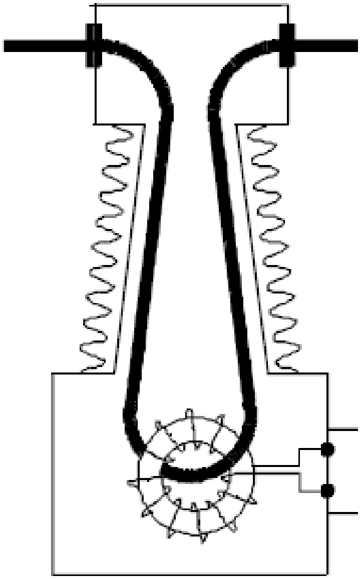
Relay Location Bus	Maximum Load Current (A)	CT Ratio Selected
1	500	800/5
2	350	500/5
3	150	200/5
4	50	100/5
5	50	100/5

Other Types of CT Construction

The other principal type of C.T. construction is the ***Free Standing***, or ***Post*** type. These can be either ***Straight-Through*** or ***Hairpin*** construction.



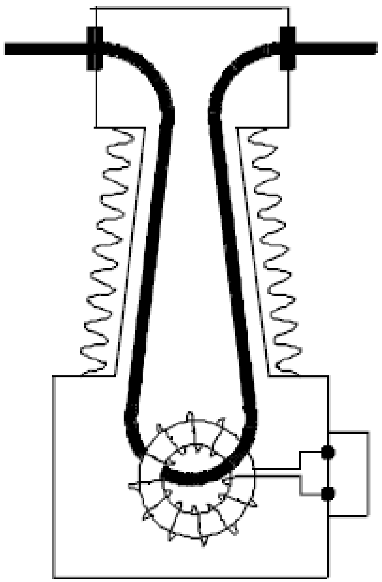




Straight-Through ***Hairpin***

Hairpin **- CT**

The second kind of ***Free-Standing*** or ***Post*** type current transformer is the ***Hairpin*** construction as shown below:



The ***HAIRPIN*** C.T. gets it's name from the shape of the primary conductor within the porcelain. With this type, the tank housing the toroid is at ground potential. The primary conductor is insulated for the full line voltage as it passes into the tank and through the toroid.

CT-Instrument Transformer

Current transformers of this type are commonly used on H.V. transmission systems at voltage levels of 500kV and 230kV.

Free standing current transformers are very expensive, and are only used where it is not possible to install `Doughnut' C.T.'s in Oil Breakers or transformer bushing turrets.

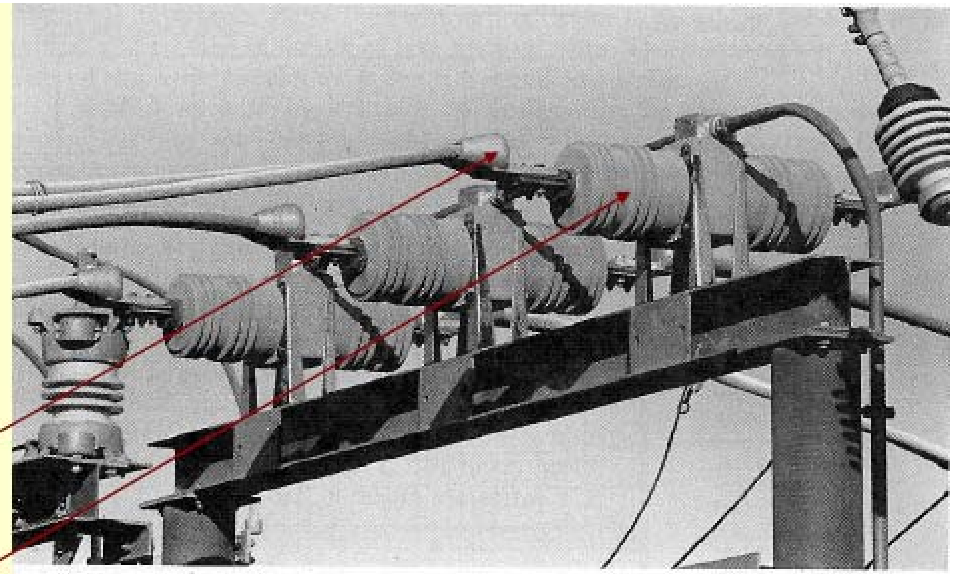
As an example, C.T.'s cannot easily be accommodated in Air Blast circuit breakers, or some outdoor SF_6 breakers.

Free Standing current transformers must therefore be used with these types of switchgear. Current transformers often have multiple ratios. This is achieved by having taps on various points of the secondary winding, to provide the different turns ratios.

High Voltage CTs

High Voltage CTs

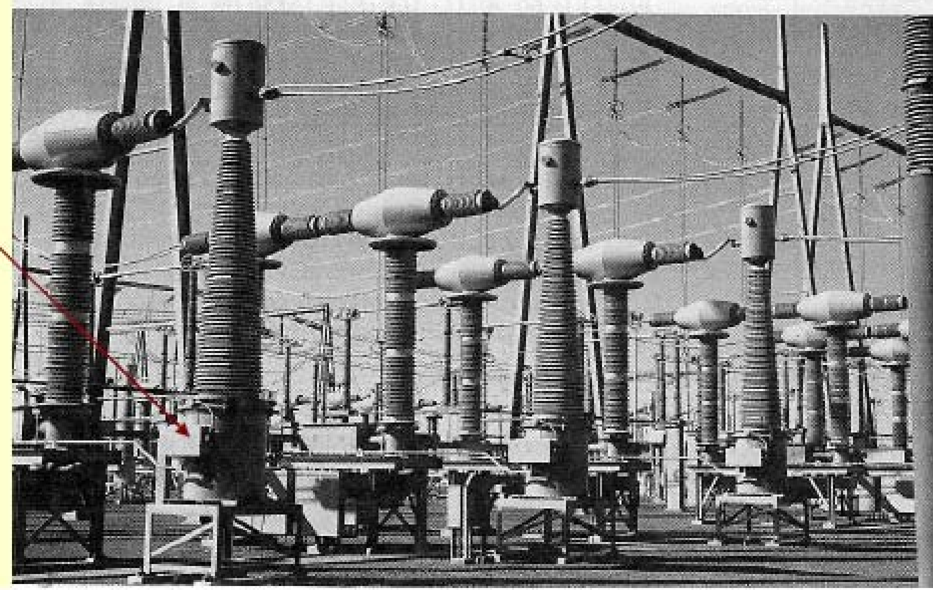
33kV single bar primary type CTs



Bar primary

Secondary coil

500kV "hairpin" single bar primary type CTs, with SF6 CB's behind



Current Transformers - CT

- As with all transformers-Ampere-turns balance must be achieved
- e.g. 1000Amps × 1 turn (bar primary) =

  1 Amp × 1000 turns (secondary side)
- Error introduced into measurement by magnetising current
- **Current Transformers for protective relaying purposes must reproduce the primary current accurately for all expected fault currents.**

CT-Instrument Transformers

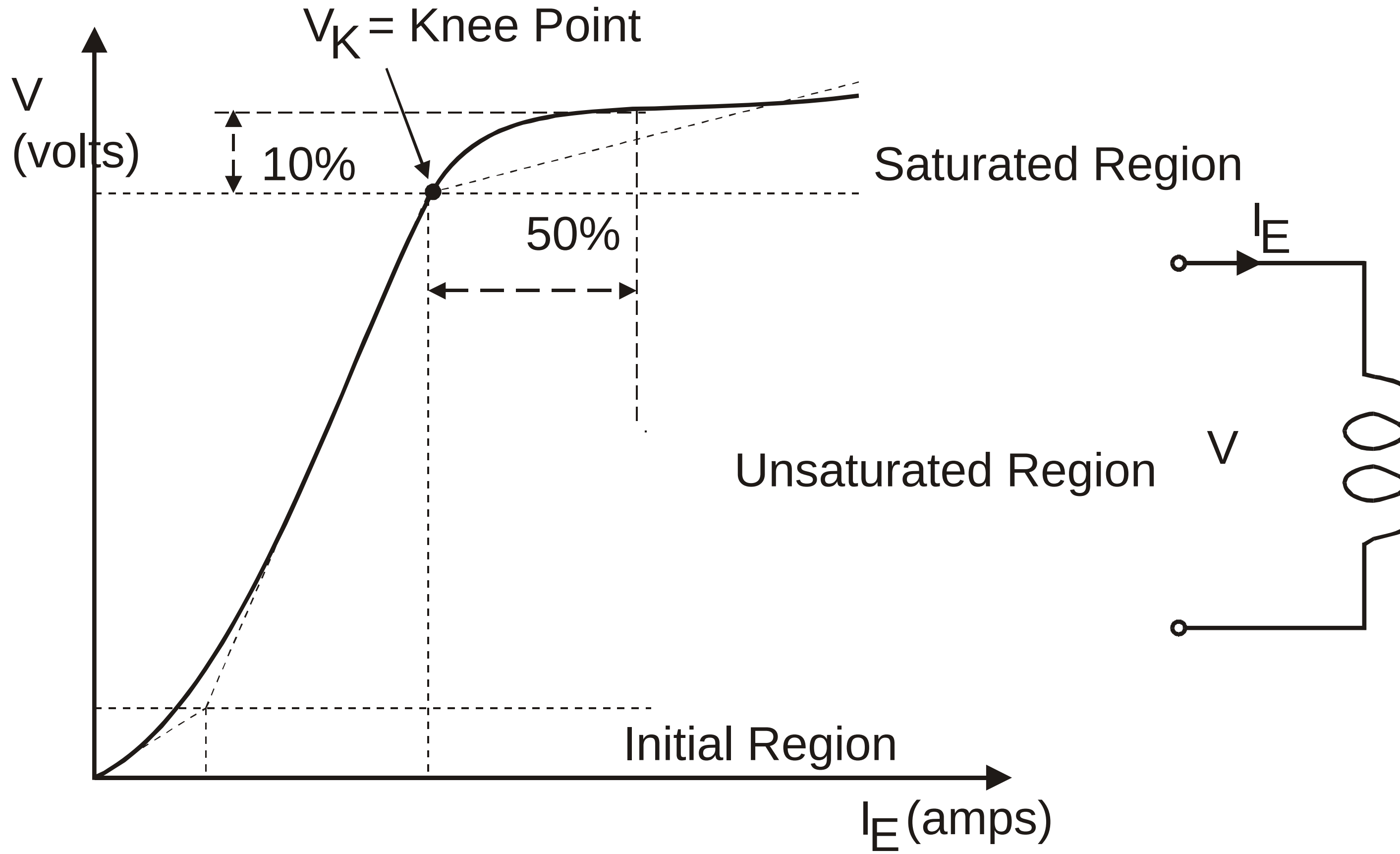
If we have a 33 kV C.T. with a ratio of 1200 : 5A, the secondary winding is continuously rated for 5 Amps.

If the maximum fault current that can flow through the C.T. is 12,000 Amps, then the C.T. must accurately produce a secondary current of 50 Amps to flow through the relay during this fault condition.

This current will, of course, flow for only about 0.2 seconds, until the fault current is interrupted by the tripping of the circuit breaker.

CT-Excitation Characteristics

The C.T. must be designed such that the iron core does not saturate for currents below maximum fault current. For a typical C.T.

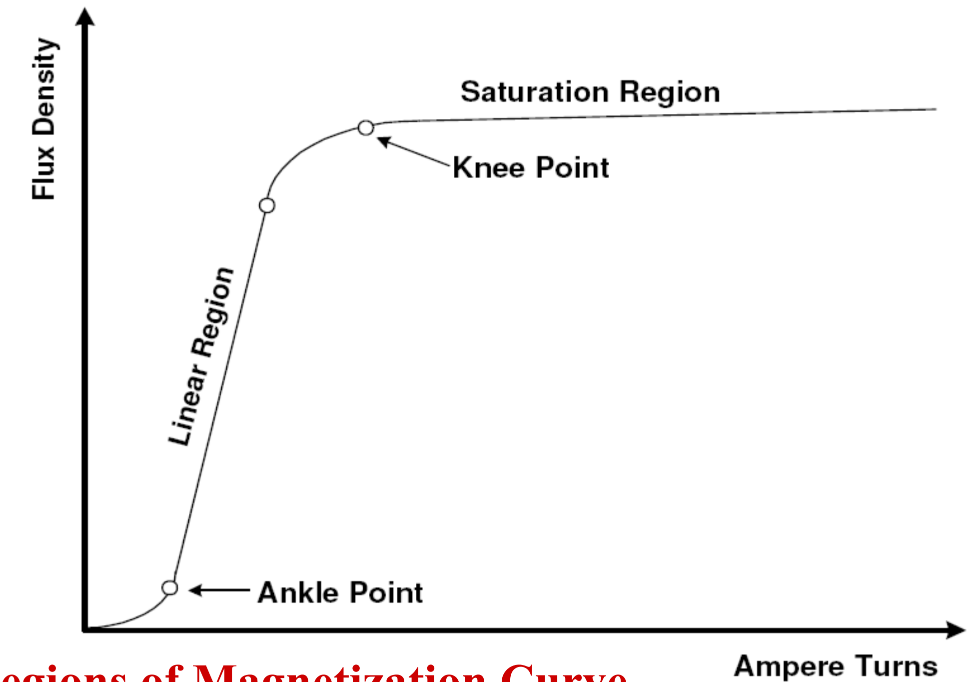


CT-Excitation Characteristics

For a C.T. to operate satisfactorily at maximum fault currents, it must operate on the linear part of the magnetizing curve.

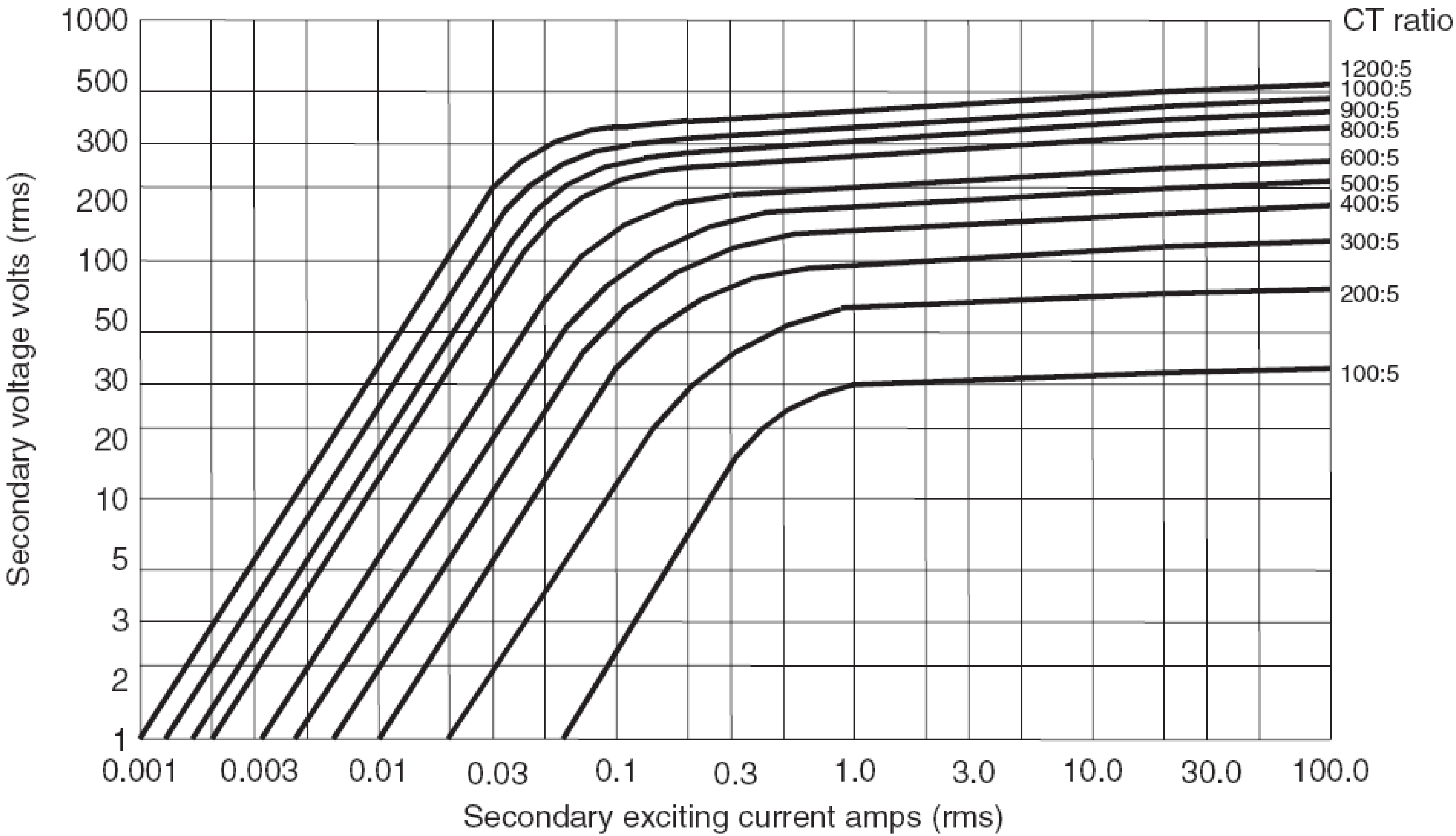
i.e. Below the point at which saturation occurs, which is known as the **KNEE POINT**. The **KNEE POINT** is defined as:

<***the point at which a 10% increase in voltage produces a 50% increase in magnetizing current***.>



Different Regions of Magnetization Curve

CT-Excitation Characteristics



Magnetizing characteristic of a typical CT

Two Main Types of CTs

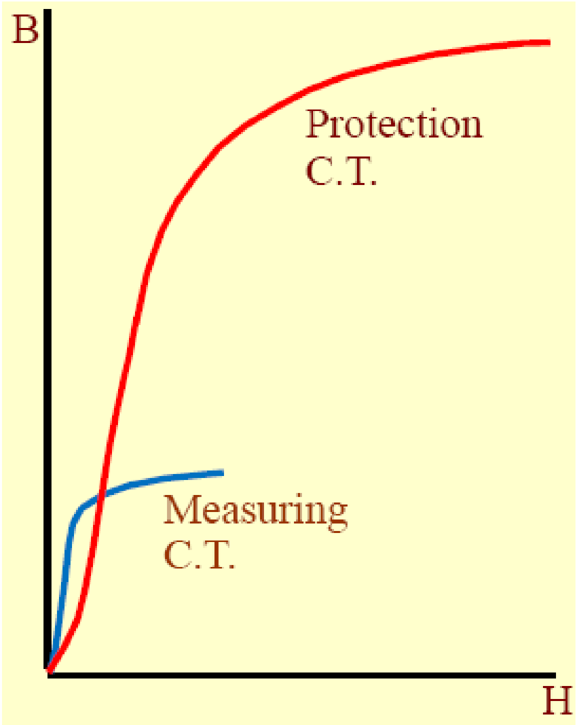
- Measuring CTs (type 'M')
- Protection CTS (type 'P')

Measurement class CT

- Designed to operate at rated current

Protection class CT

- Designed to operate at fault current

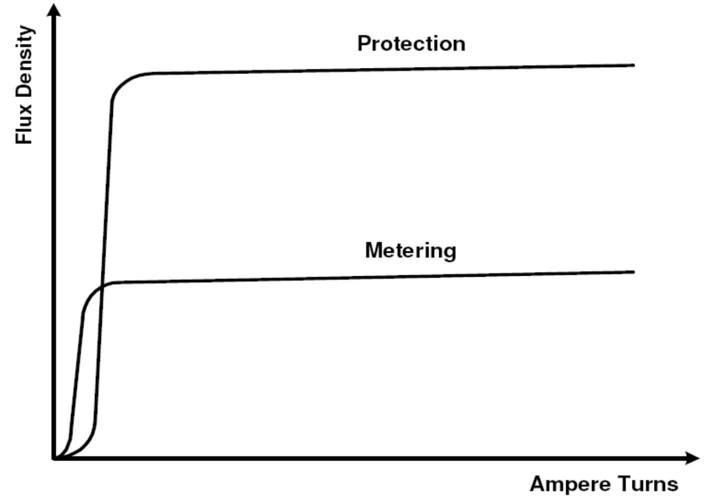


Characteristics and Specification are very different

Protection vs Metering CT's

- Protection CT's have to measure fault currents many times in excess of full load current without saturating to drive relays to trip - Accuracy is not as important

- Metering CT's have to be accurate as customers will be billed on the information provided by measuring the current from the metering CT's - special alloys are used for the cores so that they saturate quickly



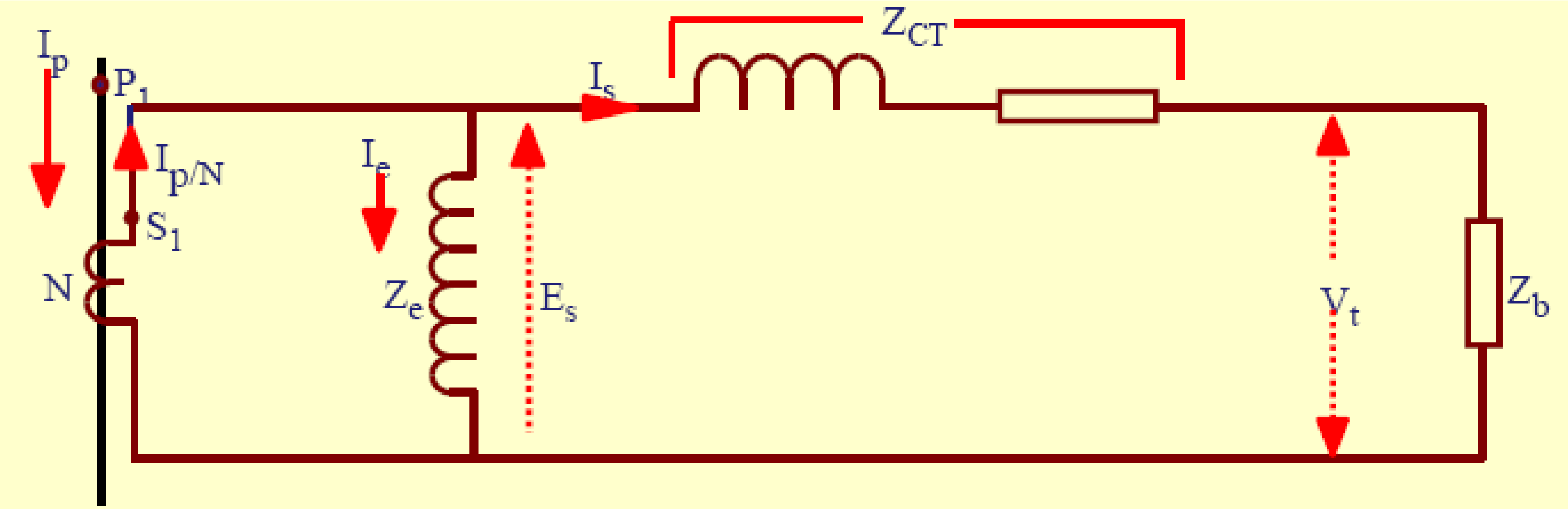
Protection vs Metering CT's

When C.T.'s are used for metering purposes, they must have a high degree of accuracy only at **LOAD** currents. i.e. 0 to 5 Amps secondary.

There is no need for a high degree of accuracy for fault currents, and it is quite acceptable for a metering C.T. to saturate when fault current flows through it.

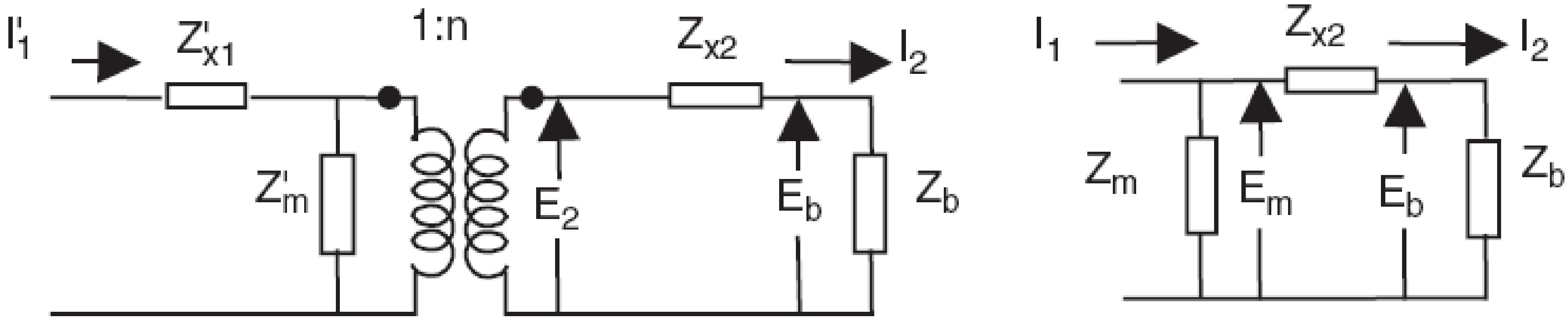
A C.T. for protective relaying purposes may typically have a ***knee point*** at 500 volts, whereas a metering C.T. may saturate at well below 100 volts.

CT- General Equivalent Circuit



I_p	=	Primary rating of C.T.	I_e	=	Secondary excitation current
N	=	C.T. ratio	I_s	=	Secondary current
Z_b	=	Burden of relays in ohms (r+jx)	E_s	=	Secondary excitation voltage
Z_{CT}	=	C.T. secondary winding impedance in ohms (r+jx)	V_t	=	Secondary terminal voltage across the C.T. terminals
Z_e	=	Secondary excitation impedance in ohms (r+jx)			

CT- Equivalent Circuit and its Simplification



(a)

(b)

Since the primary winding of a CT is connected in series with the power network, its primary current I_1 is dictated by the network. Consequently, the leakage impedance of the primary winding $Z`x_1$ has no effect on the performance of the transformer, and may be omitted.

$$I_1 = \frac{I'_1}{n} \qquad E_m = E_b + Z_{x2} I_2$$

and the magnetizing current I_m is given by

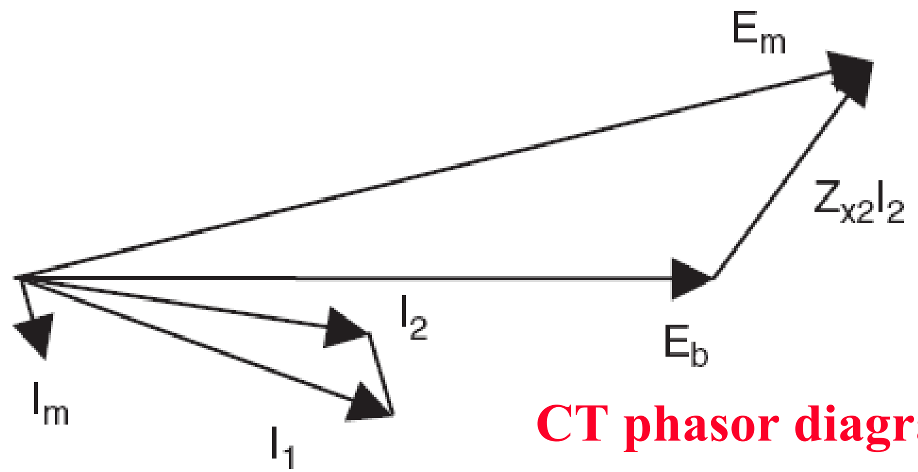
$$I_m = \frac{E_m}{Z_m}$$

$$Z_m = n^2 Z'_m$$

CT- Equivalent Circuit and its Simplification

The primary current I_1 (referred to the secondary winding) is given by

$$I_1 = I_2 + I_m$$



CT phasor diagram

For small values of the burden impedance, E_b and E_m are also small, and consequently I_m is small. The per unit current transformation error defined by

$$\varepsilon = \frac{I_1 - I_2}{I_1} = \frac{I_m}{I_1}$$

is, therefore, small for small values of Z_b. In other words, CTs work at their best when they are connected to very low impedance burdens.

CT- Equivalent Circuit and its Simplification

More often, the CT error is presented in terms of a ratio correction factor ***R*** instead of the per unit error **ε.** The ratio correction factor ***R*** is defined as the constant by which the nameplate turns ratio ***n*** of a CT must be multiplied to obtain the effective turns ratio.

$$R = \frac{1}{1 - \varepsilon}$$

Example 6: CT Performance

Consider a current transformer with a turns ratio of 500:5, a secondary leakage impedance of (0.01+j0.1) and a resistive burden of 2.0. If the magnetizing impedance is (4.0+j15), then for a primary current (referred to the secondary) of I_l

$$E_{\mathrm{m}} = \frac{I_1(0.01 + \mathrm{j}0.1 + 2.0)(4.0 + \mathrm{j}15.0)}{(0.01 + \mathrm{j}0.1 + 2.0 + 4.0 + \mathrm{j}15.0)} = I_1 \times 1.922\angle 9.62^\circ$$

$$I_{\mathrm{m}} = \frac{I_1 \times 1.922\angle 9.62^\circ}{(4.0 + \mathrm{j}15.0)} = I_1 \times 0.1238\angle -65.45^\circ$$

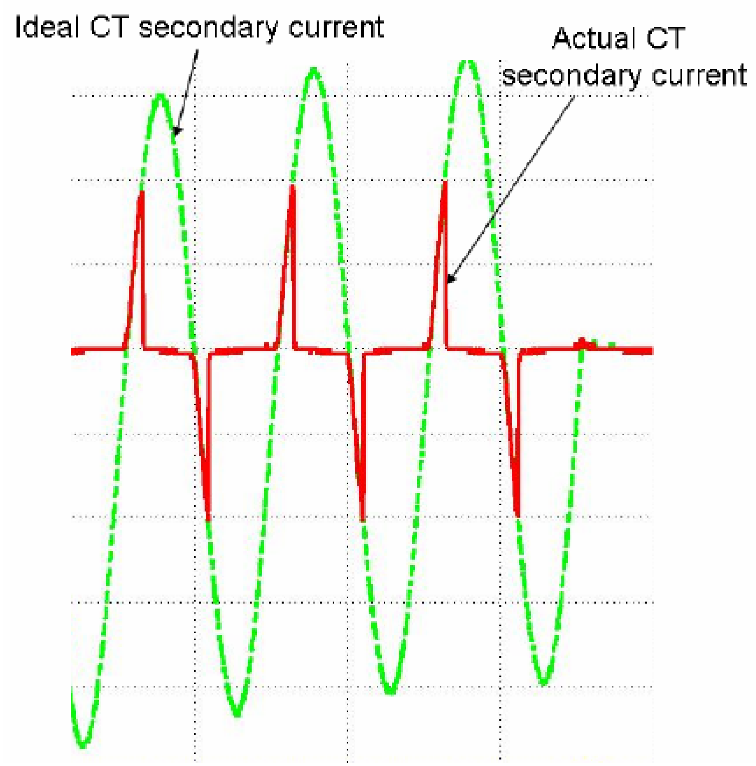
CT error $$\varepsilon = \frac{I_{\mathrm{m}}}{I_1} = 0.1238\angle -65.45^\circ$$

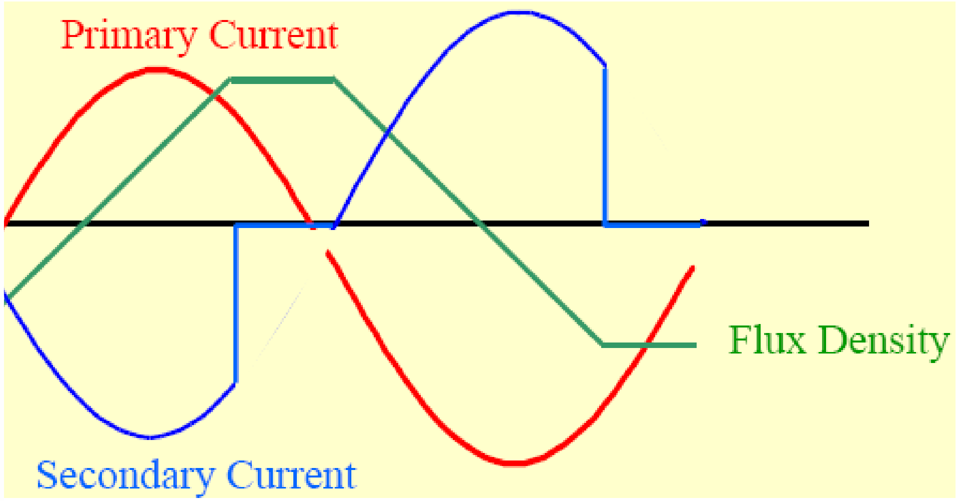
The corresponding ratio correction factor ***R***:

$$R = \frac{1}{(1.0 - 0.1238\angle -65.45^\circ)} = 1.0468\angle -6.79^\circ \quad \text{for } Z_{\mathrm{b}} = 2 \text{ ohms}$$

Effect of CT Saturation

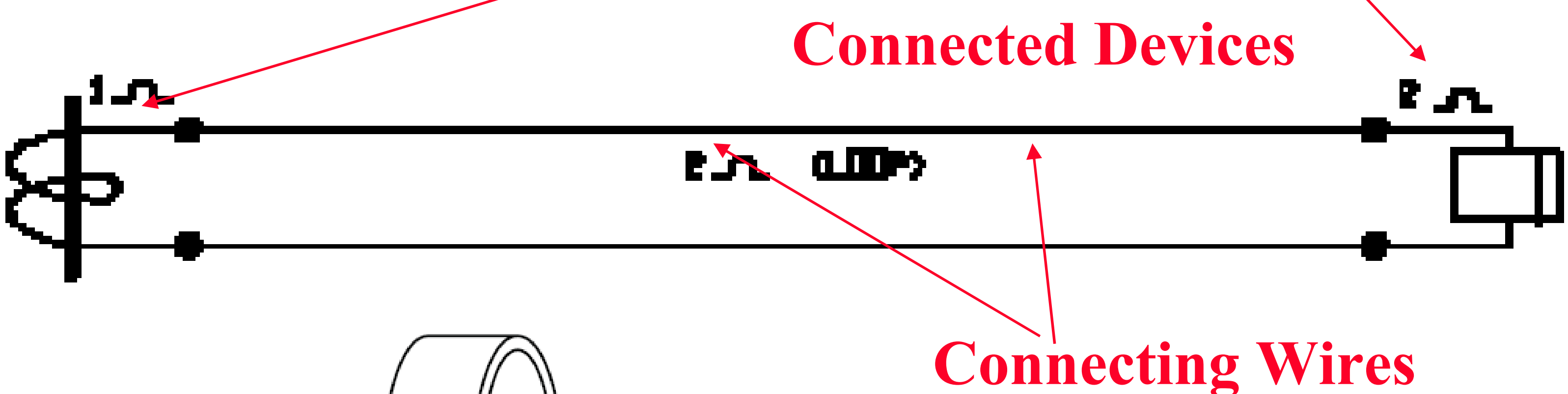
Output current drops to zero when flux is constant (core saturated)

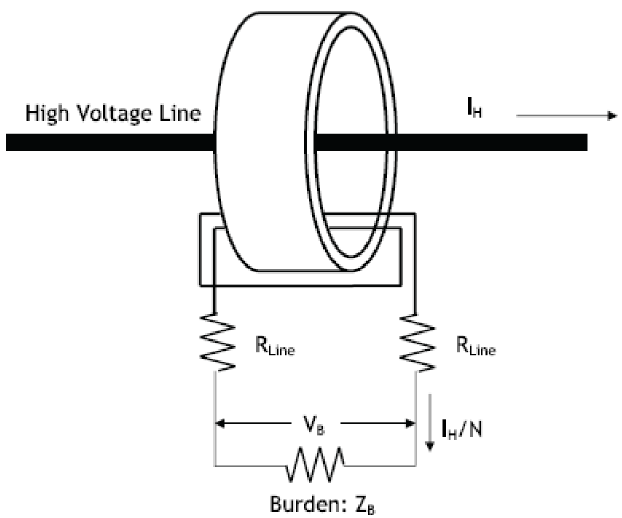




CT Knee-point Voltage

The point on the magnetizing curve at which the C.T. operates is dependent upon the resistance of the C.T. secondary circuit, as shown below:



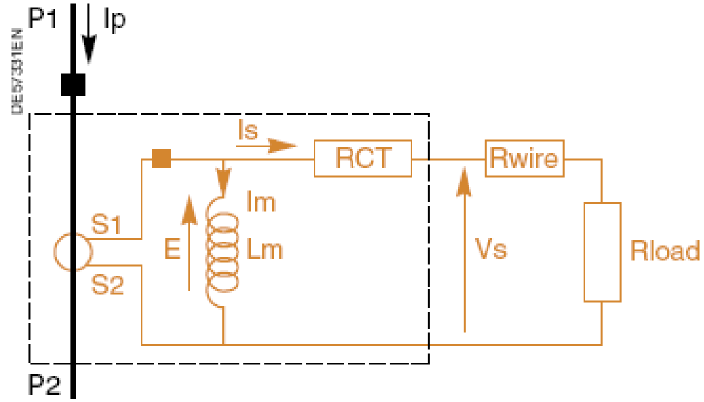


Current Transformer Circuit with Burden

CT Knee-point Voltage

Kee-point Voltage ➔ $V_k = (R_{ct} + R_b + 2\times R_L).I_{sn}$

- R_{ct}= CT secondary winding resistance
- R_L= secondary wiring (leads) resistance
- R_b = burden (load). (meter or relay)
- I_{sn} = rated secondary current
- V_k= required knee-point voltag

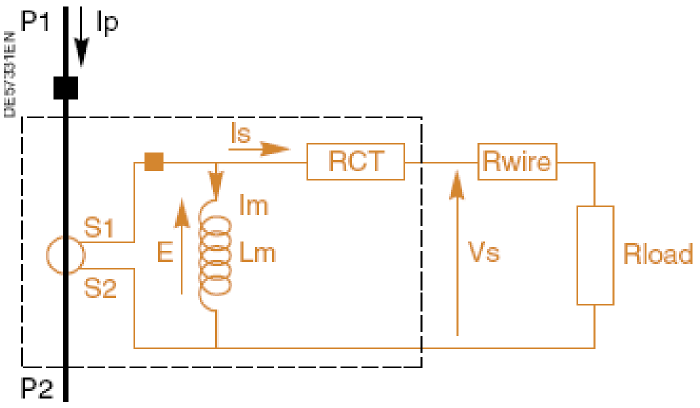


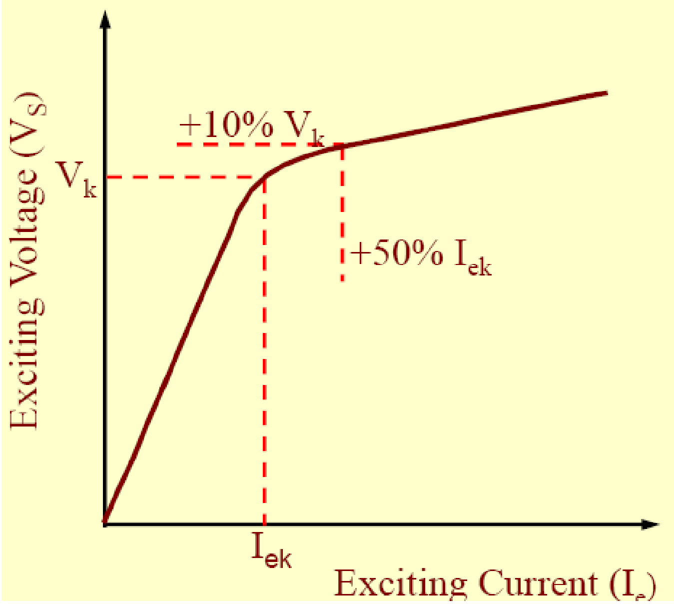
Example 1: CT Knee-point Voltage

In this example the resistance of the C.T. secondary circuit, or C.T. burden is:

C.T. Secondary Winding Resistance = 1 OHM
Resistance of Cable from C.T. to Relay = 2 OHMS
Resistance of Relay Coil = 2 OHMS

Total Resistance of C.T. Secondary Circuit = 5 OHMS





Example 1: CT Performance

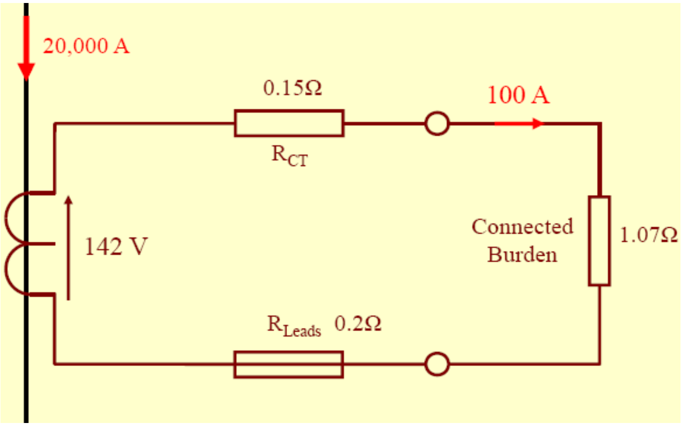
If the fault current is 12,000 Amps, and the C.T. ratio is 1200 : 5A, then the C.T. secondary current is 50 Amps. At this secondary current and the above C.T. burden of 5 OHMS, the C.T. must produce a terminal voltage of 250 volts.

For the C.T. to operate with good accuracy, without saturating for the maximum fault current, the ***knee point*** must be well above **250 volts**.

The importance of the C.T. maintaining good accuracy, and not saturating at the maximum fault current, is most critical on differential protection.

Example 2: CT Performance

- Find maximum allowable secondary burden
  - **CT Ratio = 1000/5**
  - **R_{CT}= 0.15 Ω**
  - **R_{leads} = 0.1 Ω (R_{leads}= 2×0.1 Ω = 0.2 Ω)**
  - **Max Flux Density (B) = 1.6 Tesla**
  - **Core Cross Sectional Area (A) = 20cm²**



- **Solution:**
  - **V_k= 4.44×B×A×f×N (the "transform erequation")**
  - **N = 1000/5 = 200 Turns**
  - **A = 20cm²= 20×10⁻⁴ m²**
  - **V_k= 4.44×1.6× 20×10⁻⁴ ×50×200 = 142 V (alternatively, if *A* not known, curves of V_k may be available)**
  - **Max Fault Current = 20kA (Primary) = 100 A (secondary)**
  - **Max resistance = 142V/100A = 1.42 Ω**
  - **R_{CT}= 0.15Ω; Lead resistance = 0.2 Ω**
  - **Maximum connected burden 1.42 - 0.15 - 0.2 = <u>1.07 Ω</u>**

Example 3: CT Performance

- A current transformer is specified as being 600 A:5 A class C200. Determine it's characteristics. This designation is based on ANSI Std. C57.13-1978. 600 A is the continuous primary current rating, 5 A is the continuous secondary current rating, and the turns ratio is 600/5=120. C is the accuracy class, as defined in the standard. The number following the C, which in this case is 200,is the voltage that the CT will deliver to the rated burden impedance at 20 times rated current without exceeding 10 percent error. Therefore, the rated burden impedance is

$$Z_{\text{rated}} = \frac{\text{Voltage class}}{20 \cdot \text{Rated secondary current}} = \frac{200\text{ V}}{20 \cdot 5\text{ A}} = 2\ \Omega$$

- This CT is able to deliver up to 100 A secondary current to load burdens of up to 2 with less than 10 percent error. Note that the primary source of error is the saturation of the CT iron core and that 200 V will be approximately the knee voltage on the CT saturation curve.

Example 4: CT Performance

- The circuit of Fig.1 has 600:5 class C100 CTs. The peak-load current is a balanced 475 A per phase.

1. Determine the Relay Currents for the Peak-Load Conditions.

The A phase CT secondary current is

$$I_A = \frac{475\text{A}}{120} = 3.96\ \text{A}\underline{/0^\circ}$$

- Here, the A phase current is taken to be at 0°. The B and C phase currents are the same magnitude, shifted by 120°,

$$I_B = 3.96\ \text{A}\underline{/-120^\circ},\ I_C = 3.96\ \text{A}\underline{/120^\circ}$$

- The residual current is

$$I_R = I_A + I_B + I_C = 3.96\ \text{A}\underline{/0^\circ} + 3.96\ \text{A}\underline{/-120^\circ} + 3.9.6\ \text{A}\underline{/120^\circ} = 0\ \text{A}$$

Example 4: CT Performance

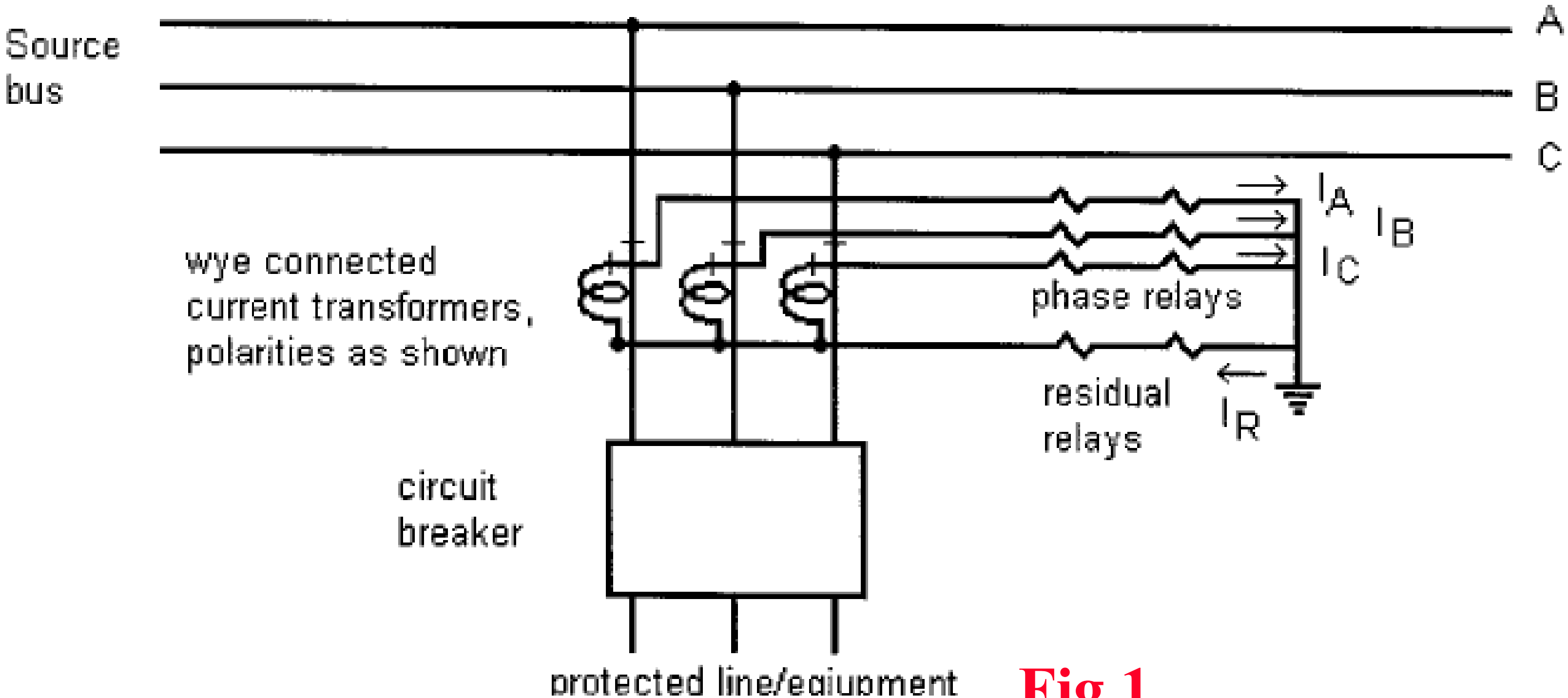
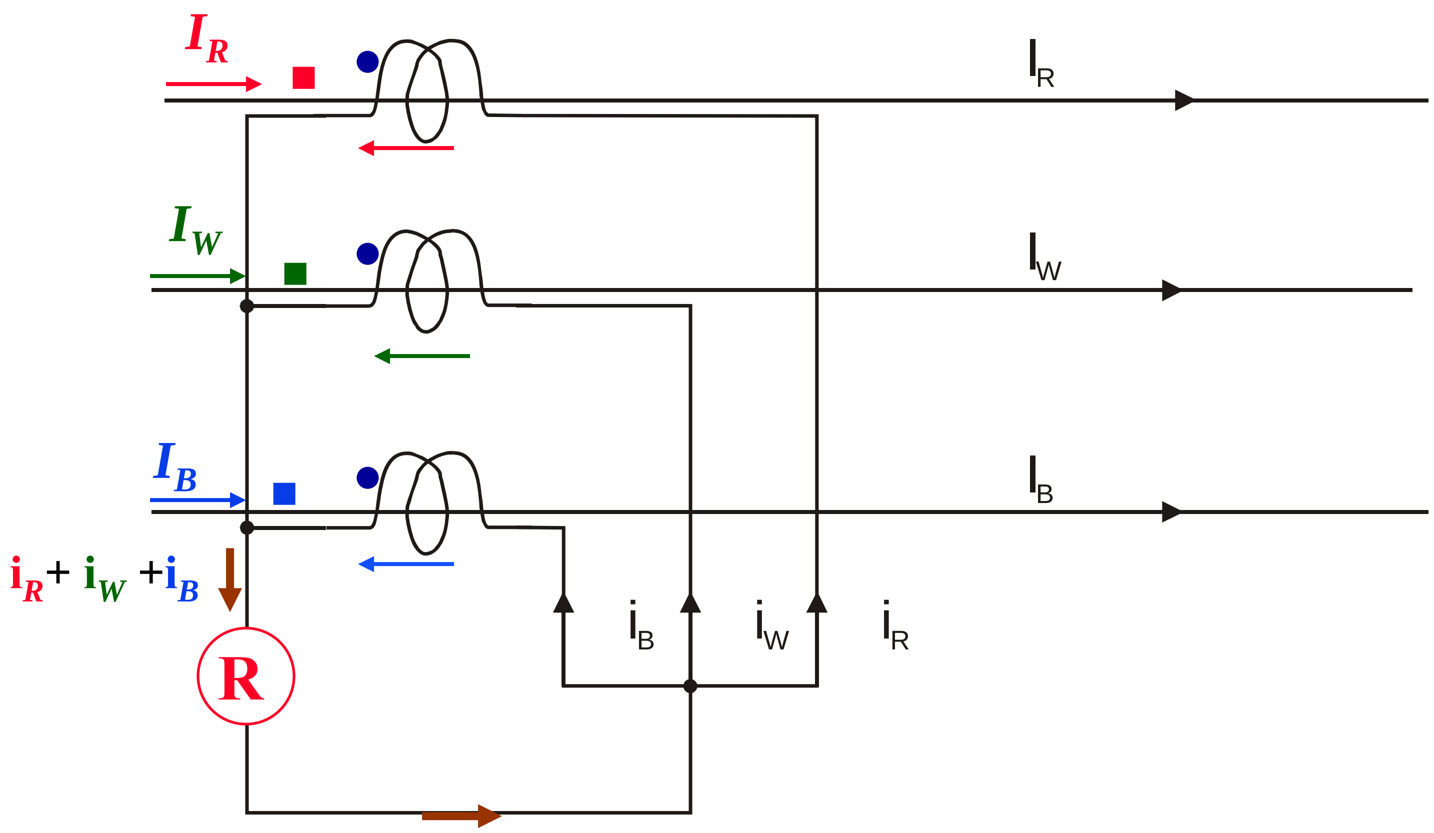


Fig.1

Typical setup for wye-connected CTs protecting a line or piece of equipment.

Connection of CT's in 3 phase systems

Most common connection is star (below) - residual current will spill through neutral and through relay **R** during faults



Example 4: CT Performance

2. The circuit has an A phase to ground fault on the line, with fault current magnitude of 9000 A. Find the phase and residual relay currents. Again, assume that the A phase current is at 0°.

$$I_A = \frac{9000\text{ A}}{120} = 75\text{ A}\underline{/0^\circ}$$

$$I_B = 0\text{ A}$$

$$I_C = 0\text{ A}$$

$$I_R = I_A + I_B + I_C = 75\text{ A}\underline{/0^\circ} + 0\text{ A} + 0\text{ A} = 75\text{ A}\underline{/0^\circ}$$

❖ The current path is therefore through the A phase lead and back through the residual lead.

Example 4: CT Performance

3. The circuit has a two-phase fault with 5000 amps going out B phase and back in on C phase. Choose B phase current to be at 0°.

$$I_A = \frac{0\text{ A}}{120} = 0\text{ A}$$

$$I_B = \frac{5000\text{ A}\angle 180°}{120} = 41.7\text{ A}\angle 0°$$

$$I_C = \frac{5000\text{ A}\angle 180°}{120} = 41.7\text{ A}\angle 180° = -I_B$$

$$I_R = I_A + I_B + I_C = 0\text{ A} + 41.7\text{ A}\angle 0° + 41.7\text{ A}\angle 180° = 0\text{ A}$$

❖ This current path involves the B and C phase leads, with no current in either the A phase lead or residual.

Example 4: CT Performance

4. The circuit has a three-phase fault with 8000 A per phase.

$$I_A = \frac{8000\text{ A}\underline{/0^\circ}}{120} = 66.7\text{ A}\underline{/0^\circ}$$

$$I_B = \frac{8000\text{ A}\underline{/-120^\circ}}{120} = 66.7\text{ A}\underline{/-120^\circ}$$

$$I_C = \frac{8000\text{ A}\underline{/120^\circ}}{120} = 66.7\text{ A}\underline{/-120^\circ}$$

$$I_R = I_A + I_B + I_C = 66.7\text{ A}\underline{/0^\circ} + 66.7\text{ A}\underline{/-120^\circ} + 66.7\text{ A}\underline{/120^\circ} = 0\text{ A}$$

- The phase currents sum to zero, so no current flows in the residual for this fault.
- The path of current flow for these various situations must be considered in calculating the CT excitation voltage and subsequent saturation.

Example 5: CT Performance

- For part 2, 3, and 4 of Example 4, calculate the CT voltage if the phase relay burden is 1.2 , the residual relay burden is 1.8 , the lead resistance is 0.4 ,and the CT resistance is 0.3 . Neglect CT saturation in this calculation.

1. Single-Phase Fault

   The A phase CT will have an excitation voltage of

$$\begin{aligned} V_{exA} &= I_{Asec}(Z_{CT} + 2Z_{lead} + Z_{phase} + Z_{residual}) \\ &= 75\text{ A}(0.3\ \Omega + 2 \cdot 0.4\ \Omega + 1.2\ \Omega + 1.8\ \Omega) \\ &= 307\text{ V} \end{aligned}$$

- The impedances are primarily resistive, and phase angle is often neglected in the voltage calculations. The impedances can be determined by tracing the path of the current through the CT secondary circuit.

Example 5: CT Performance

2. Two-Phase Fault

The B phase CT will have an excitation voltage of

$$\begin{aligned} V_{exB} &= I_{Bsec}(Z_{CT} + Z_{lead} + Z_{phase}) \\ &= 41.7\text{ A}(0.3\ \Omega + 0.4\ \Omega + 1.2\ \Omega) \\ &= 79.2\text{ V} \end{aligned}$$

- The C phase CT will see a similar voltage. Note that the A phase CT will also see a significant voltage, although it is carrying no current.

3. Three-Phase Fault

$$\begin{aligned} V_{exA} &= I_{Asec}(Z_{CT} + Z_{lead} + Z_{phase}) \\ &= 66.7\text{ A}(0.3\ \Omega + 0.4\ \Omega + 1.2\ \Omega) \\ &= 126.7\text{ V} \end{aligned}$$

- The worst-case fault for this example is therefore the single-phase fault. It is clear that a CT with a saturation voltage of 200 V would experience substantial saturation for this fault. This saturation would cause a large reduction in the current delivered. In the other two cases, the CT remains unsaturated, so the CT will deliver the expected current at this voltage level.

Safety when working with CT’s

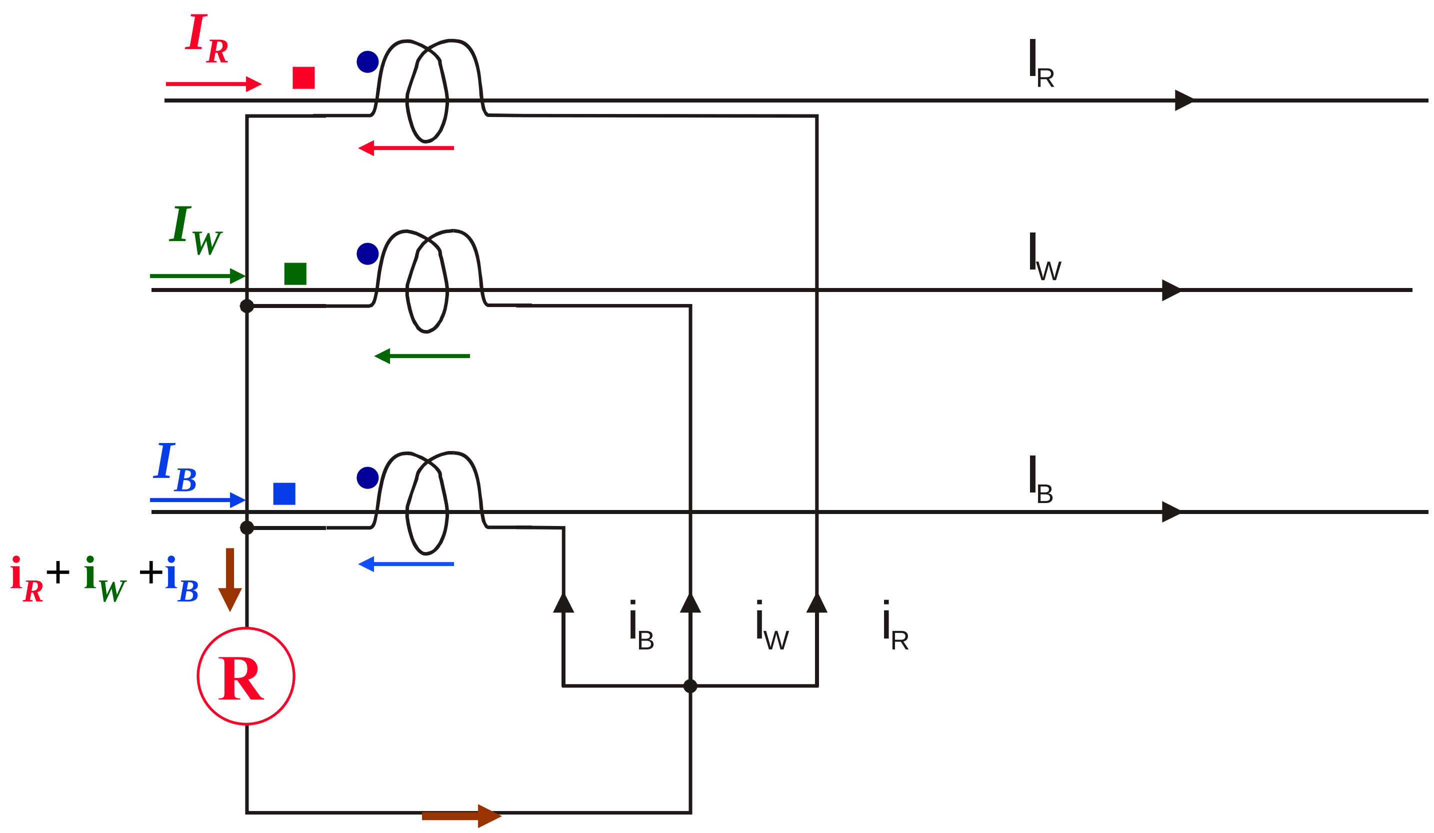
CAUTION:

When C.T.'s are in service they MUST have a continuous circuit connected across the secondary terminals. If the C.T. secondary is `open circuit' Whilst primary current is flowing, dangerously high voltages will appear across the C.T. secondary terminals.

Extreme care must be exercised when performing ‘on load' tests on C.T. circuits, to ensure that a C.T. is not inadvertently ‘open circuited'.

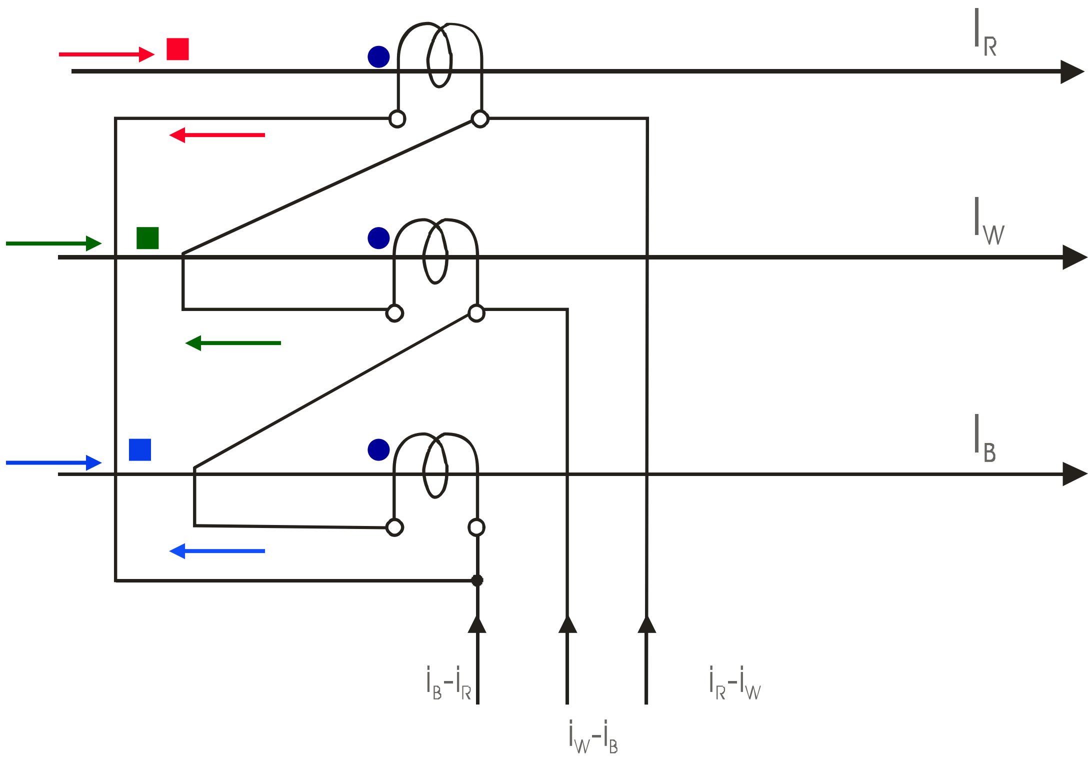
Connection of CT's in 3 phase systems

Most common connection is star (below) - residual current will spill through neutral and through relay **R** during faults



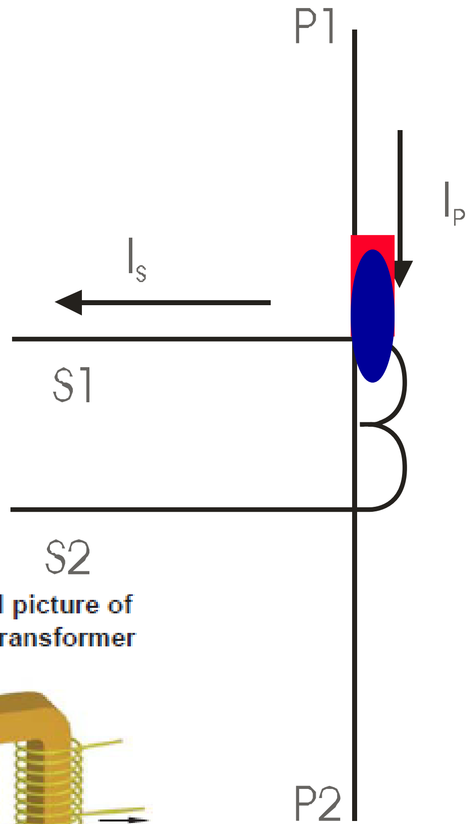
Connection of CT's in 3 phase systems

Delta connections are used when a phase shift with respect to the CT's on the other side of a ΔY transformer is required

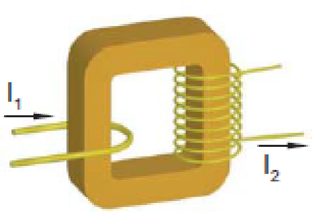


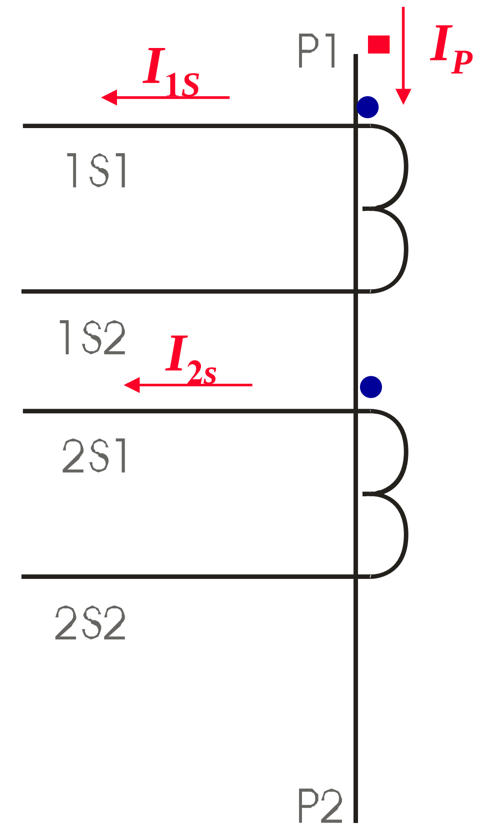
Terminal designations for CT's

IEC185 - terminals to be marked as follows: P1, S1 , C1 to all have same polarity e.g. See below

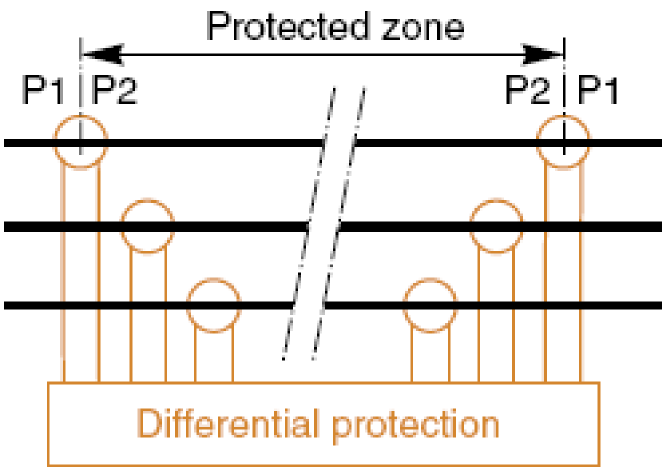


Conceptual picture of a Current Transformer

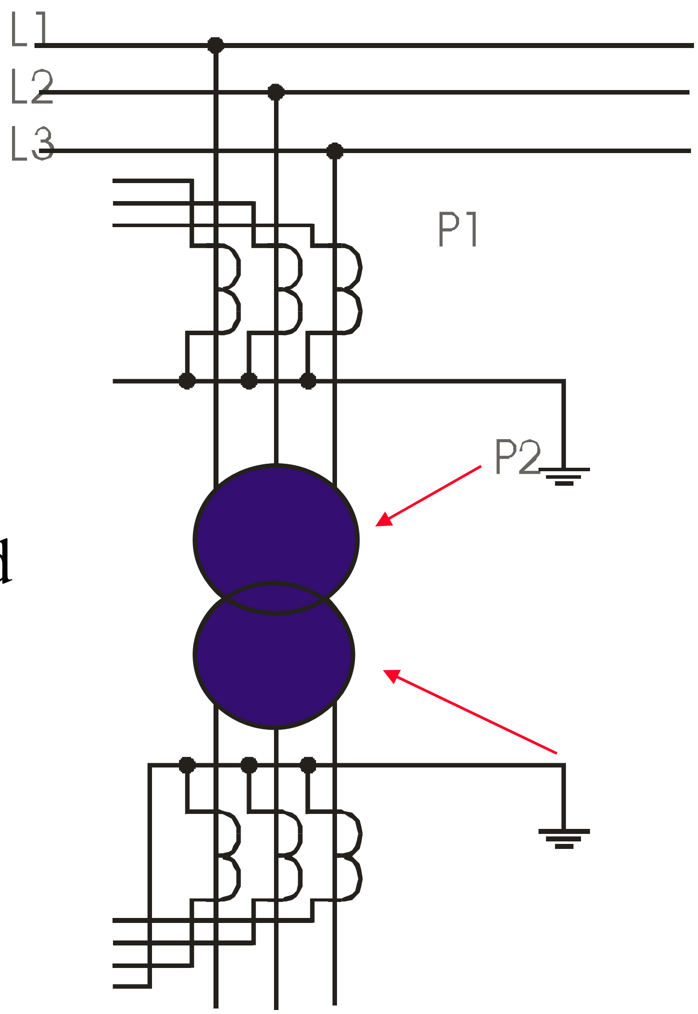




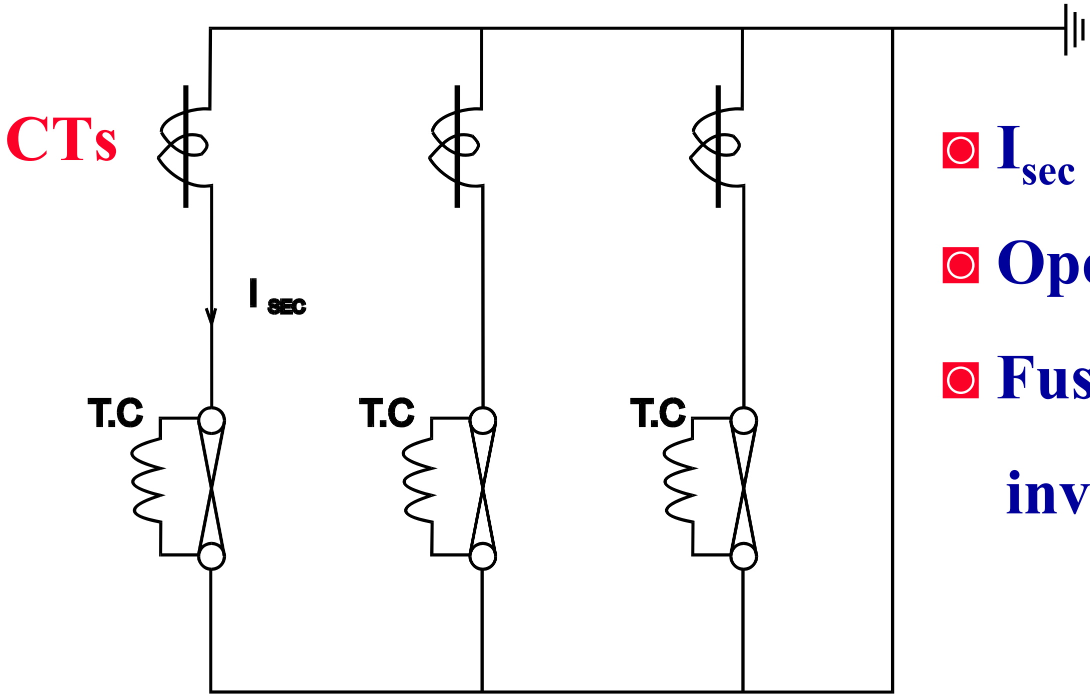
Earthing of CT's



Star and earth closest to protected equipment

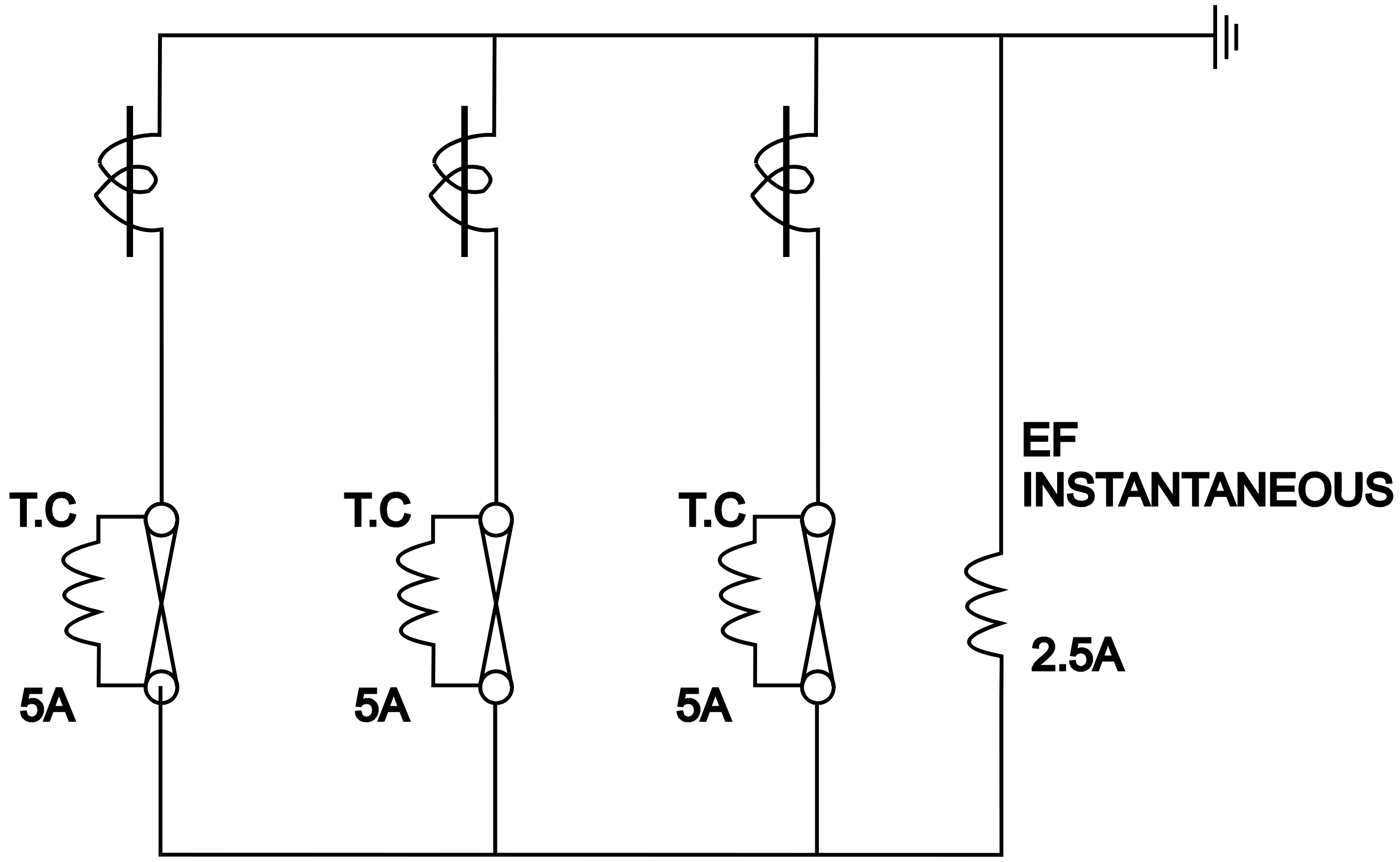


Application of CT's - Overcurrent

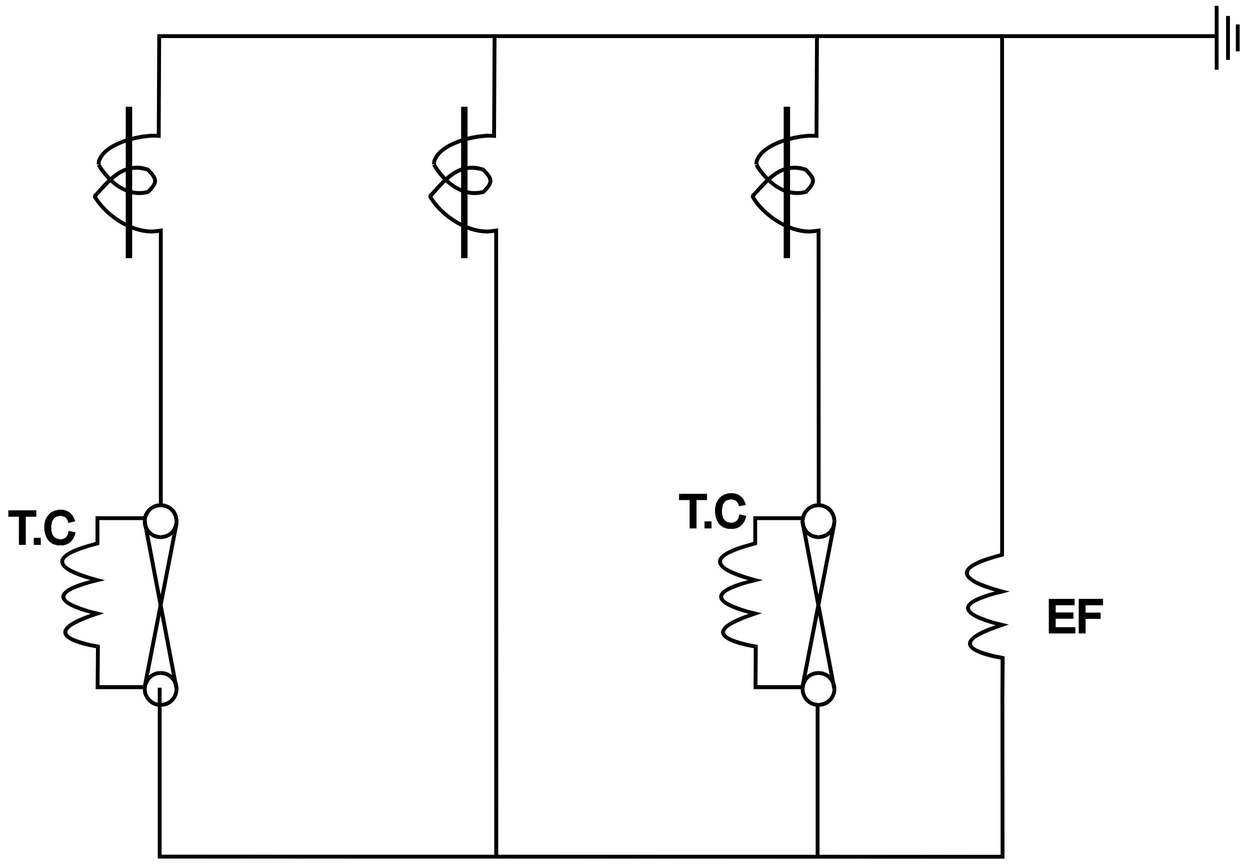


- I_{sec} blows fuse
- Operates trip coil (T.C.)
- Fuse characteristic is inverse

Overcurrent and Earth Fault

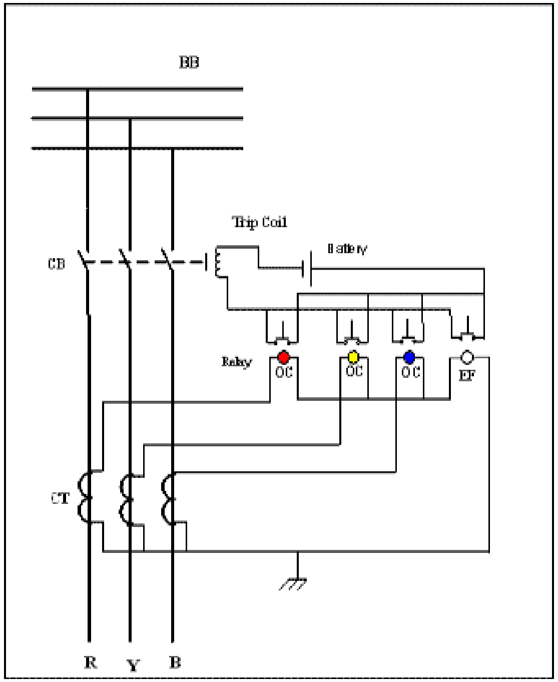


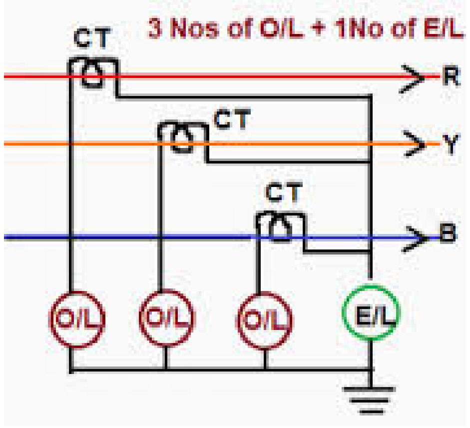
A more economical Arrangement

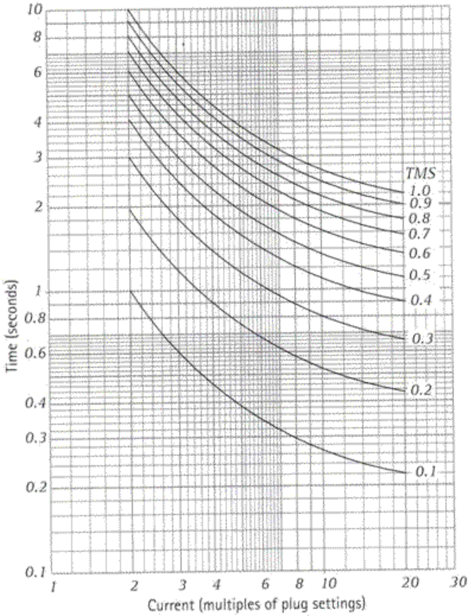


Topic 1: Power System Protection

1.4: Prt 1
IDMT Overcurrent Protection





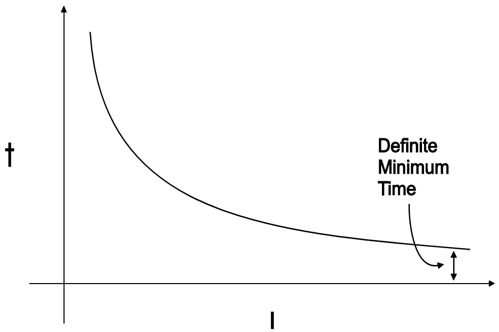


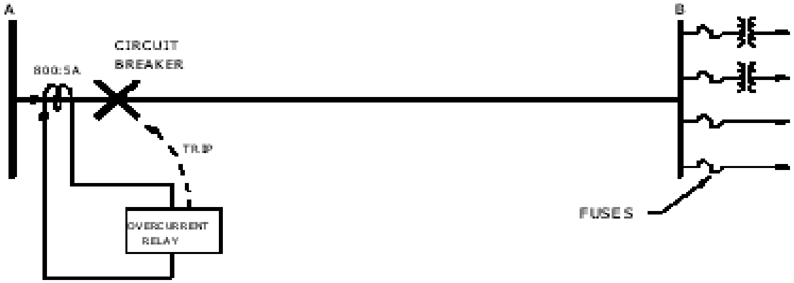
Over-Current Protection

- OC protection is that protection in which the relay picks up when the magnitude of current exceeds the pickup level.
- The basic element in OC protection is an OC relay.

- The OC relays are connected to the system, normally by means of **CT's**
- HRC fuses, drop out fuses, etc. are used in low voltage medium voltage and high voltage distribution systems, generally up to 11 kV.

- Thermal relays are used widely for over-current protection





Primary Requirements of OC Protection

- OC protection includes the protection from overloads which is generally provided by thermal relays.
- OC protection includes short-circuit protection. SC currents are generally several times (5 to 20) full load current. Hence fast fault clearance is always desirable on short-circuits

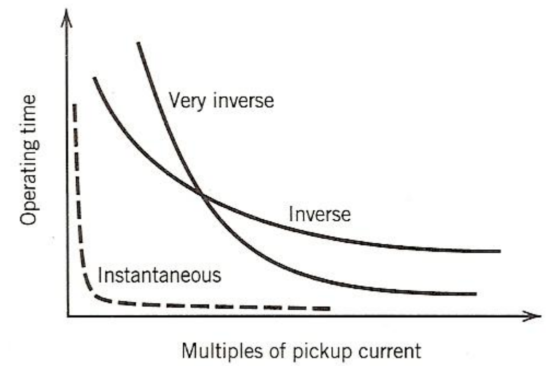
- OC protection should not operate for starting currents, permissible over-current, and current surges. To achieve this, the time delay is provided (in case of inverse relays). If time delay cannot be permitted, high-set instantaneous relaying is used.
- The protection should be coordinated with neighboring over-current protections so as to discriminate.

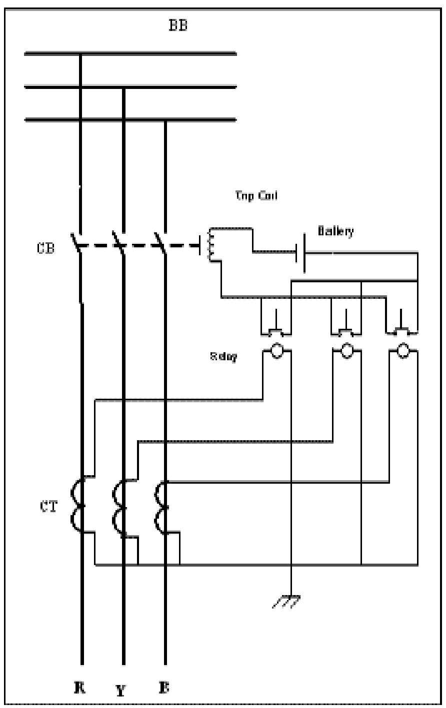
Applications of OC Protection

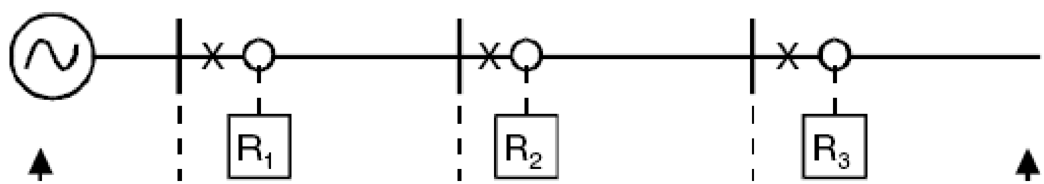
Line Protection

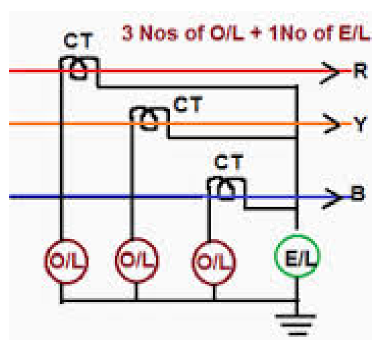
The lines (feeders) can be protected by

1. Instantaneous over-current relays.
2. Definite time Over-current relays
3. Inverse time over-current relays.
4. Directional over-current relay.





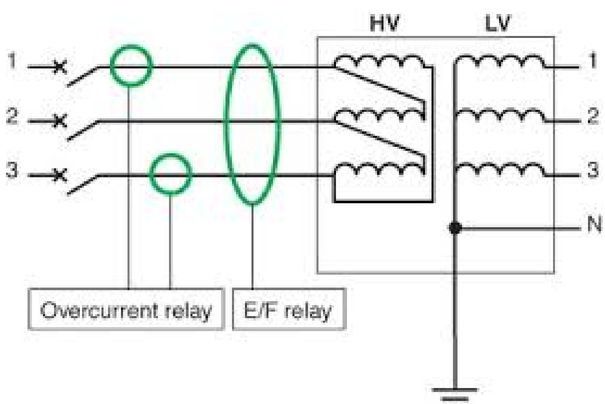


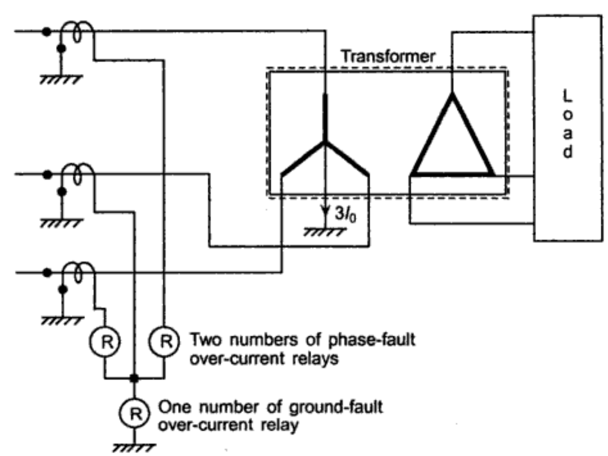


Applications of OC Protection

➢ Transformer Protection

- Transformers are provided with OC protection against faults, only, when the cost of differential relaying cannot be justified.
- OC relays are provided in addition to differential relays to take care of through faults. Temperature indicators and alarms are always provided for large transformers.
- Small transformers below 500 kVA installed in distribution system are generally protected by fuses, as the cost of relays plus circuit-breakers is not generally justified.

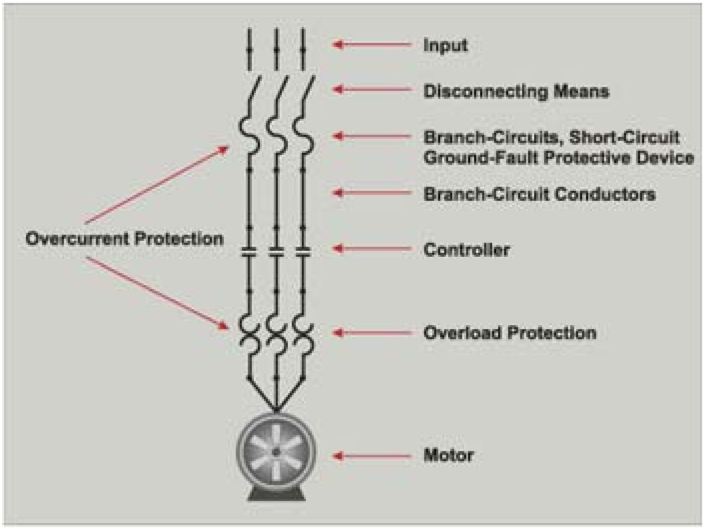


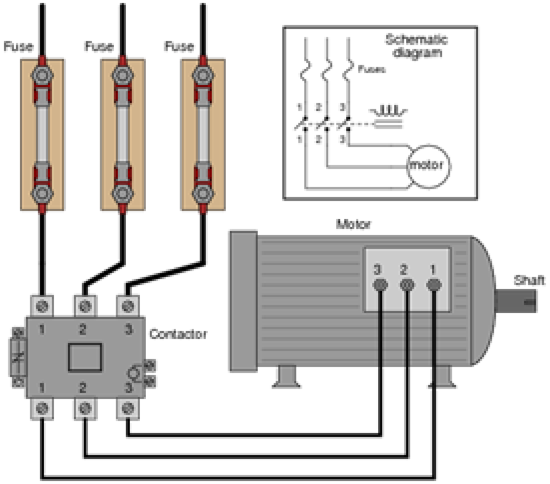


Applications of OC Protection

➢ Motor Protection

- OC protection is the basic type of protection used against overloads and short-circuits in stator windings of motors. Inverse time and instantaneous phase and ground OC relays can be employed for motors above 1200 H.P.
- For small/medium size motors where cost of CT's and protective relays is not economically justified, thermal relays and HRC fuses are employed, thermal relays used for overload protection and HRC fuses for short-circuit protection.

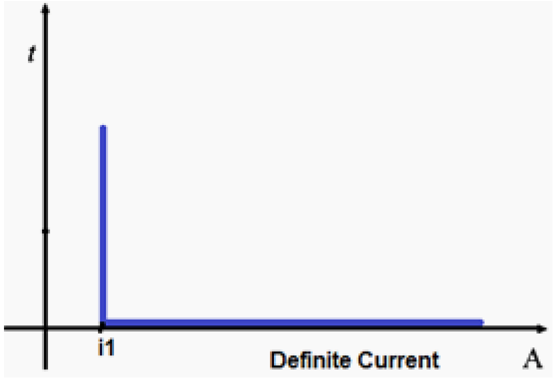


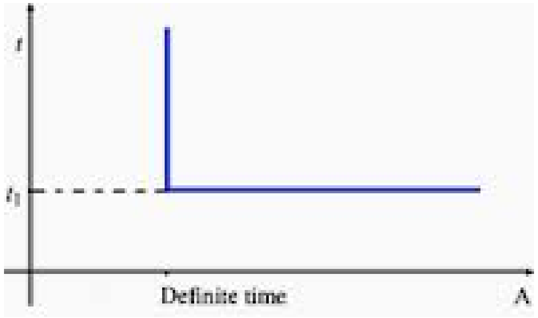


Types of Overcurrent Relays

a. Instantaneous Overcurrent Relays.

These relays operate, or *pick-up* at a specific value of current,
with no intentional time delay.The *pick-up* setting is usually
adjustable by means of a dial, or by plug settings.

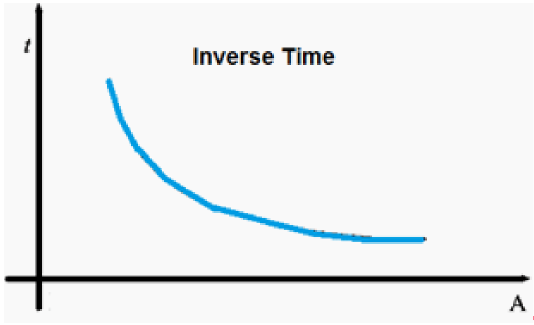




b. Timed Overcurrent Relays.

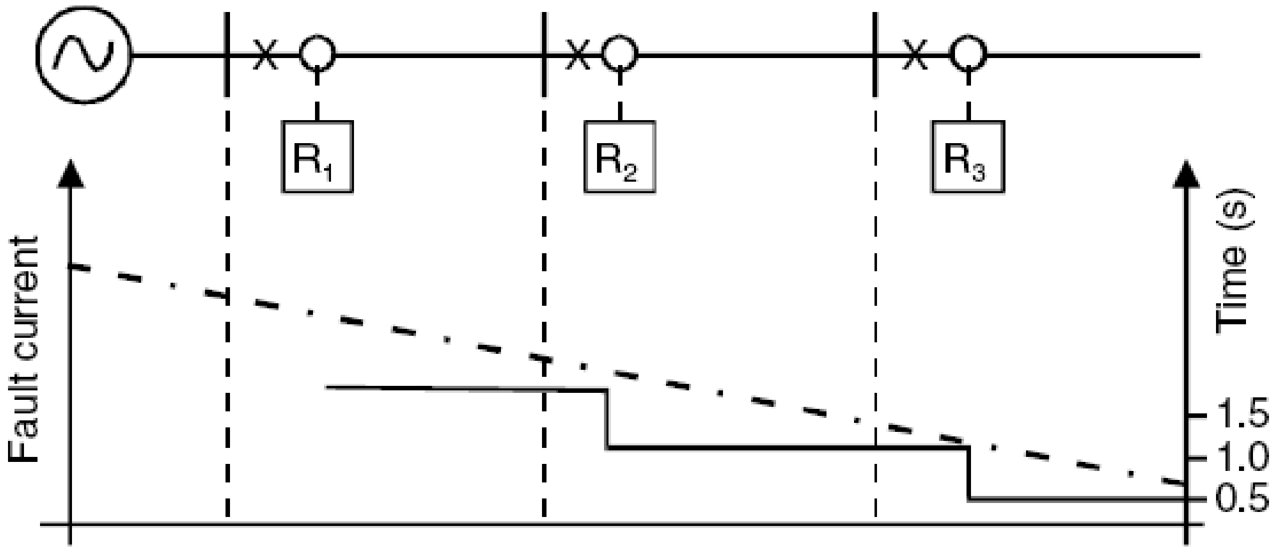
Two types:

1. Definite Time Lag
2. IDMT Relay
   (Inverse Definite Minimum Time)



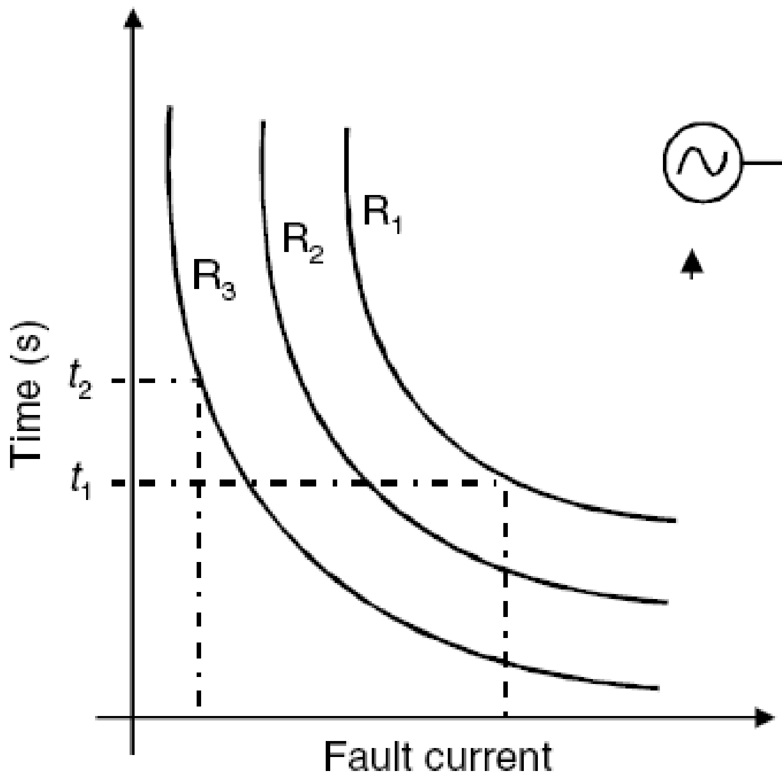
Definite Time Lag - O/C Relay

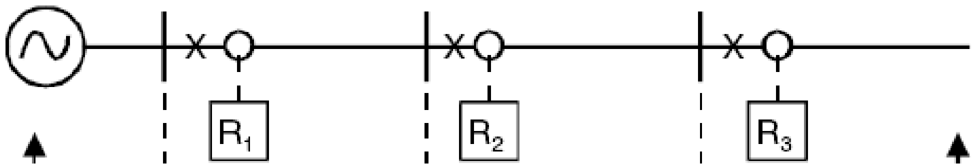
- For the first option, the relays are graded using a definite time interval of approximately 0.5 s. The relay R3 at the extremity of the network is set to operate in the fastest possible time, whilst its upstream relay R2 is set 0.5 s higher. Relay operating times increase sequentially at 0.5 s intervals on each section moving back towards the source as shown
- The problem with this philosophy is, the closer the fault to the source the higher the fault current, the slower the clearing time – exactly the opposite to what we should be trying to achieve.



IDMT - O/C Relay

- On the other hand, inverse curves as shown operate faster at higher fault currents and slower at the lower fault currents, thereby offering us the features that we desire. This explains why the **IDMT** philosophy has become standard practice throughout many countries over the years.





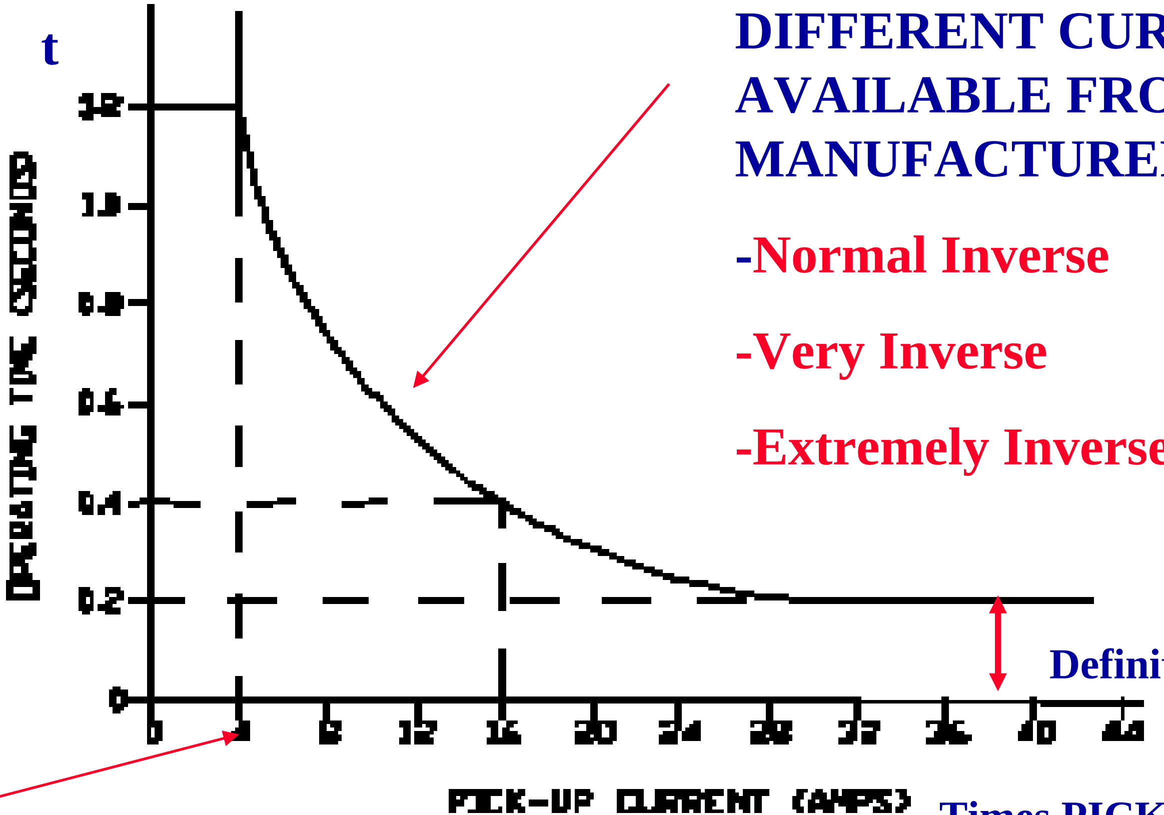
Time $$t \propto \frac{1}{I^2}$$

This gives an inverse characteristic. (the higher the current - the shorter the rotating time)

OC IDMT Relay

Inverse Definite Minimum Time C/C

t



DIFFERENT CURVES AVAILABLE FROM MANUFACTURERS

- -Normal Inverse
- -Very Inverse
- -Extremely Inverse

Definite Minimum Time

Times PICK-UP CURRENT

DEFINITE MINIMUM

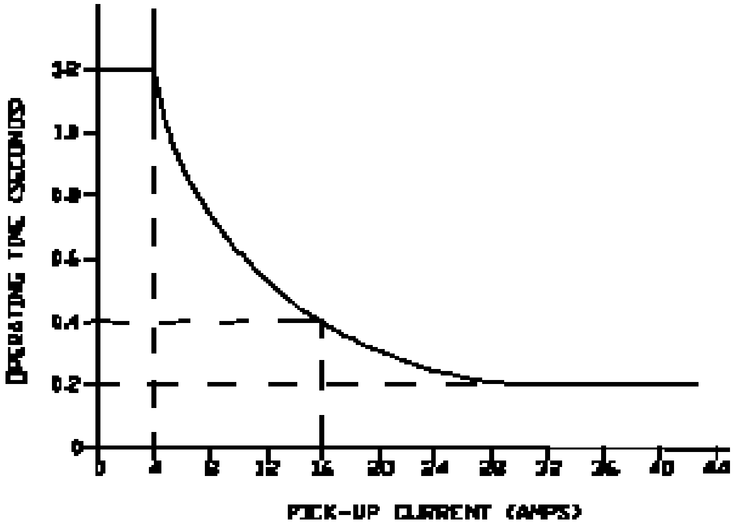
PICK-UP CURRENT

I

IDMT Relay

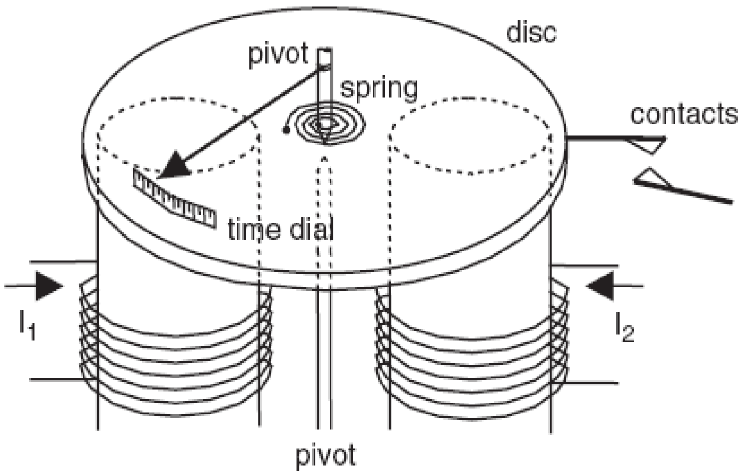
The relay characteristic is such that for very high fault currents the relay will operate in it's ***definite minimum time of 0.2*** seconds. For lower values of fault current the operating time is longer.

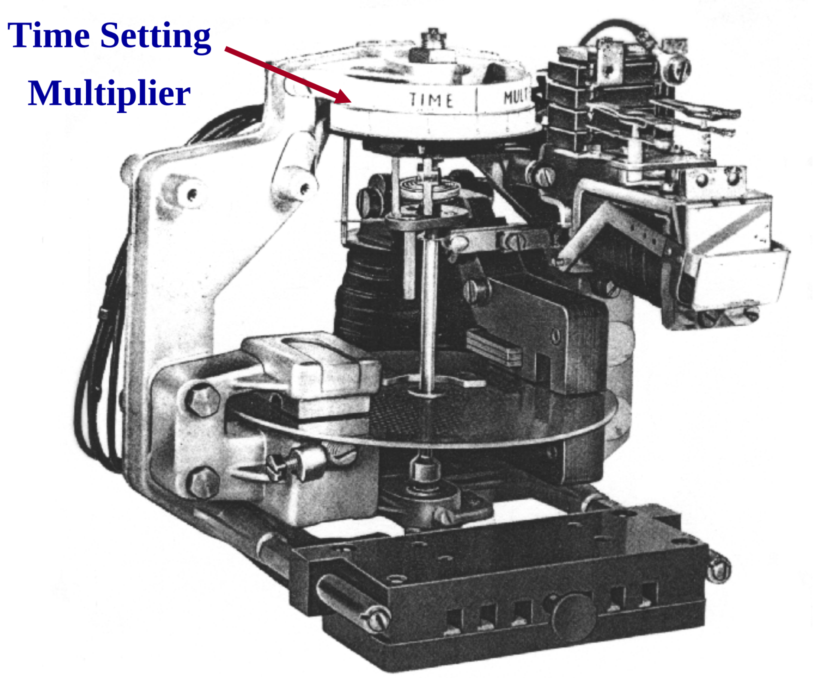
For example, at a relay current of 16 Amps, the operating time is 0.4 seconds. The relay has a ***definite minimum pick-up*** current of 4 Amps. This minimum ***pick-up*** current must, of course, be greater than the maximum load on the feeder.

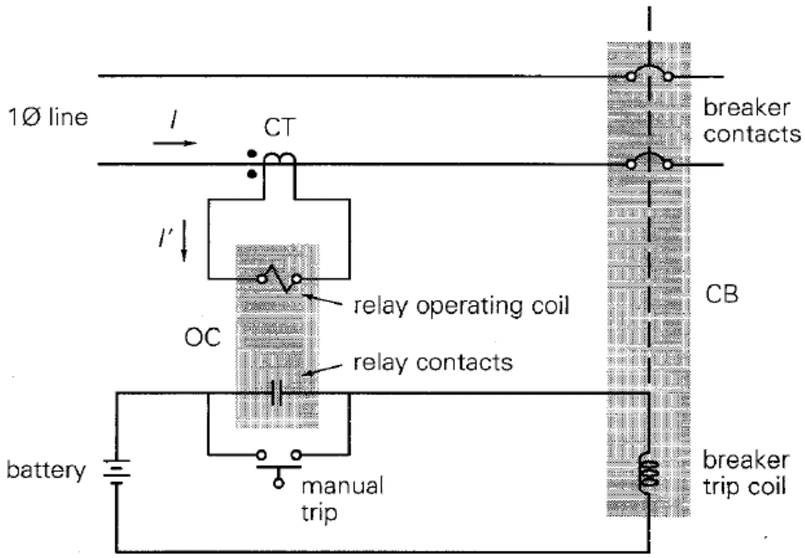


Overcurrent IDMT Relays

- The electro-mechanical version of the **IDMT** relay has an induction disc. The disc must rotate through a definite sector before the tripping contacts are closed.

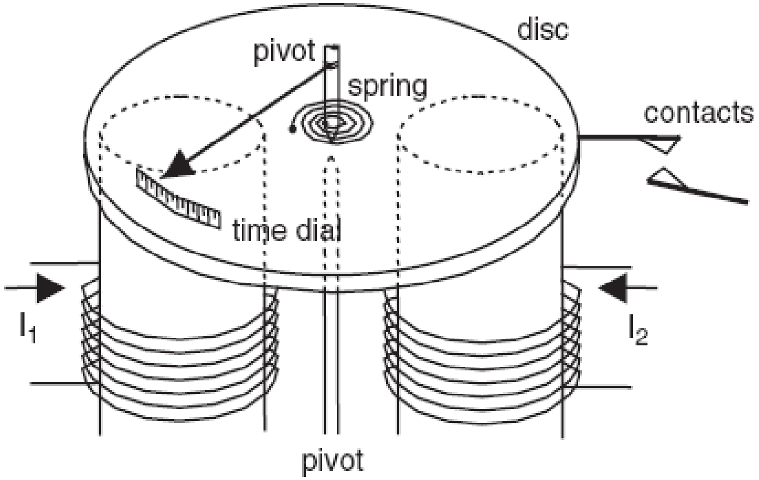






Adjustments of OC IDMT Relay

a) The ***time multiplier setting***:
This adjusts the operating time at a given ***multiple of setting current***, by altering by means of the torsion head, the distance that the disc has to travel before contact is made.



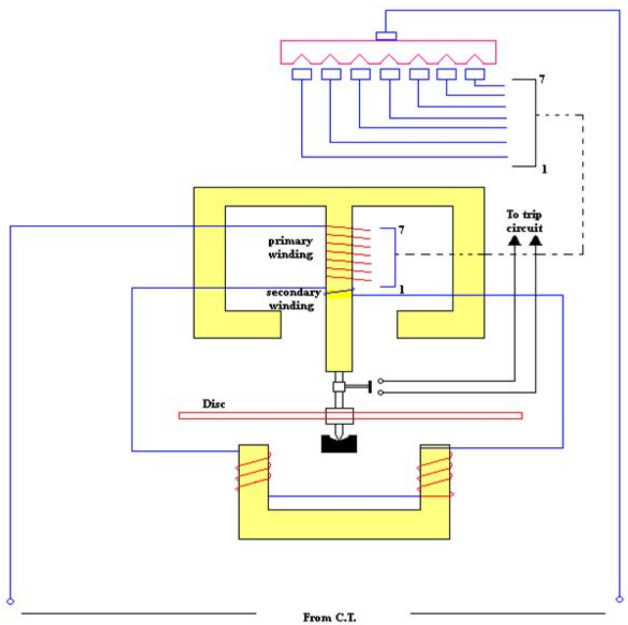
This dial rotates the disc and its accompanying moving contact closer to the fixed contact, thereby reducing the amount of distance to be traveled by the moving contact, hence speeding up the tripping time of the relay.

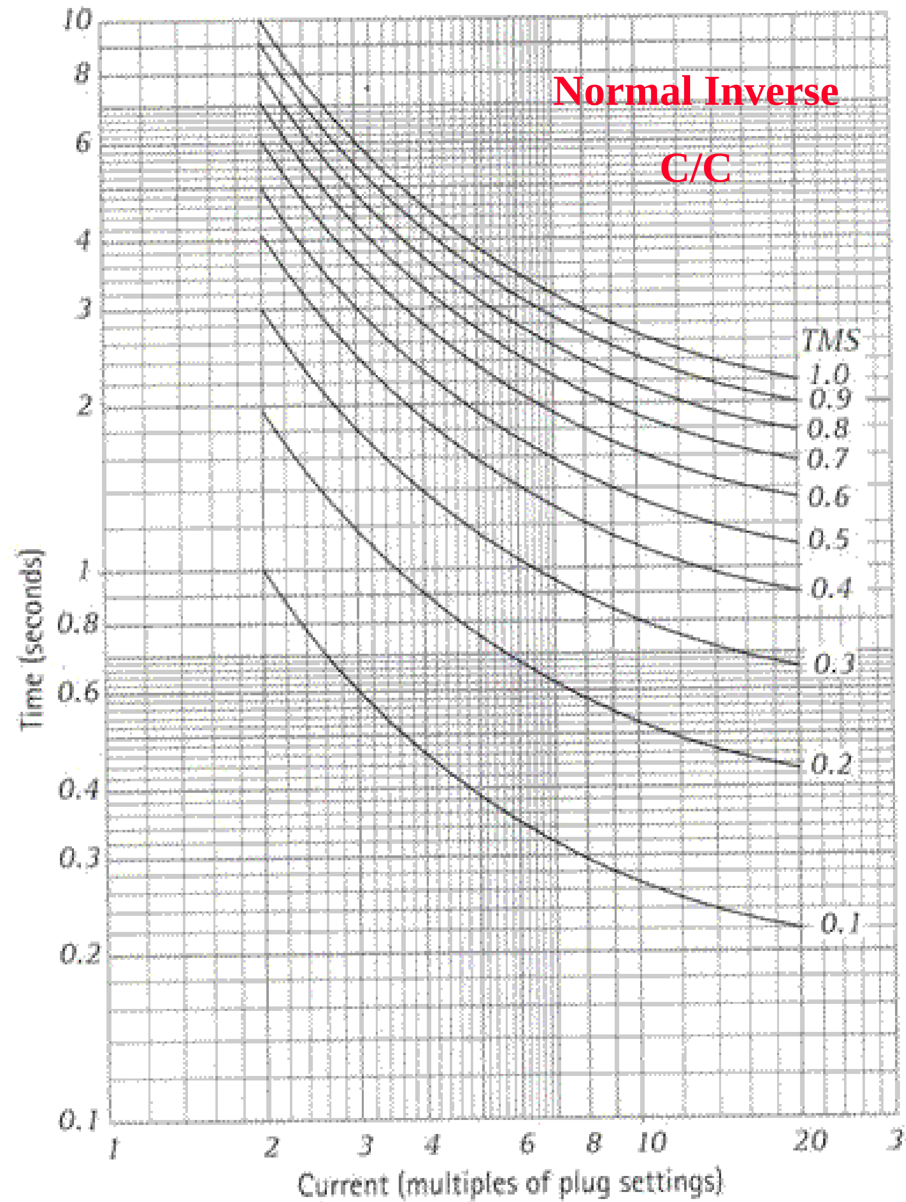
This has the effect of moving the inverse curve down the axis as shown below.

TM setting	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Degrees of travel before contacts operate	0°	18°	36°	54°	72°	90°	108°	126°	144°	162°	180°

OC IDMT Relay

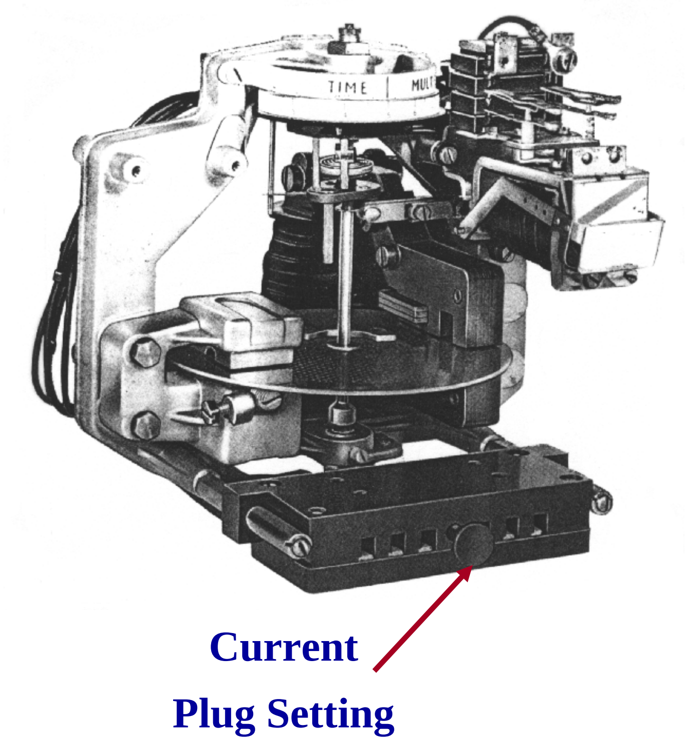
This gives the relay a very wide range of setting characteristics, and allows the relay setting to be coordinated with other protection devices, such as fuses, on adjacent power system elements

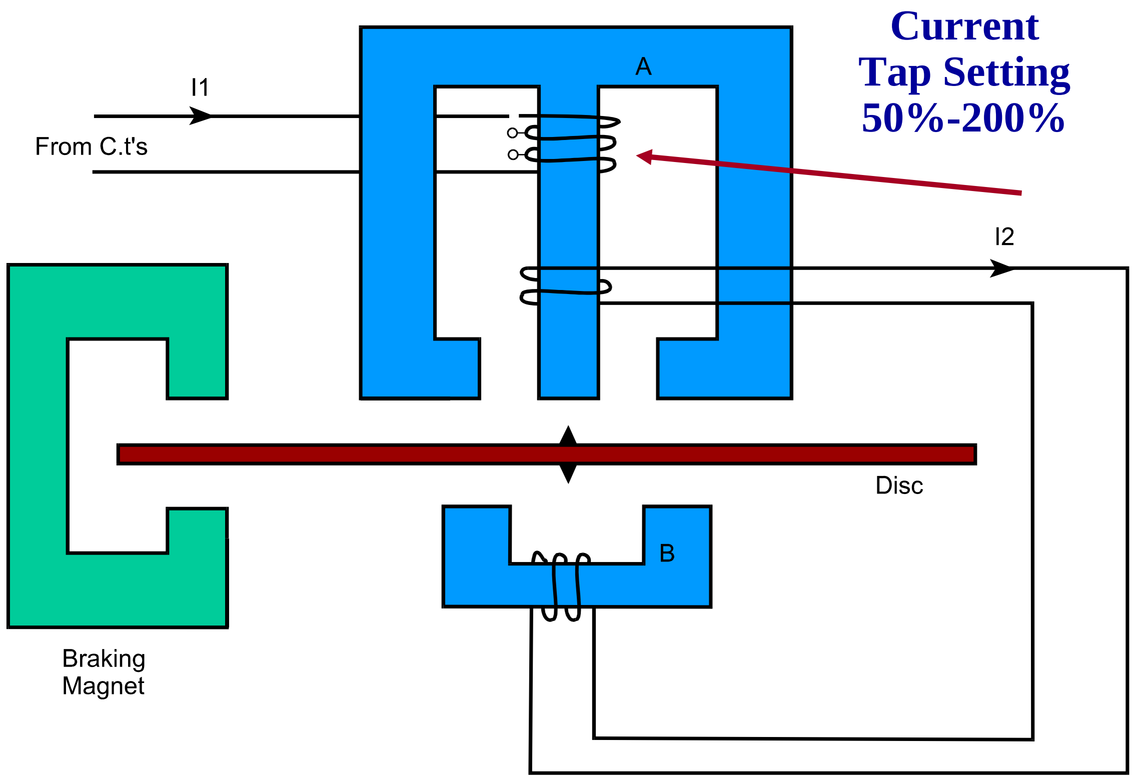




Adjustments of OC IDMT Relay

a) The ***current pick-up*** or ***plug setting***: This adjusts the setting current by means of a plug bridge, which varies the effective turns on the upper electromagnet.

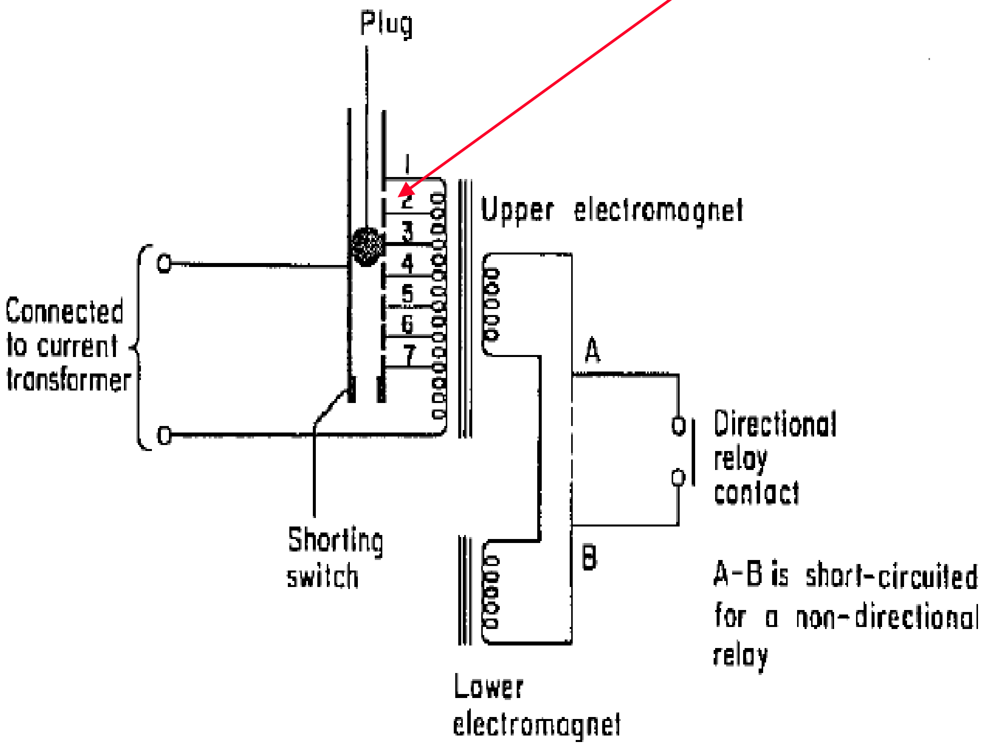


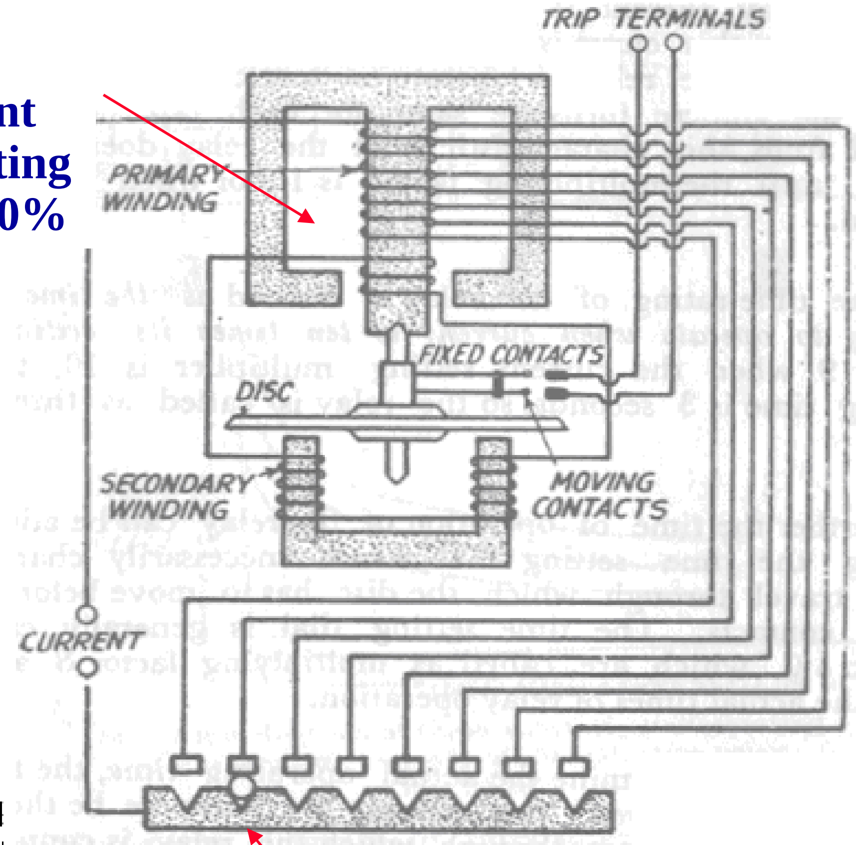


This setting determines the level of current at which the relay will start or pick-up

OC IDMT Relays

Current Tap Setting 50%-200%

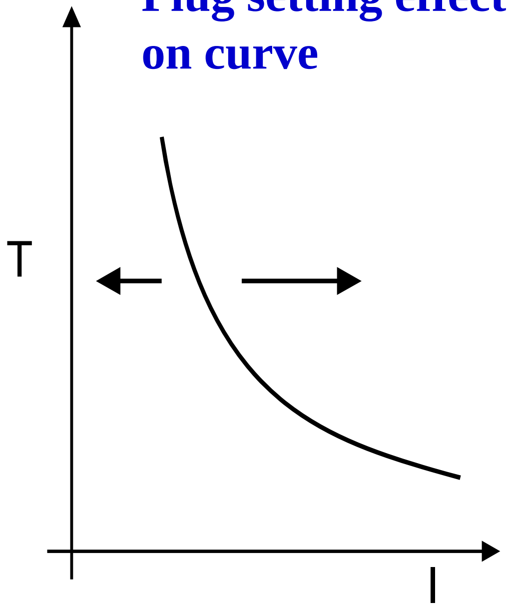




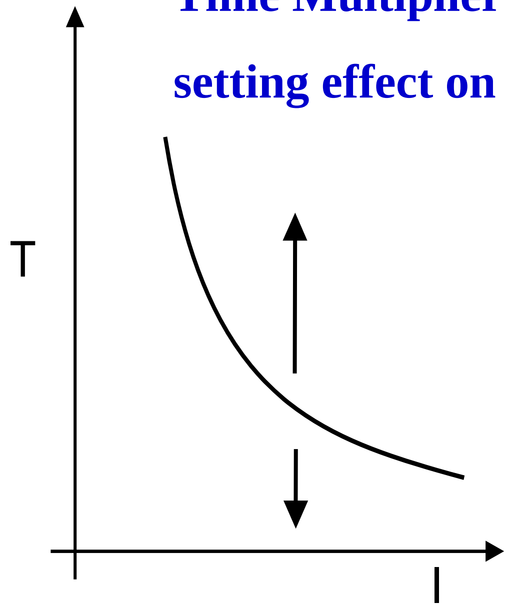
Plug Bridge

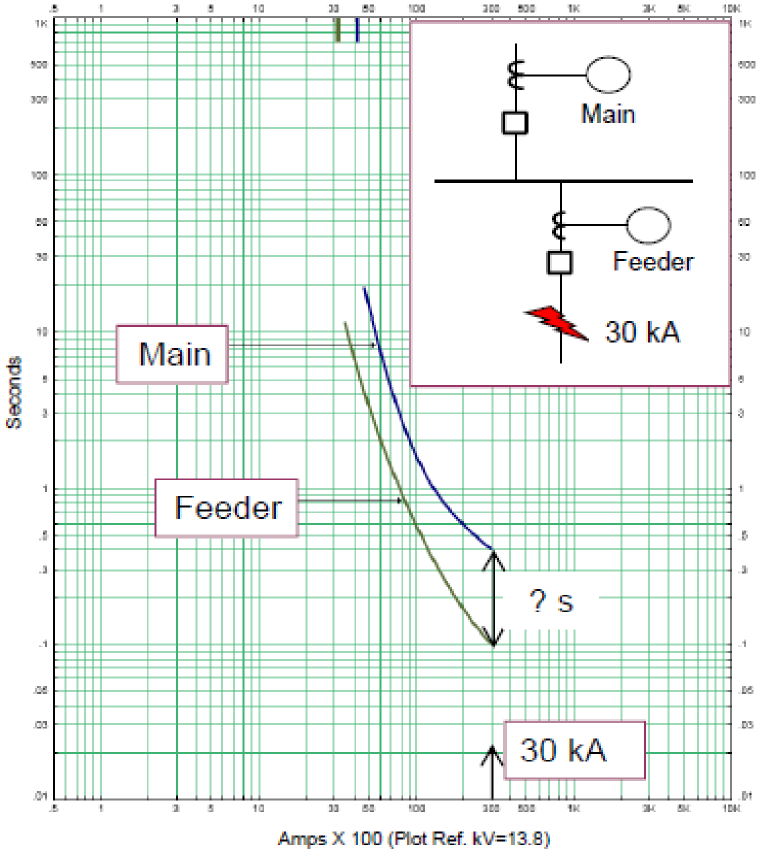
Effect of Settings and Coordination Curves

Plug setting effect on curve



Time Multiplier setting effect on curve





OC IDMT Relay

Percentage plug settings (Reyrolle)

Overcurrent:	50%	75%	100%	125%	150%	175%	200%
Earth fault:	20%	30%	40%	50%	60%	70%	80%
Or	10%	15%	20%	25%	30%	35%	40%

Current plug settings (GEC)–For 5 amp relay

Overcurrent:	1.5A	3.5A	5.0A	6.25A	7.5A	8.75A	10A
Earth fault:	1.0A	1.5A	2.0A	2.5A	3.0A	3.5A	4.0A
Or	0.5A	0.75A	1.0A	1.25A	1.5A	1.75A	2.A

Normally, the highest current tap is automatically selected when the plug is removed, so that adjustments can be made on load without open-circuiting the current transformer.

Magneto-motive-force*: *mmf = N.I

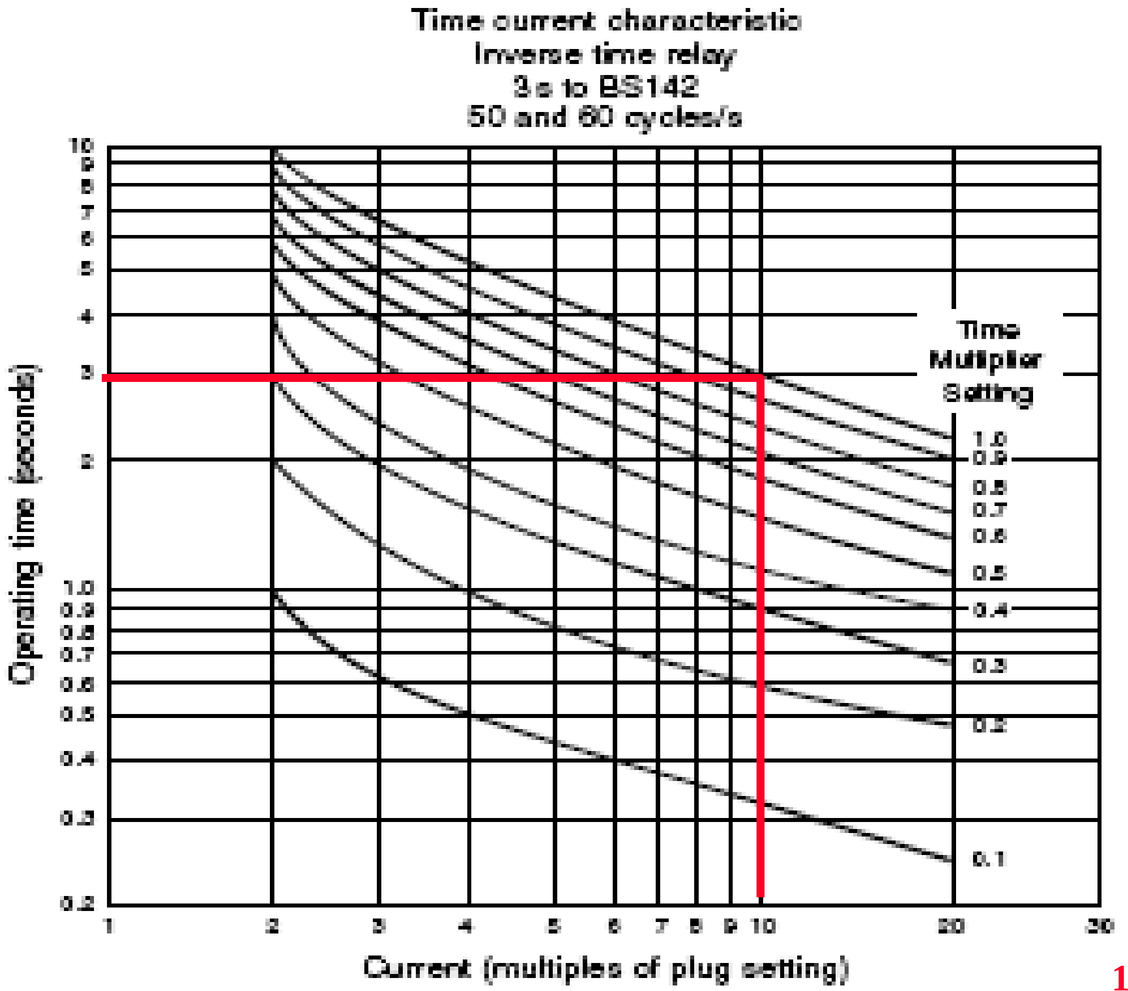
OC IDMT Relay

This curve shows the relay will operate in 3 seconds at 10 times the plug setting (with the time multiplier =1)

$$t = \frac{3.0}{\log PSM} \times TMS$$

$$t = \frac{3.0}{\log PSM}\bigg|_{TMS=1}$$

$$PSM = \frac{I_f}{I_{pickup}}$$



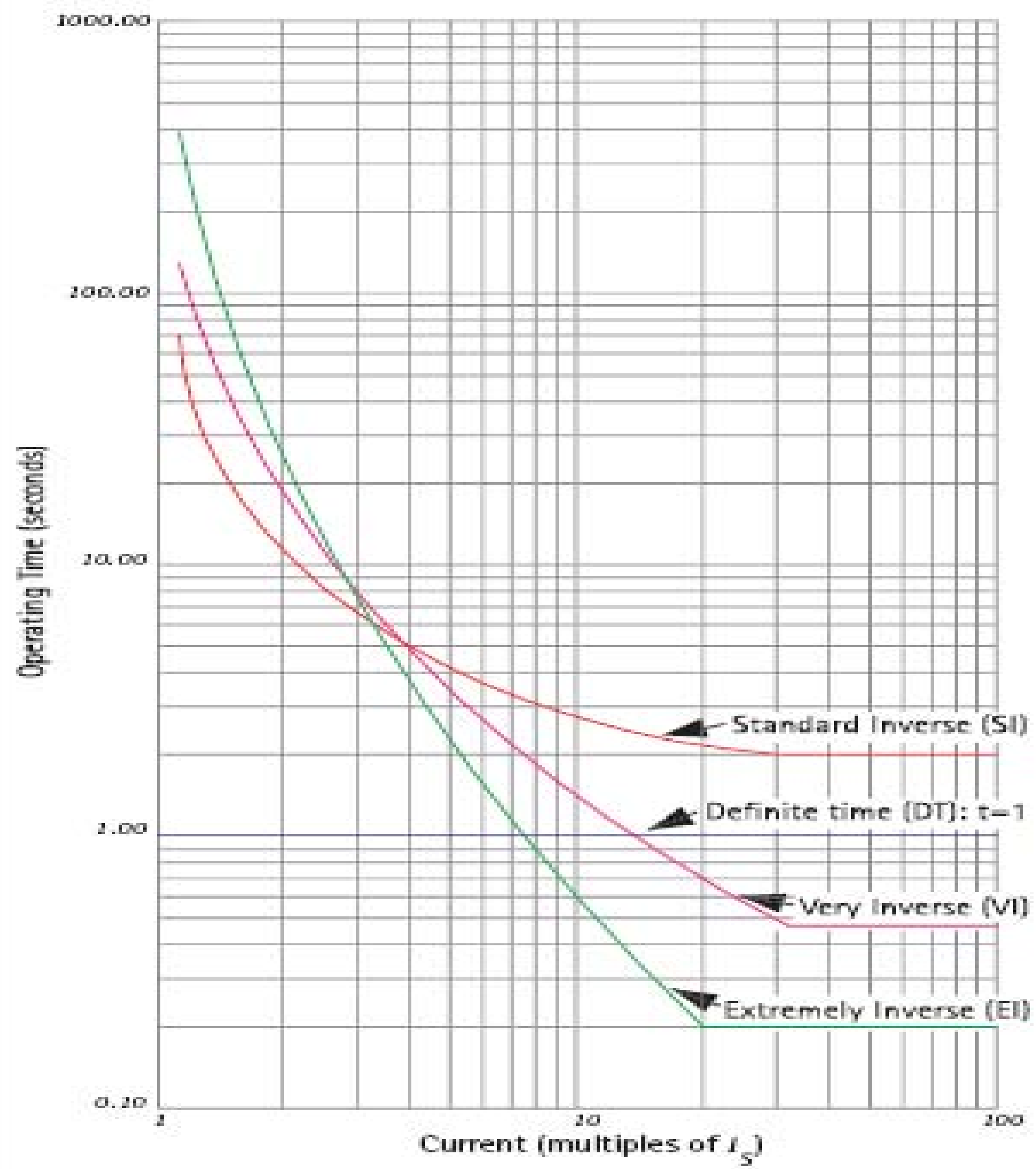
Different Curves

- The most common type used is:
  - ➔ **NORMAL INVERSE CURVE**.
- Characteristic shows a **3 second operation at 10 $\times$ the current plug setting**

  i.e. if the plug bridge is set at 1 A and when 10 A flows through, the relay will close its contacts after 3 seconds - sometimes called a 3/10 relay

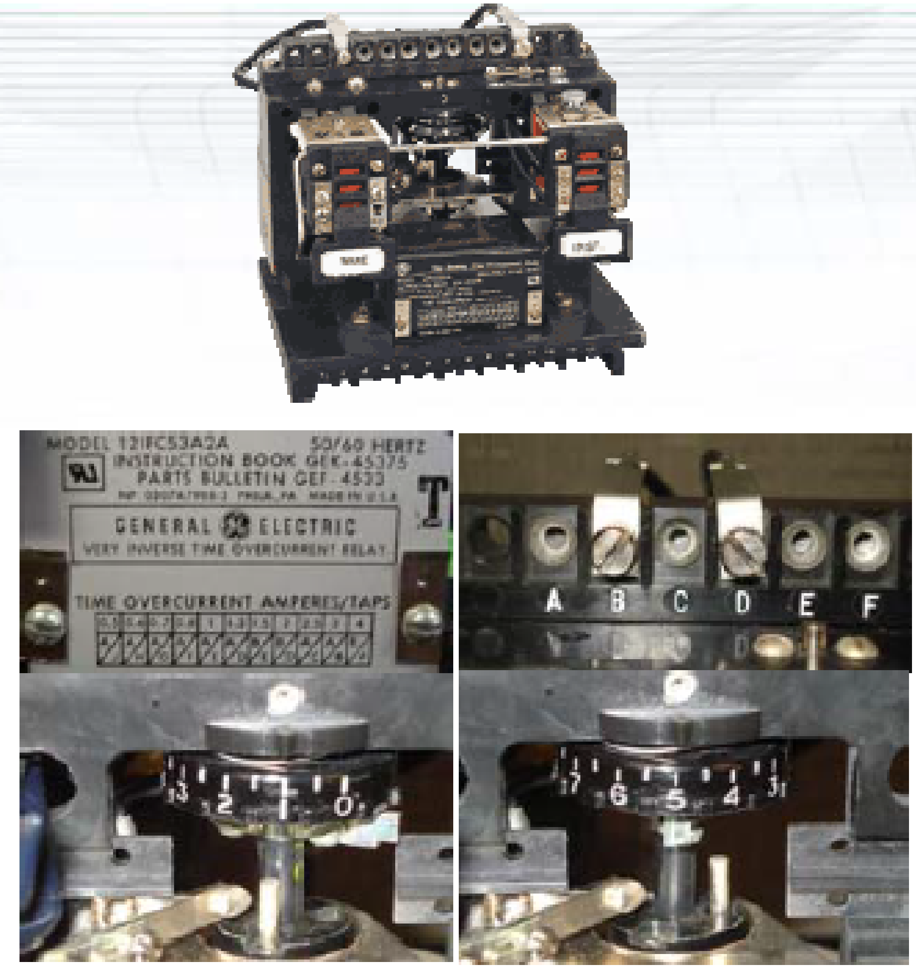
- Other characteristic curves are also available:
  - **Very Inverse**
  - **Extremely Inverse**

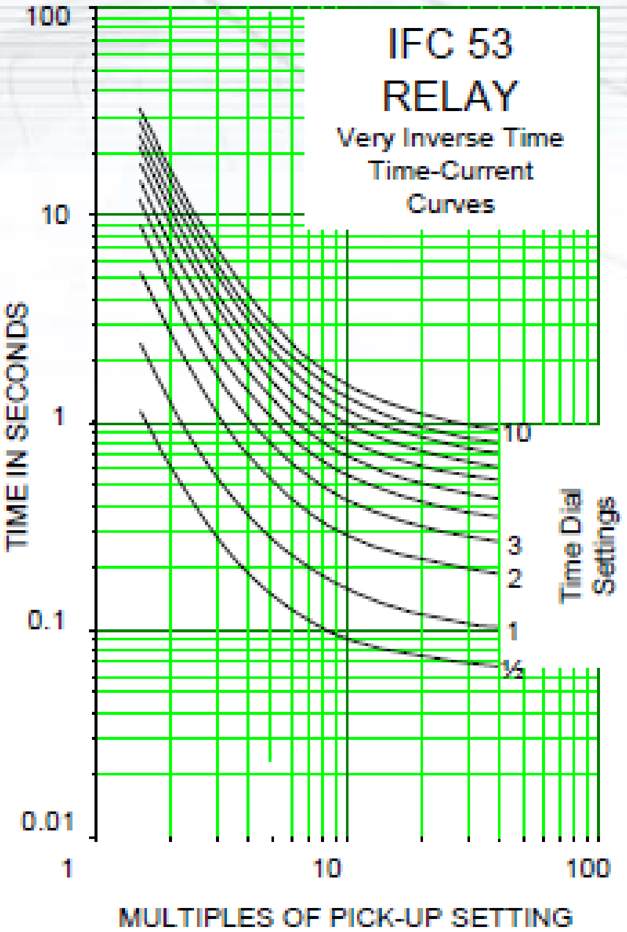
Relay characteristics to IEC 60255



(a) IEC 60255 characteristics ; TMS=1.0

IFC Relay (VI characteristics)



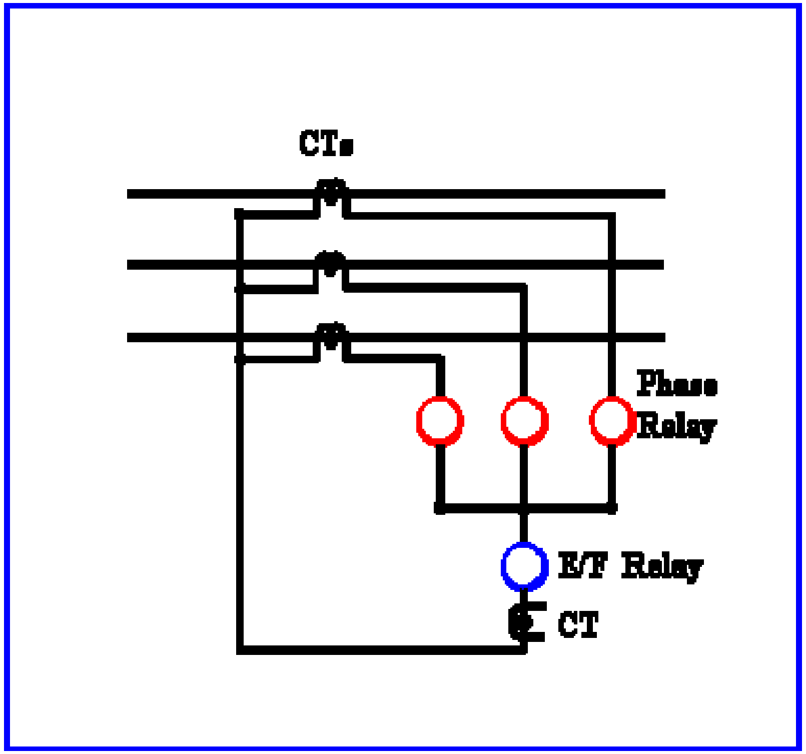


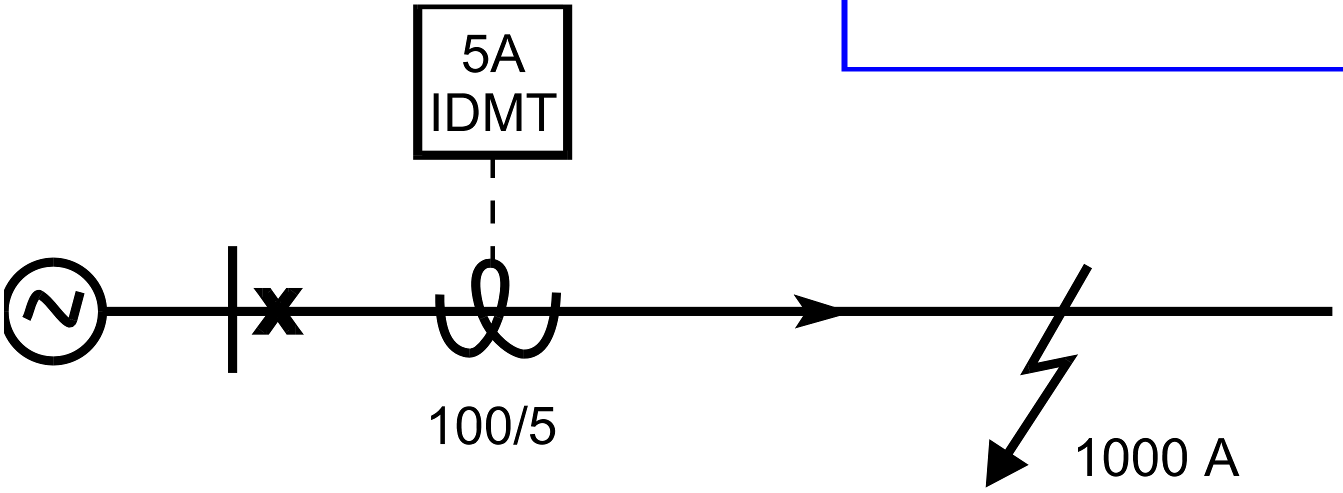
Example

Calculate:

- Plug setting (PS)
- Time multiplier setting (TMS)

for an IDMTL relay on the following network so that it will trip in 2.4 seconds.





Answer

- Fault Current = 1000 A
- CT Ratio = 100/5
- Hence current into relay = $\dfrac{I_f}{CTR}$ $= 1000 \times 5 / 100 = 50A$
- Choose plug setting (PS) of 5 A (100%) ➔PS=1.0
- Therefore current into relay as a multiple of plug setting

$$PSM = \frac{I_f}{Ipickup} = \frac{50}{5} = 10$$ times ➔PSM=10

- Referring to curves on the next page, read off Time Multiplier setting where 10 times and 2.4 seconds cross …namely 0.8. ➔TSM=0.8

➔Relay settings = Plug Setting PS = 5 A (PS = 100%)

= Time S ettingMultiplier (TSM = 0.8)

IDMT Relay

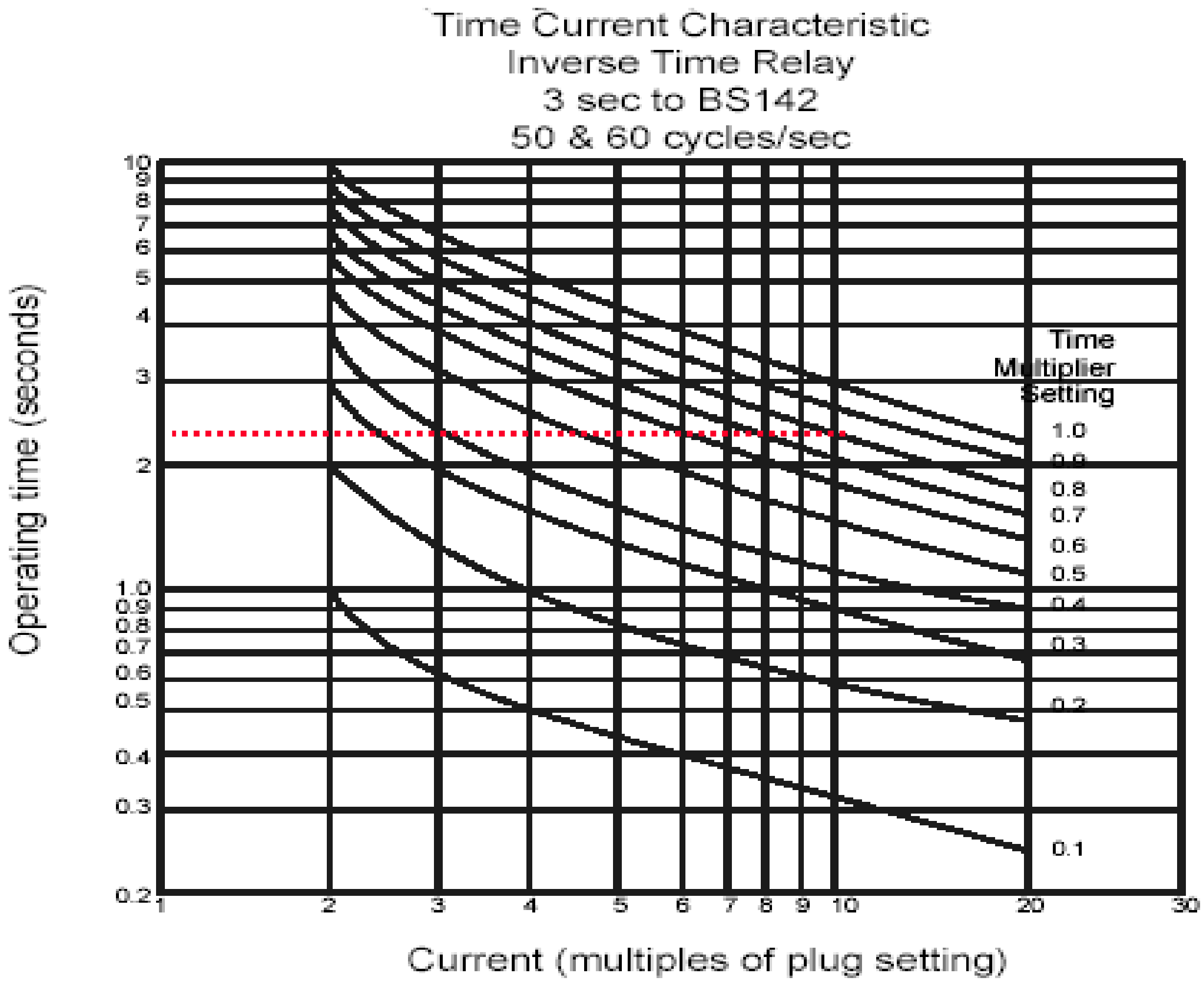


Figure 9.13
Multiples of plug setting current

IDMT Settings

- This technique is fine if the required setting falls exactly on the TM curve.
- If not….
- Go to the multiple of plug setting current and read off the seconds value corresponding to the 1.0 Time Multiplier curve. Then divide the desired time setting by this figure. This will give the exact Time Multiplier setting:
- Seconds figure at 10 times = 3 (TSM=1)
- Desired Setting = 2.4
- Therefore Time Setting Multiplier = 2.4/3 = 0.8

IDMT Relay

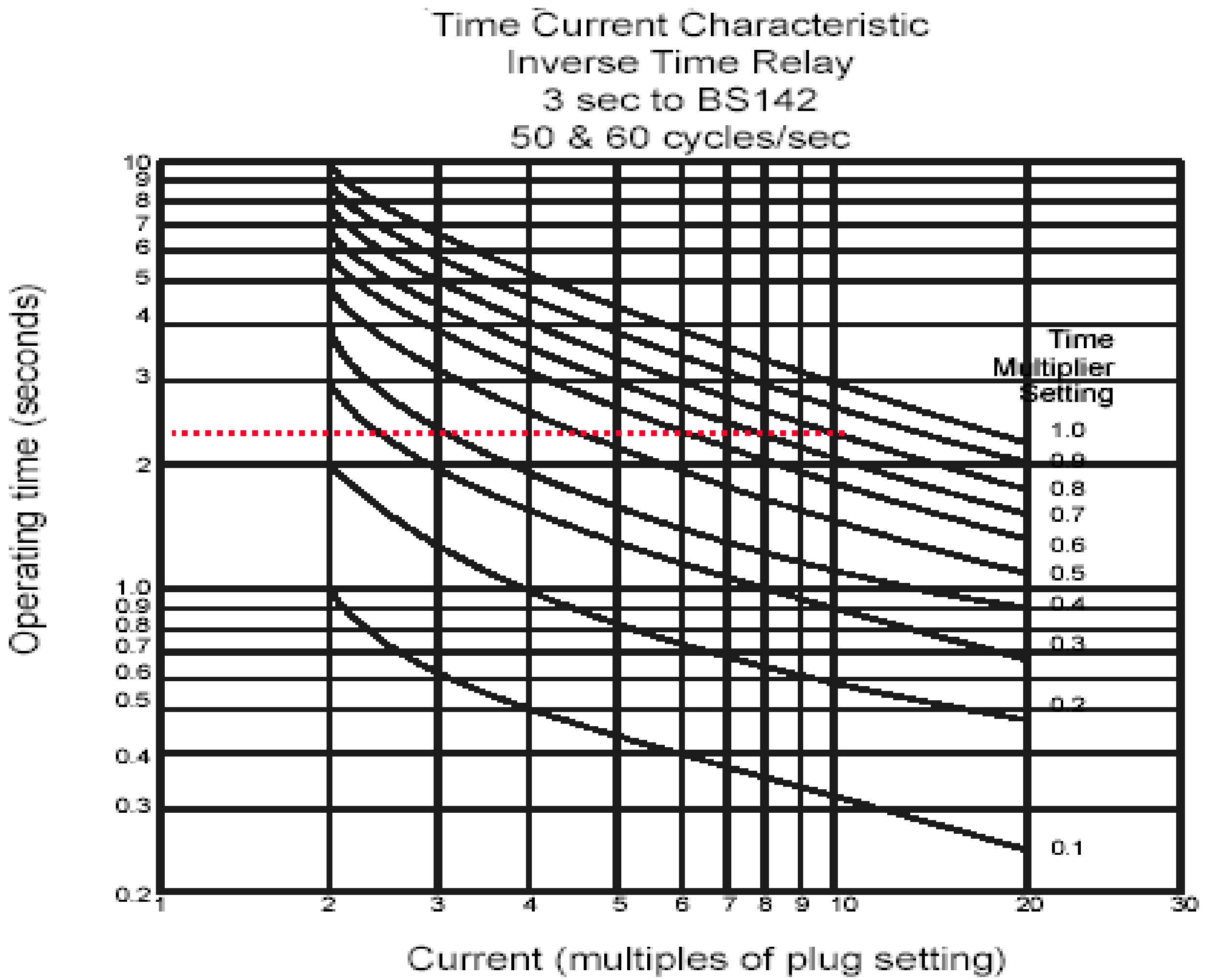


Figure 9.13
Multiples of plug setting current

IDMT Relay

- Alternatively, if the current plug setting is chosen as 125% (6.25 A), the PSM of the relay will be PSM=50/6.25=8 . The graph shows that 8 times plug setting to operate in 2.4 seconds, the time multiplier should be about 0.7.
- This technique is fine if the required setting falls exactly on the TM curve. However, if the desired setting falls between the curves, it is not easy to estimate the intermediate setting accurately as the scales of the graph are log/log. The following procedure is therefore recommended:
- Go to the multiple of plug setting current and read the seconds value corresponding to the 1.0 TM curve. Then divide the desired time setting by this figure. This will give the exact time multiplier setting:

Seconds value at 10 times =3 (at 8 times =3.4)

Desired time (setting) = 2.4

$\Rightarrow$TSM=2.4/3.0=0.8 or 2.4/3.4 =0.7 in the second case

Pickup Calculation - Electromechanical Relays

The relay should pick-up for current values above the motor FLC (~ 600 A).

For the IFC53, the available ampere-tap (AT) settings are 0.5, 0.6, 0.7, 0.8, 1, 1.2, 1.5, 2, 2.5, 3, & 4.

For this type of relay, the primary pickup current was calculated as:

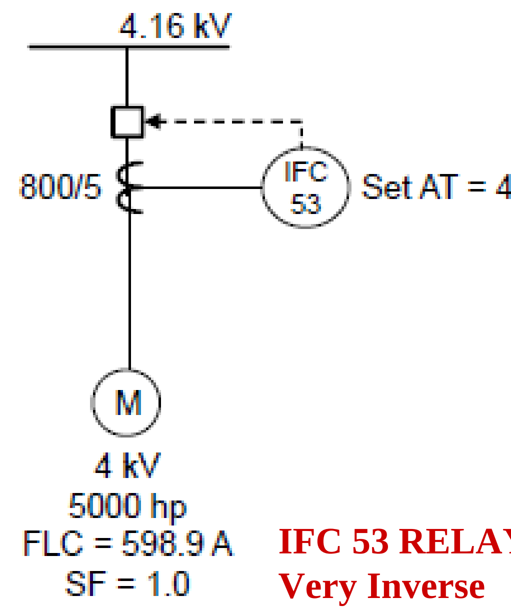
$I_{Pickup} = I_{FL}/CTR = 600/(800/5) = 3.75$

Set $I_{Pickup} = 4$ A (secondary)

$I_{Pickup} = 4 \times CTR$ (primary)

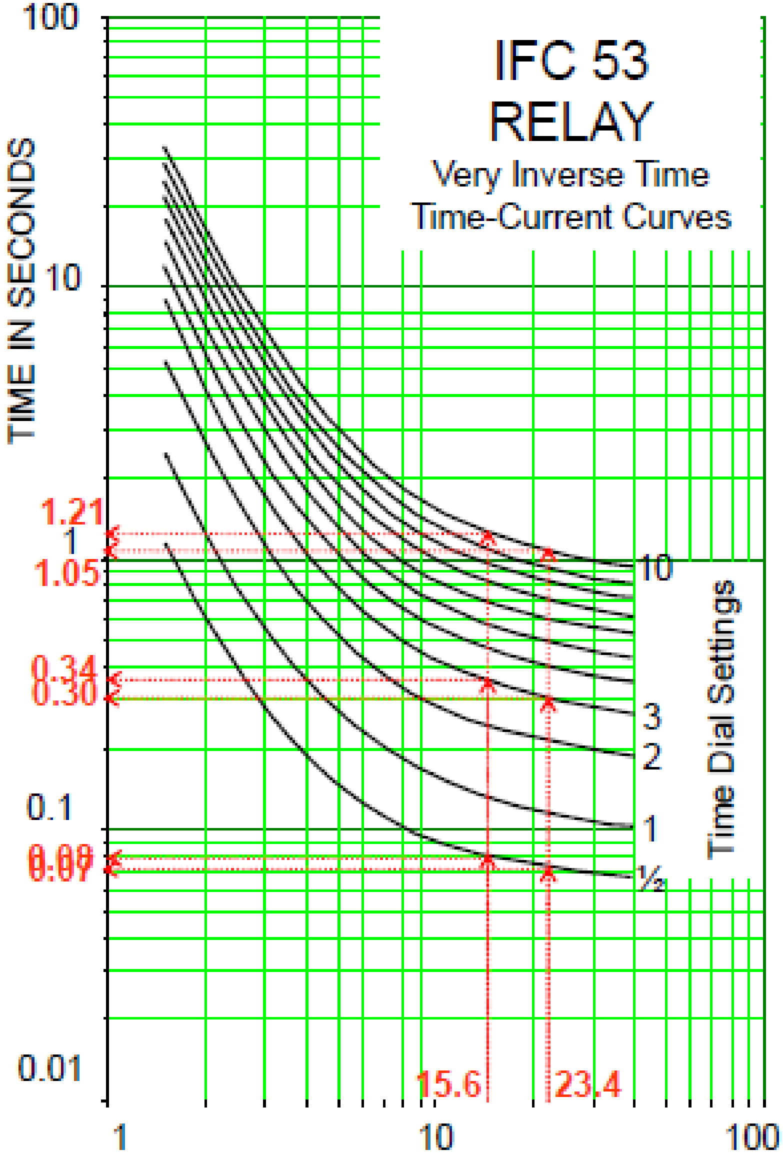
$= 4 \times (800/5)$

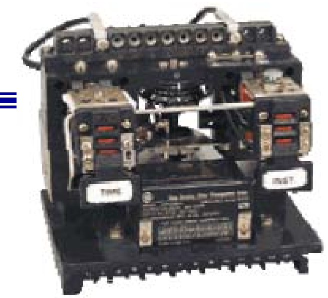
$= 640 \text{ A} > I_{FL}$

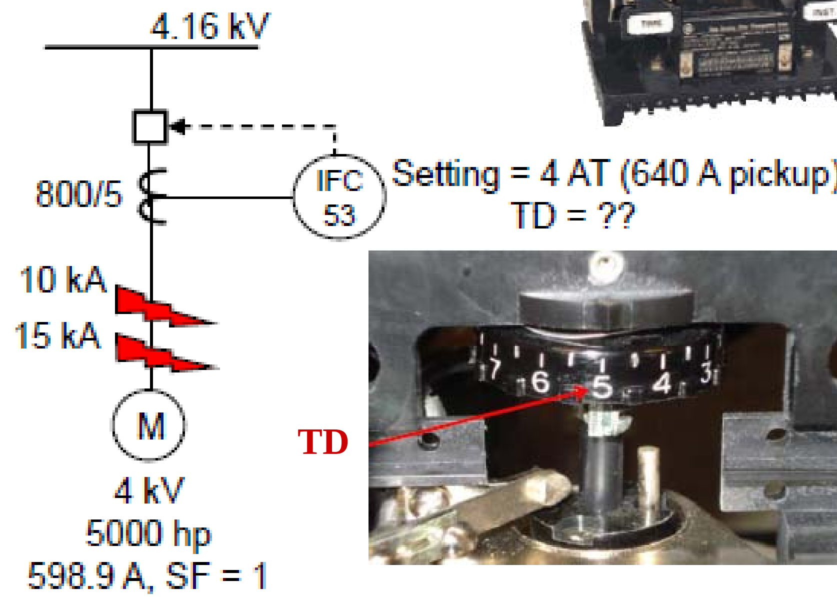


IFC 53 RELAY
Very Inverse

Pickup Calculation - EM Relays







IFC 53 Relay Operating Times		
Fault Current	15 kA	10 kA
Multiple of Pick-up	15000/640 = 23.4	10000/640 = 15.6
Time Dial ½	0.07 s	0.08 s
Time Dial 3	0.30 s	0.34 s
Time Dial 10	1.05 s	1.21 s

British Standard 142 and IEC 255 Inverse Curves

$$t = TMS \times \frac{\beta}{I_r^{\alpha} - 1}$$

where:

- t = operating time in secs.
- TMS = time multiplier setting
- $I_r = (I/I_s)$
  - I=measured current
  - I_s= relay setting current
- α & β are constants for curve selection

C/C	α	β
Normal	0.02	0.14
Very	1.00	13.50
Extreme	2.00	80.00
Long Time	1.00	120.00

Relay Characteristic	Equation (IEC 60255)
Standard Inverse (SI)	$t = TMS \times \frac{0.14}{I_r^{0.02} - 1}$
Very Inverse (VI)	$t = TMS \times \frac{13.5}{I_r - 1}$
Extremely Inverse (EI)	$t = TMS \times \frac{80}{I_r^2 - 1}$
Long time standard earth fault	$t = TMS \times \frac{120}{I_r - 1}$

IEC Standard Inverse Time Characteristic

Relay characteristics to IEC 60255

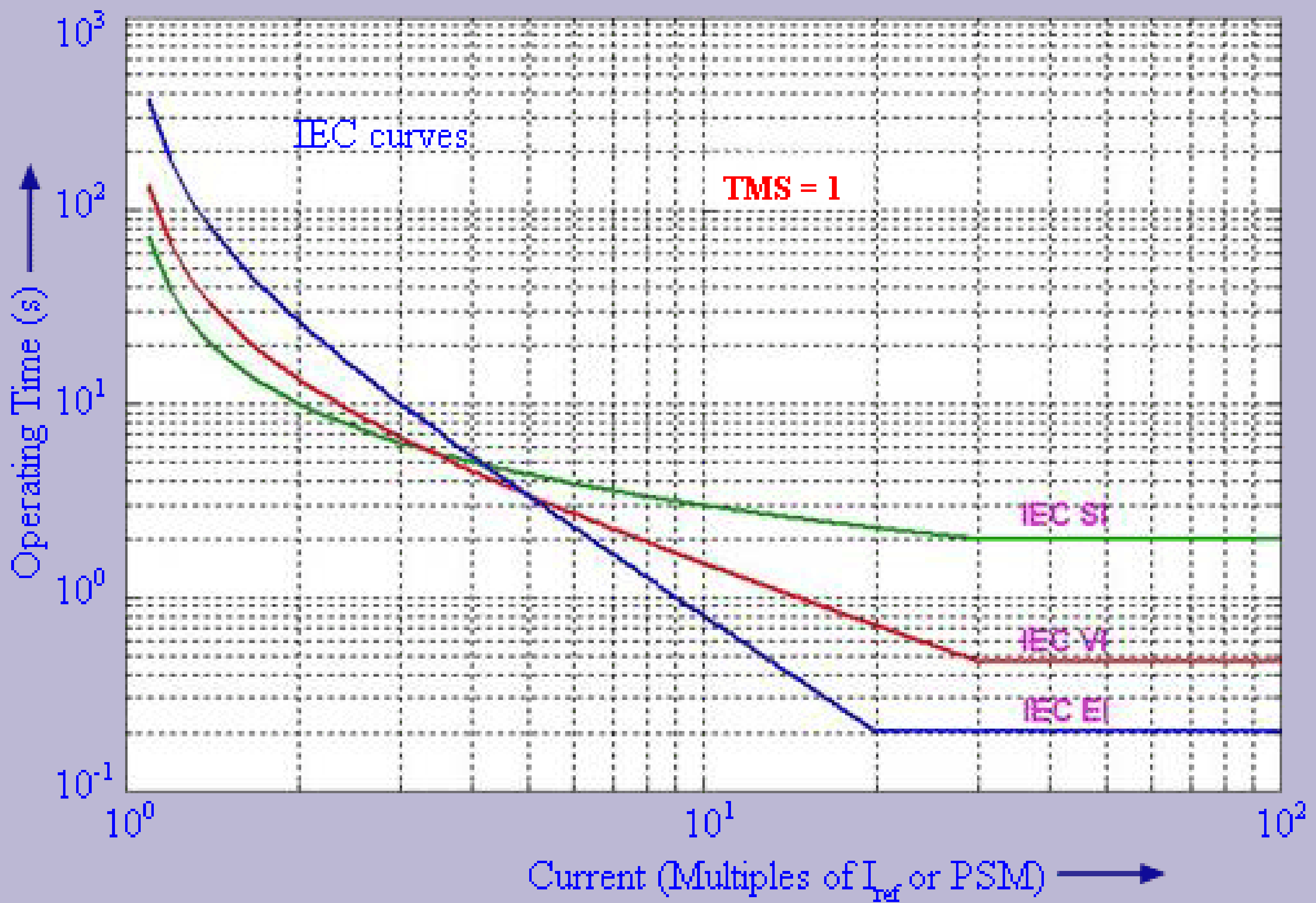


Fig 15.7a Time-current characteristics of Inverse Overcurrent Relaying

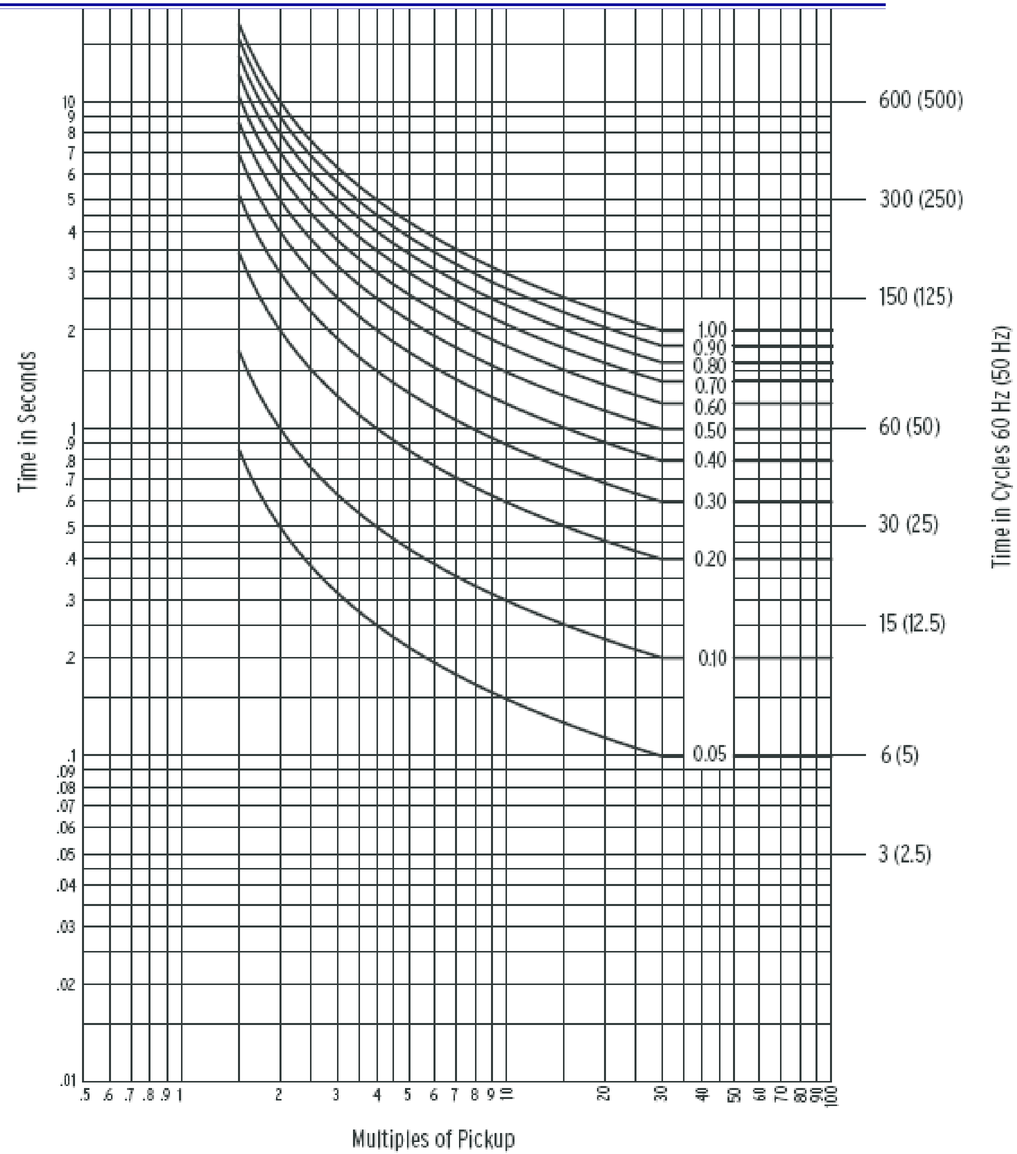
IEC Standard Inverse Time Characteristic

Relay characteristics to IEC 60255

Curve Type	Operating Time
C1 (Standard Inverse)	$T_p = TD \bullet \left(\frac{0.14}{M^{0.02} - 1}\right)$
C2 (Very Inverse)	$T_p = TD \bullet \left(\frac{13.5}{M - 1}\right)$
C3 (Extremely Inverse)	$T_p = TD \bullet \left(\frac{80}{M^2 - 1}\right)$
C4 (Long-Time Inverse)	$T_p = TD \bullet \left(\frac{120}{M - 1}\right)$
C5 (Short-Time Inverse)	$T_p = TD \bullet \left(\frac{0.05}{M^{0.04} - 1}\right)$

$$t = TMS \times \frac{0.014}{M^{0.02} - 1}$$

Standard Inverse C/C



IEEE Standard Inverse Time Characteristic

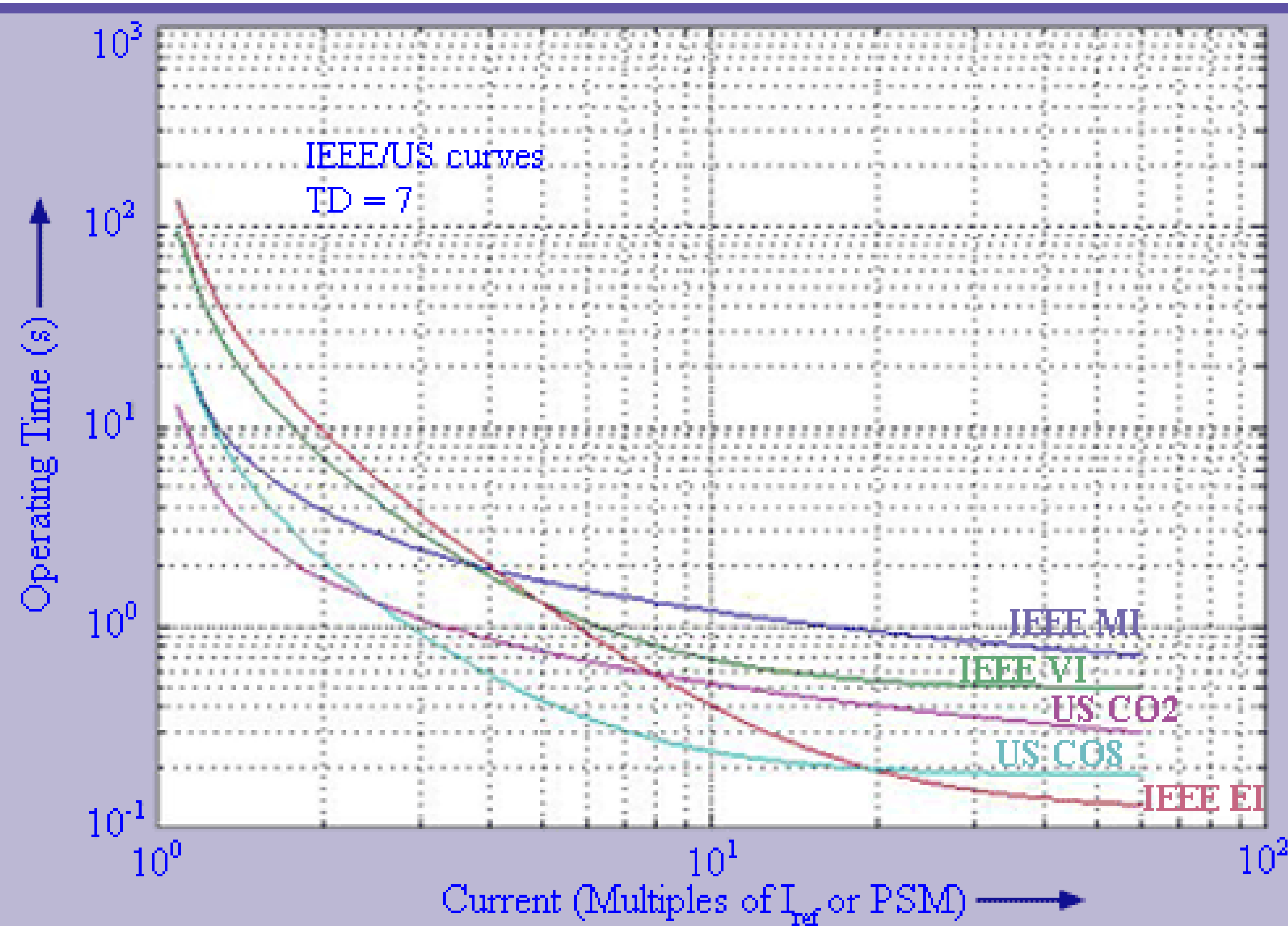
Pickup Time of an Inverse -Time Overcurrent Relay, for $M > 1$

$$t_p = \left(\frac{A}{I_r^p - 1} + B \right) \times TDS$$

- t_p: is the trip time in equation in seconds
- TDS: is the time dial setting
- I_r : is the I_{input}/I_{pickup}
  - (I_{pickup} is the relay current set point)
  - A, B, p : are constants to provide selected curve characteristics

C/C	*A*	*B*	*p*
Moderately Inverse	**0.0515**	**0.1140**	**0.020**
Very Inverse	**19.61**	**0.491**	**2.000**
Extremely Inverse	**28.20**	**0.1217**	**2.00**

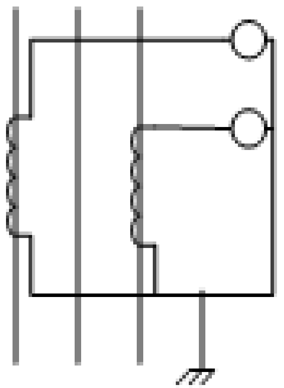
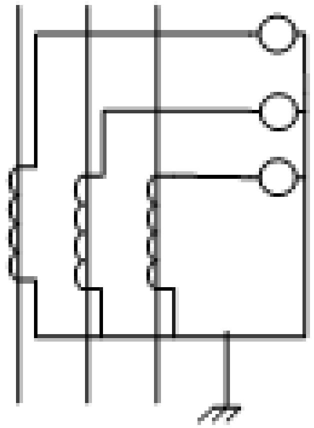
IEEE Standard Inverse Time Characteristic



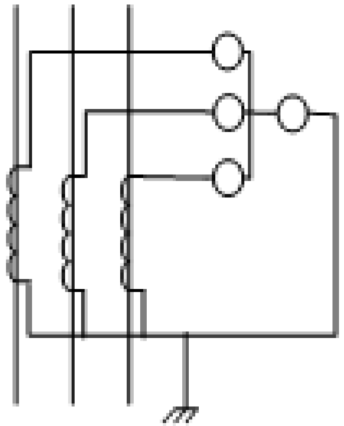
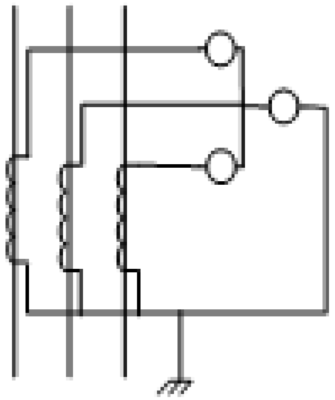
15.7b Time-current characteristics of Inverse Overcurrent Relaying (IEEE/US curves)

Methods of CT and Relay Connections in OC Protection of 3-Phase Circuits

OC protection can be achieved by means of three **OC relays** or by two OC relays

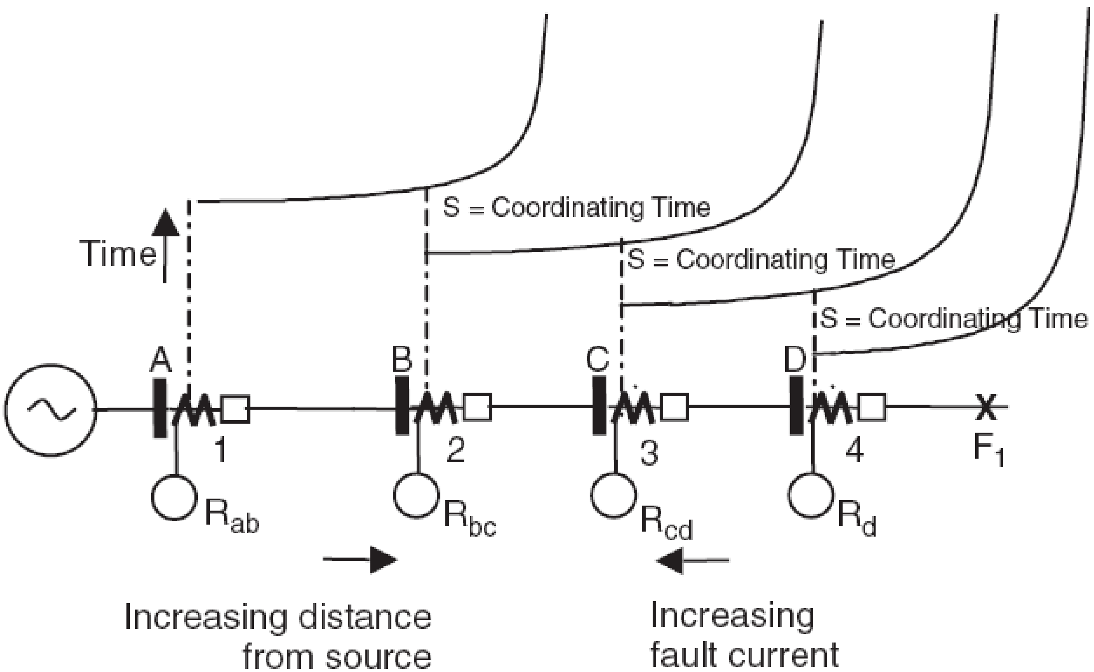
1		Two OC relays with two CT's for phase to phase fault protection.
2		Three OC relays with three CT's for phase to phase fault protection.

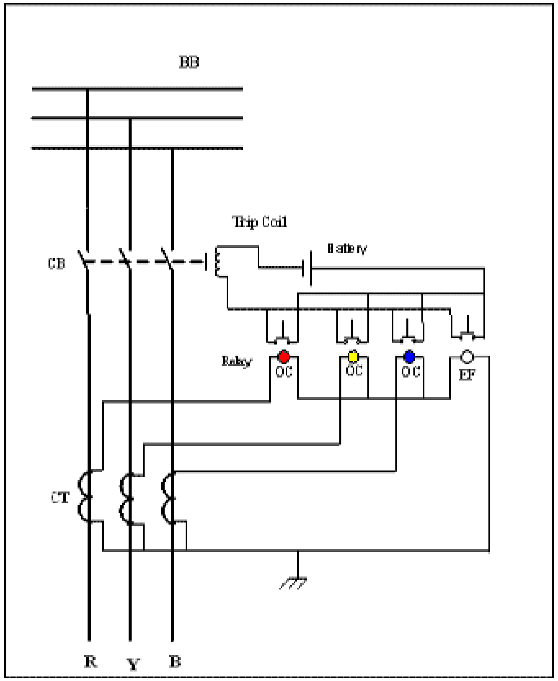
Methods of CT and Relay Connections in OC Protection of 3-Phase Circuits

3		Three OC relays with three CT's for phase to phase fault protection and phase to earth fault. EF setting less than phase fault setting
4		Two OC relays for phase to phase protection One EF relays for phase to earth fault protection

Topic 1: Power System Protection

1.4-Prt 2: Feeder O/C Protection





Co-ordination by Time Grading

- Selectivity and coordination by time grading can be achieved by two philosophies:

  1. Definite time lag (**DTL**)
  2. Inverse definite minimum time (**IDMT**)

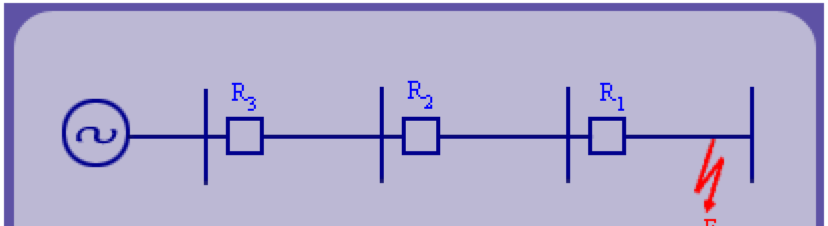
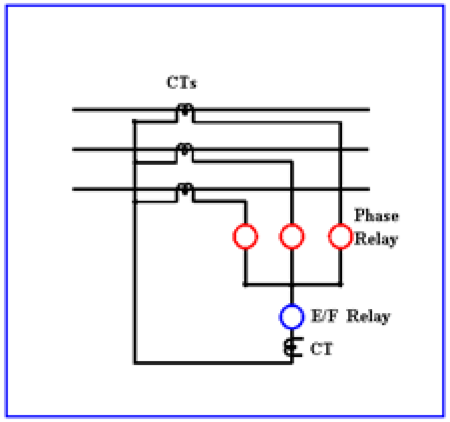


Fig 15.2 A Simple Radial Distribution System



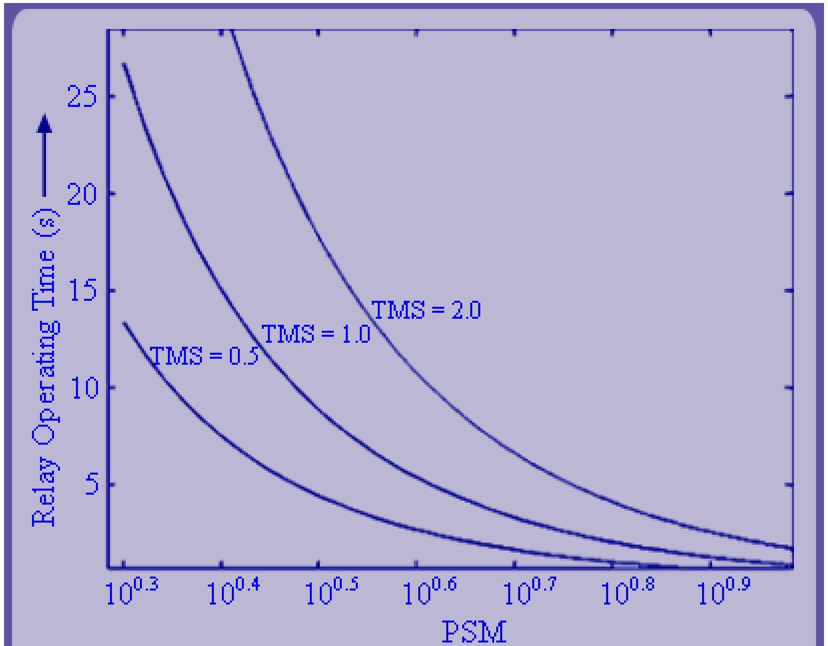


Fig 15.3 Role of TMS in Overcurrent Relay

Definite Time Lag (DTL) Philosophy

- Coordinate with a definite time of operation between successive relays (0.5 s)
- Relay R_1 is set to operate in the fastest possible time
- Relay operating times increase sequentially at 0.5 s intervals
- Disadvantage: the fault closer to source (higher fault current) cleared at the longest operating time.

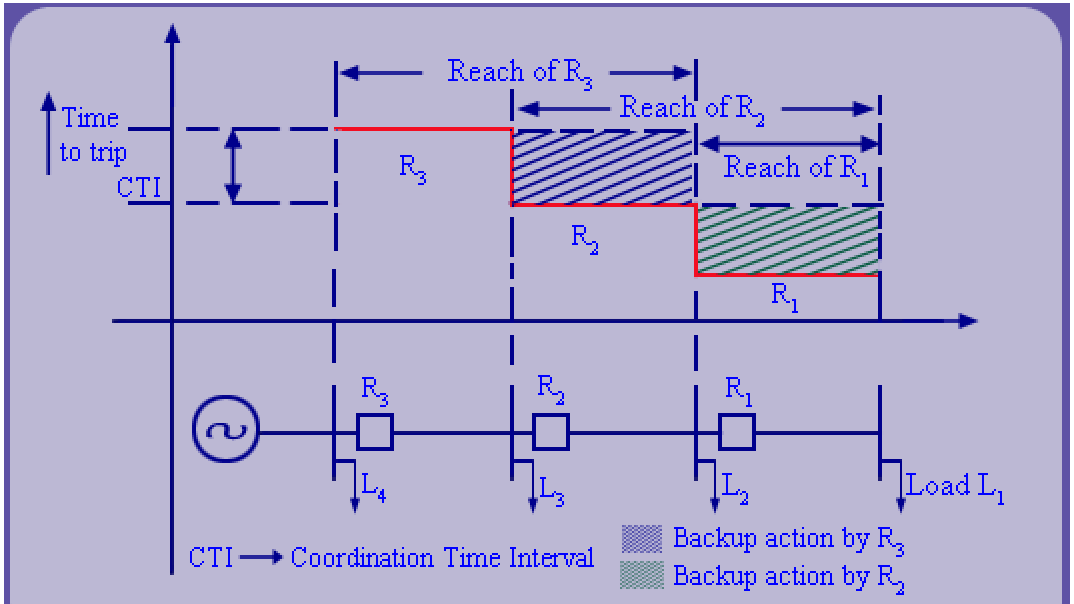
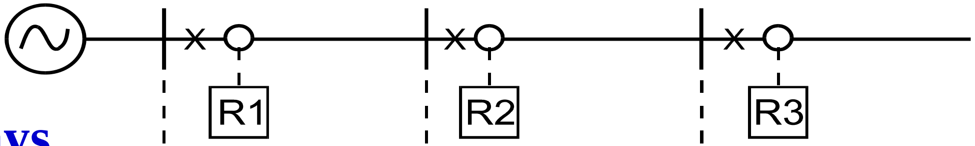


Fig 15.4 Protection of Radial Distribution System with Overcurrent Relay

Inverse Definite Minimum Time-(IDMT) Philosophy

- Use inverse *t-I* Characteristics (IDMT).
- Relay operates faster at the higher fault currents and slower at lower fault currents.
- IDMT relays have to be set to coordinate with both upstream and downstream relays



- **Use of Electromechanical Relays**
  - Need about 0.4 second interval between successive Relays due to acceptable errors
  - Imposes restrictions based on the network design
- **Use of Digital Relays**
  - Can get better coordination with considerably reduced time interval (0.3 second) due to better accuracies

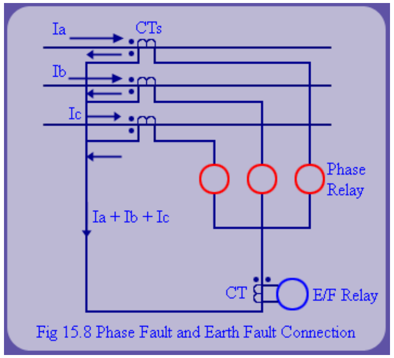
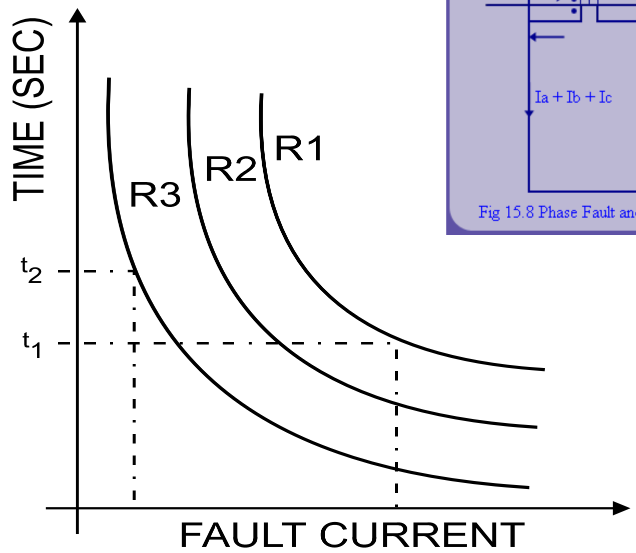
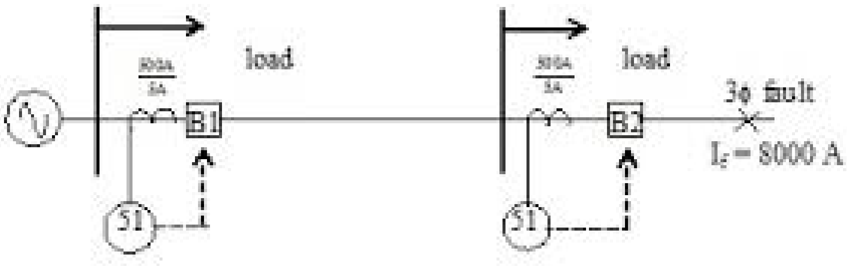


Fig 15.8 Phase Fault and Earth Fault Connection



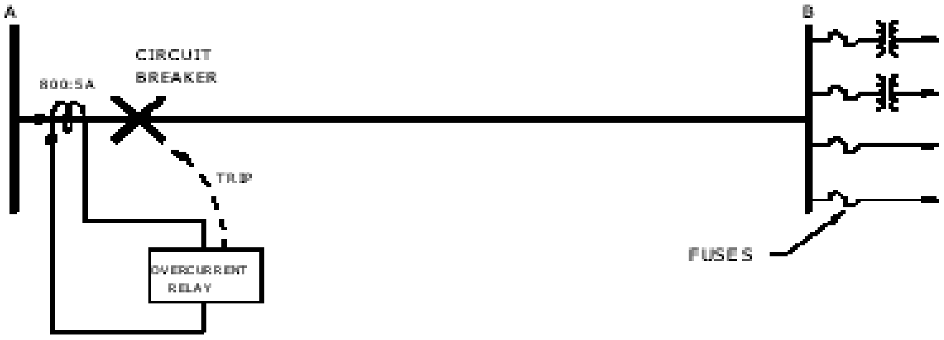
Feeder OC Protection

By far the most common type of protection for radial distribution feeders is **Overcurrent** protection.



Typical distribution system voltages are 33 kV & 11 kV

The point of supply is normally a few kilometers from the load.



Feeder O/C Protection

With ***Radial*** feeders there is only one possible point of supply, and the flow of fault current is in ***one direction only***.

Overcurrent protection can therefore be used to provide adequate protection.

The current entering the feeder at the circuit breaker is measured by means of a Current Transformer located at the base of the breaker bushing.

The C.T. secondary current is supplied to the OC relays. These OC relays must then operate and initiate tripping if a fault condition is detected on the feeder.

Feeder O/C Protection

CRITERIA FOR SETTING THE INVERSE TIMED OVERCURRENT RELAY

1. The relay must not operate for the maximum load current that will be carried by the feeder.

2. The relay setting must be sensitive enough for the relay to operate and clear faults at the very end of the feeder.

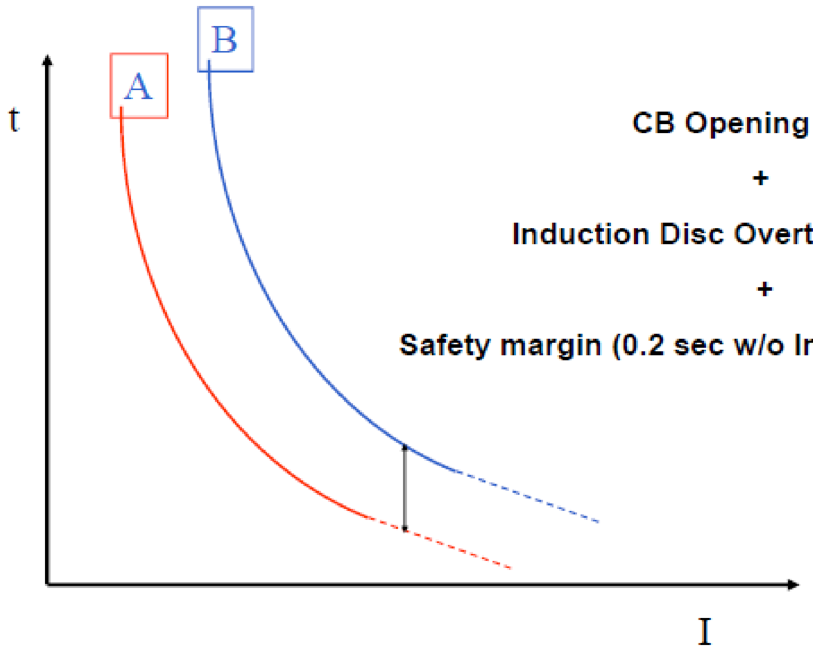
3. The relay operating characteristic must be set to coordinate with other protection devices, such as fuses, ‘downstream’ from the supply station.

Feeder O/C Protection

DIRECTIONAL OVERCURRENT PROTECTION

- If there is generation connected to a distribution feeder, the system is no longer **RADIAL.**
- Fault current can then flow in either direction – into the feeder from the power system **or** out of the feeder from the generator.
- A directional relay or element must be used to supervise the overcurrent relay elements to allow the overcurrent protection to trip **ONLY** if the fault current flows into the feeder from the power system.

Curves must not cross



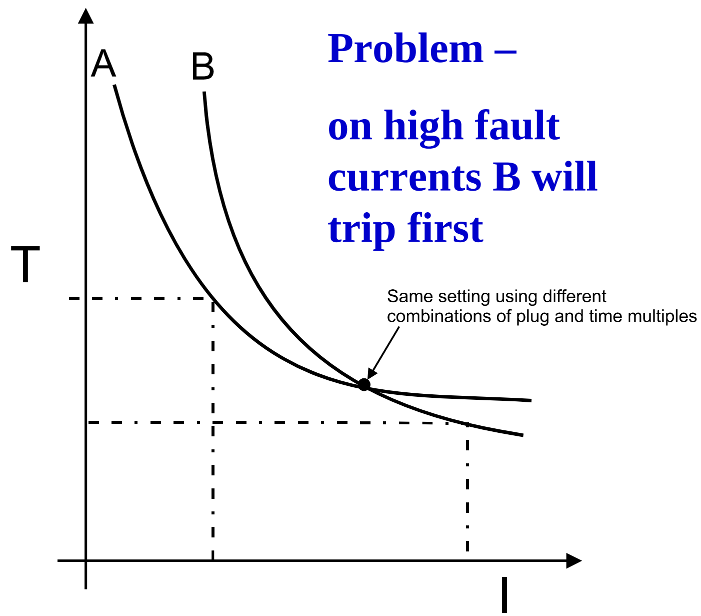
CB Opening Time

+

Induction Disc Overtravel (0.1 sec)

+

Safety margin (0.2 sec w/o Inst. & 0.1 sec w/ Inst.)



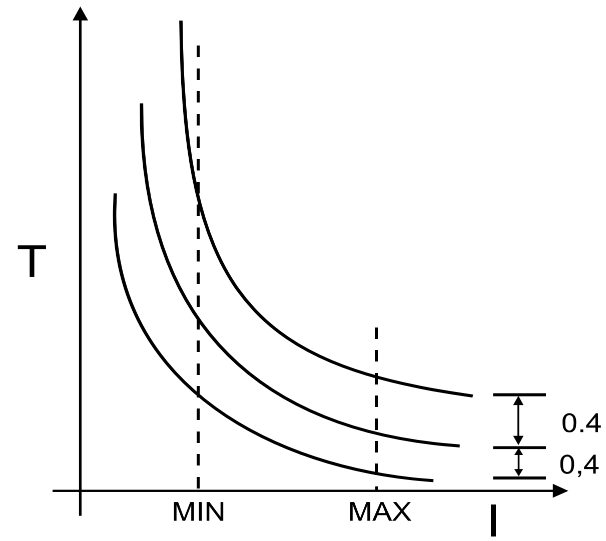
Problem –

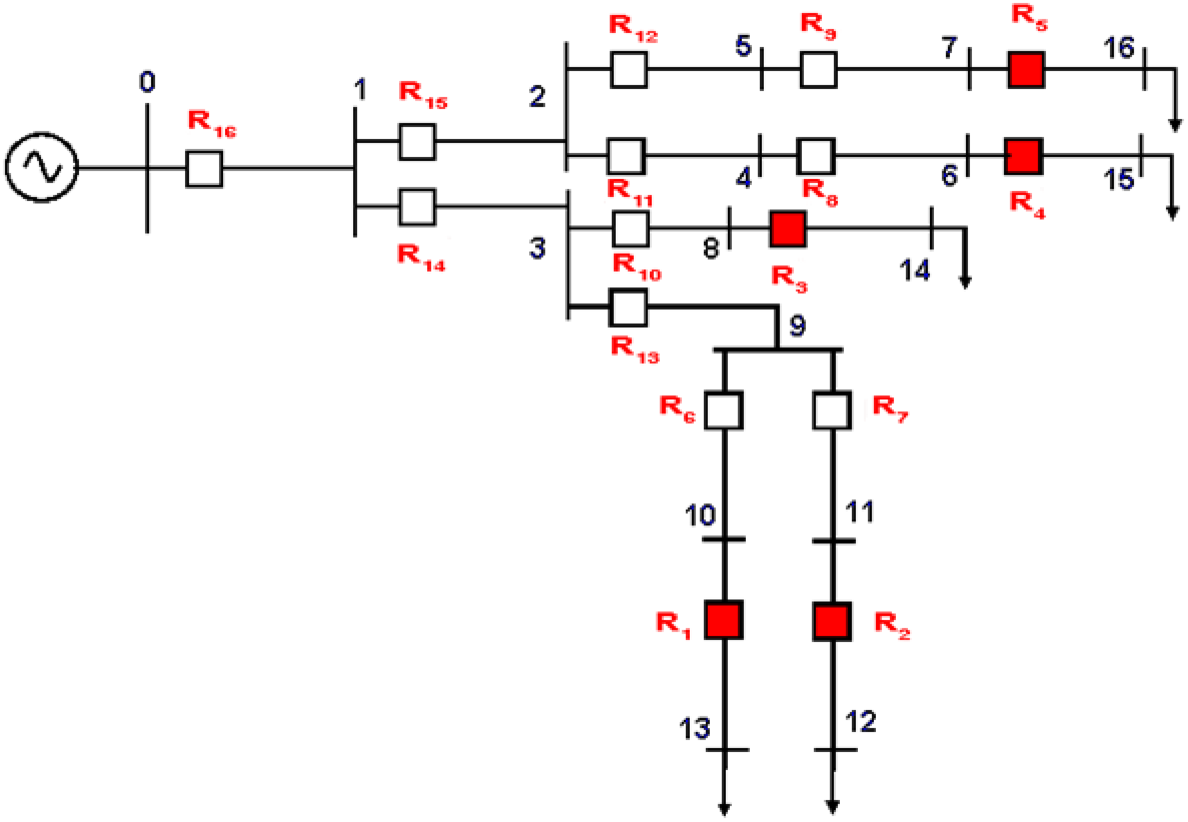
on high fault currents B will trip first

Ideal co-ordination of setting curves

Two Basic Rules !

- Pick up for lowest fault level (minimum)
- Must coordinate for highest fault level (maximum)





Ideal co-ordination of setting curves

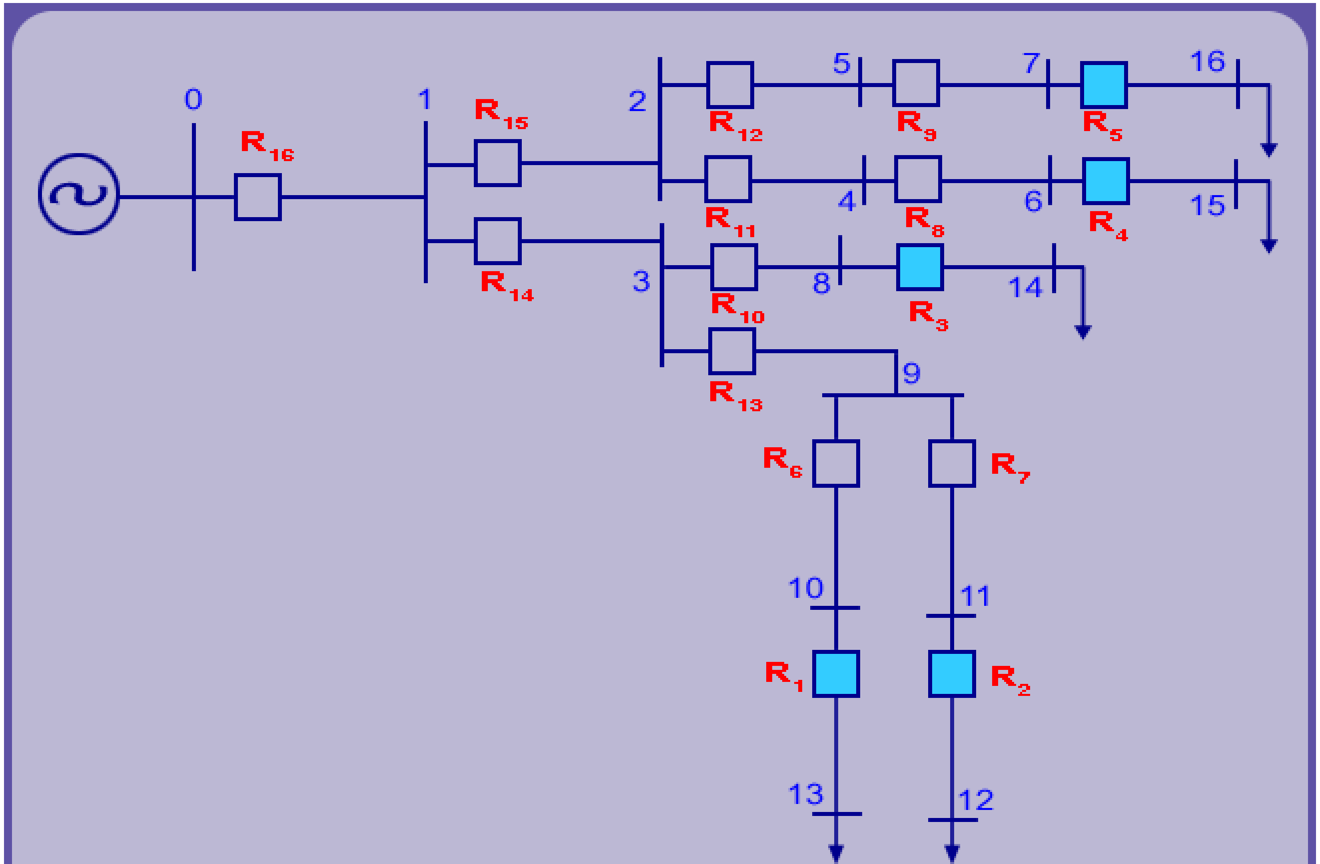
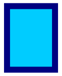
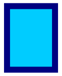


Fig 15.10 Interactive Relay Coordination Example (Primary and Backup Pairs)


Main Relay



Ideal co-ordination of setting curves

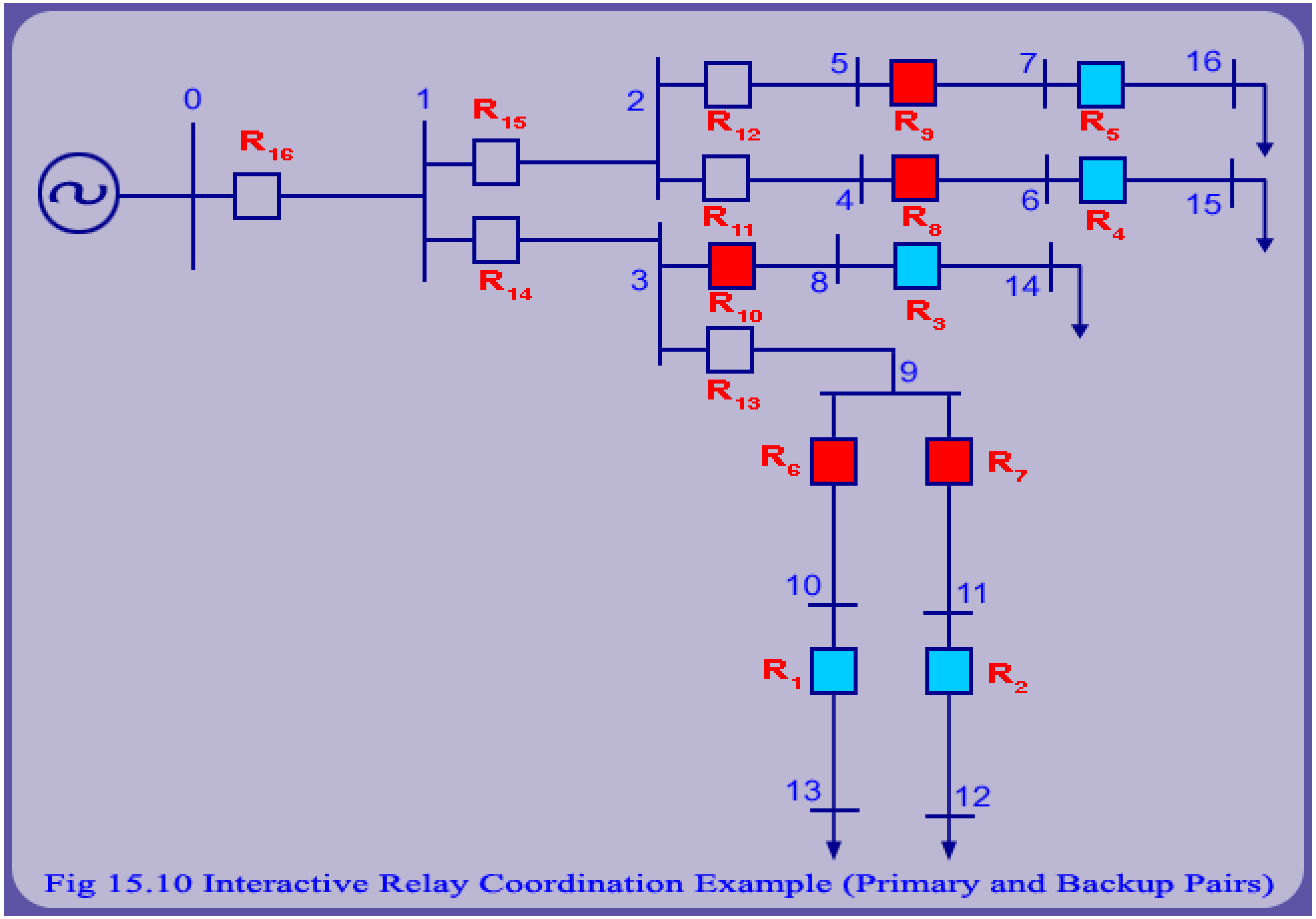


Fig 15.10 Interactive Relay Coordination Example (Primary and Backup Pairs)

Main Relay

Backup Relay

Ideal co-ordination of setting curves

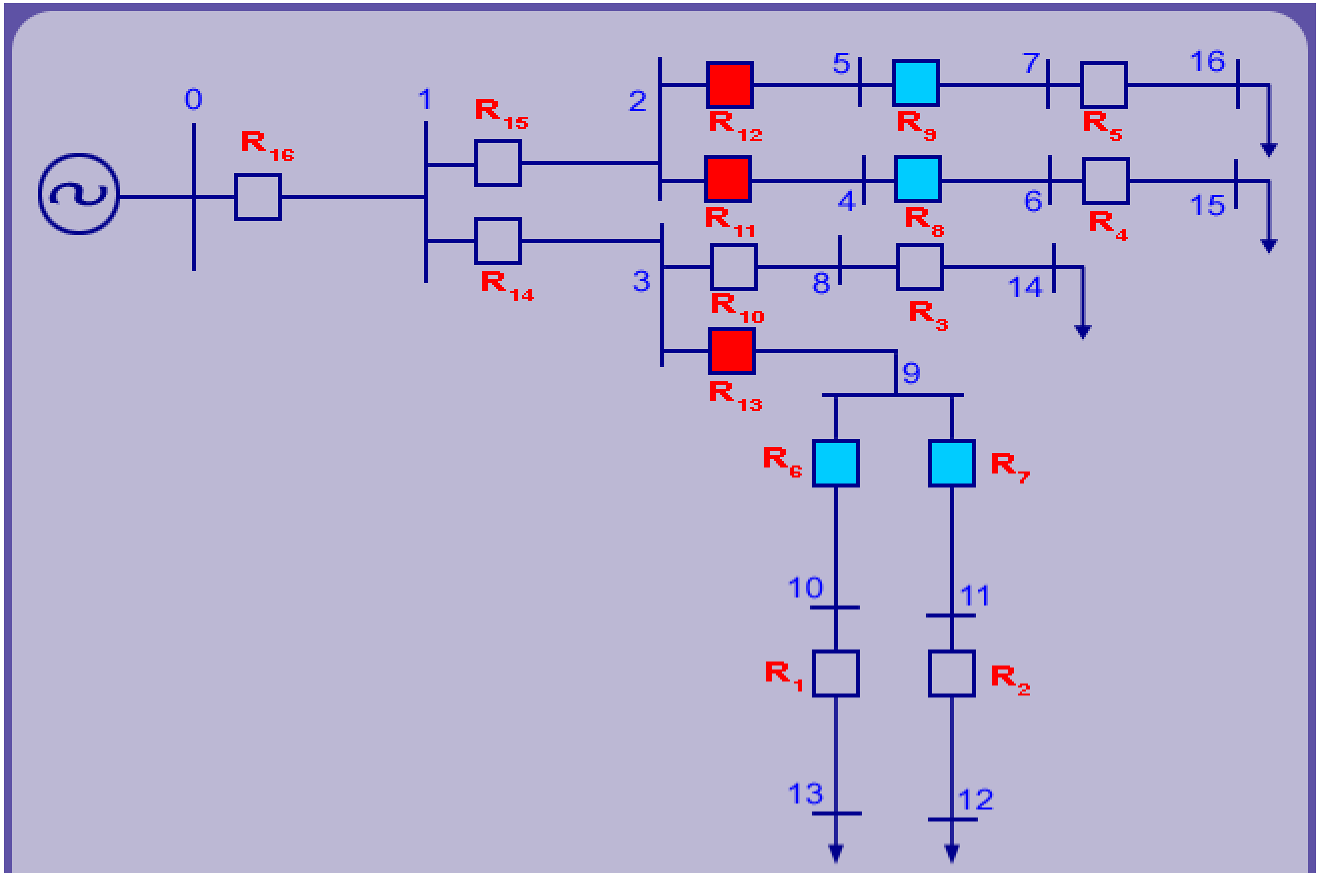


Fig 15.10 Interactive Relay Coordination Example (Primary and Backup Pairs)

Main Relay

Backup Relay

Ideal co-ordination of setting curves

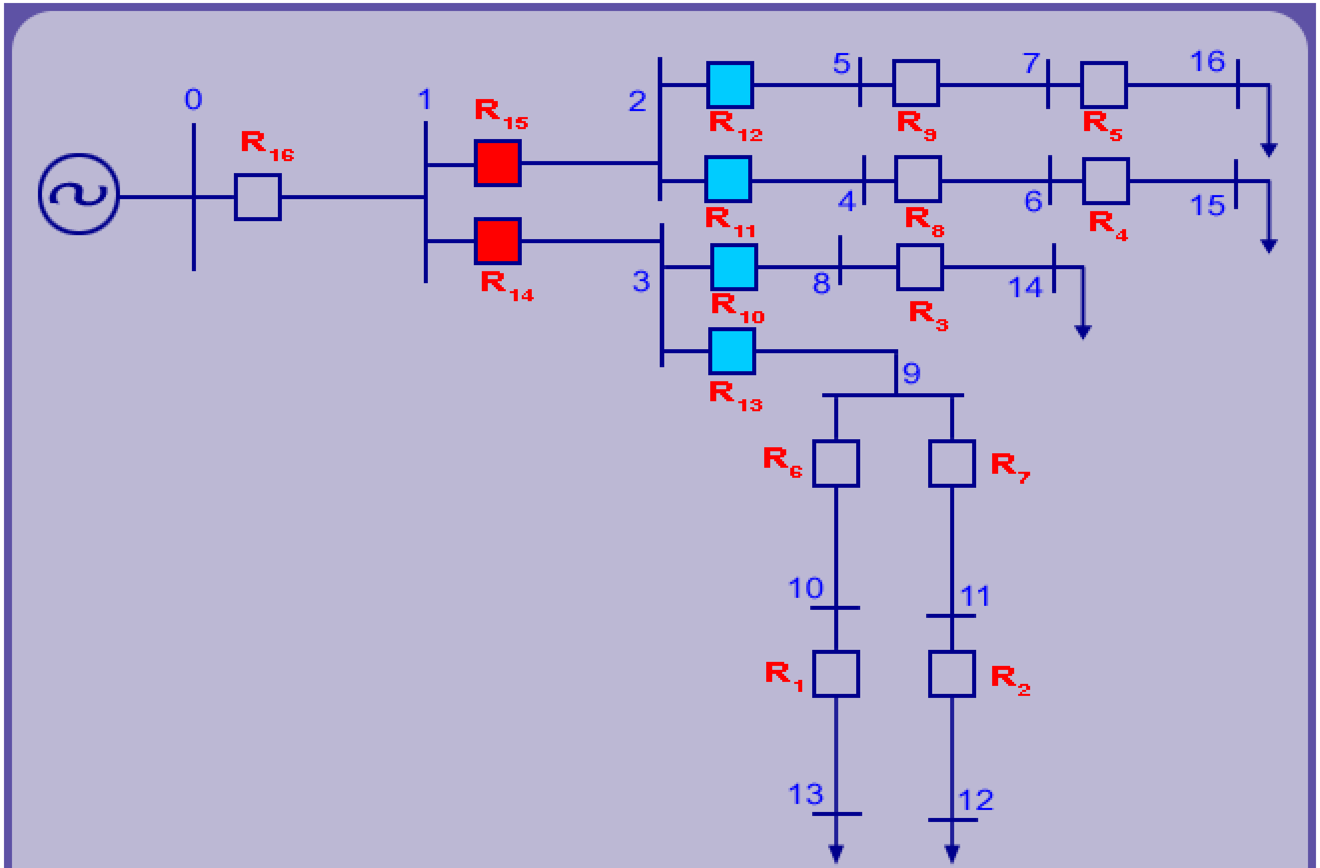


Fig 15.10 Interactive Relay Coordination Example (Primary and Backup Pairs)

Main Relay

Backup Relay

Ideal co-ordination of setting curves

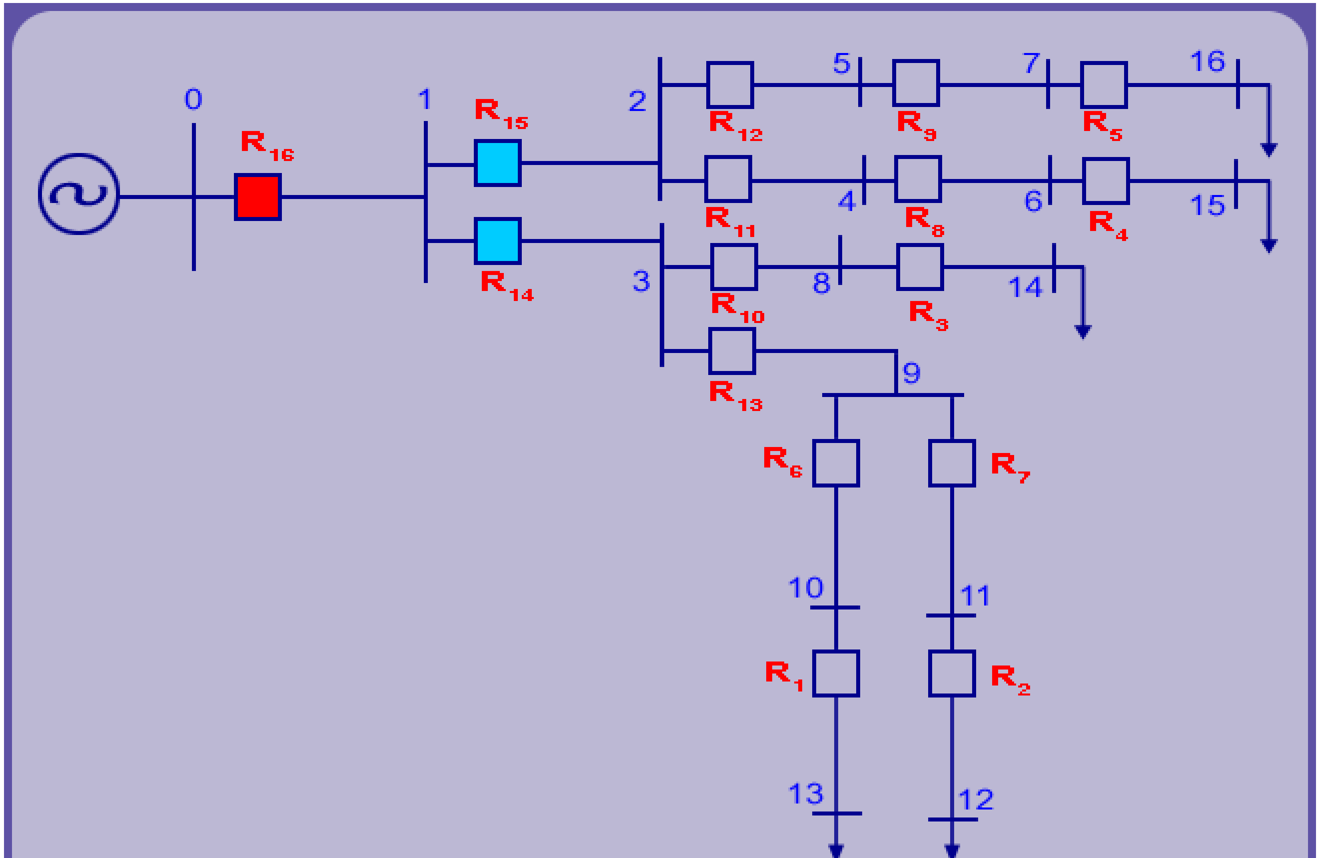
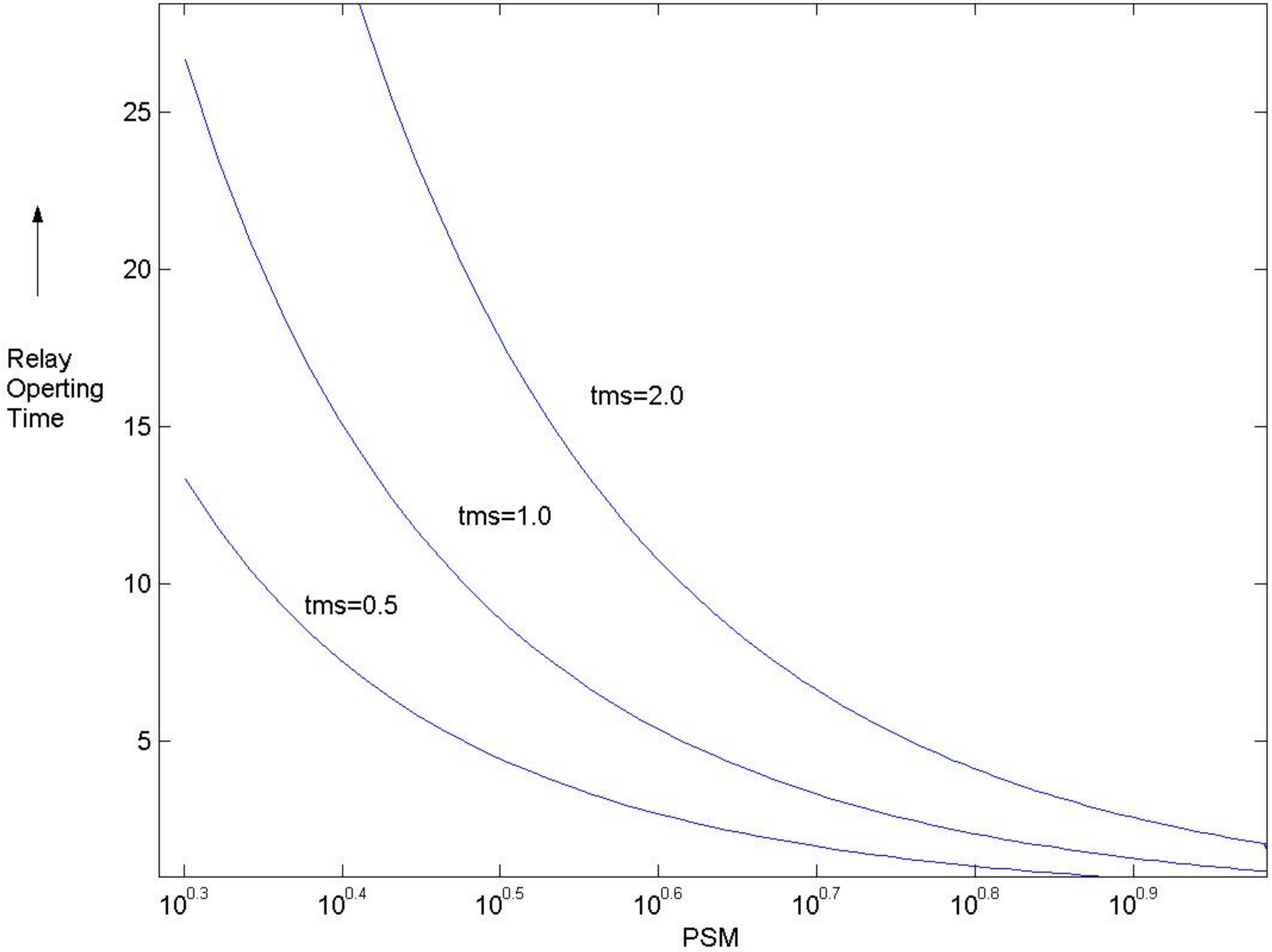


Fig 15.10 Interactive Relay Coordination Example (Primary and Backup Pairs)

Main Relay

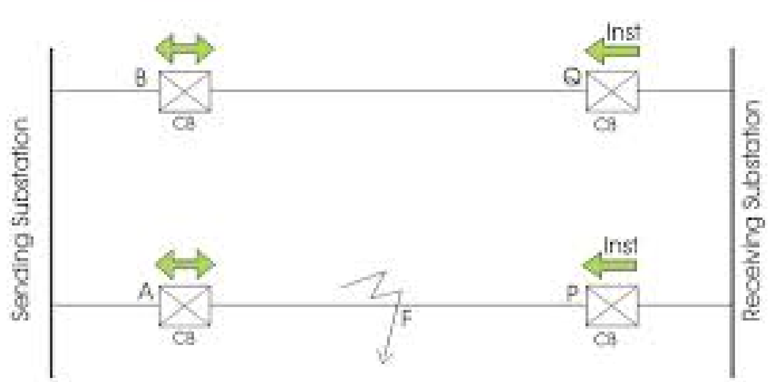
Backup Relay

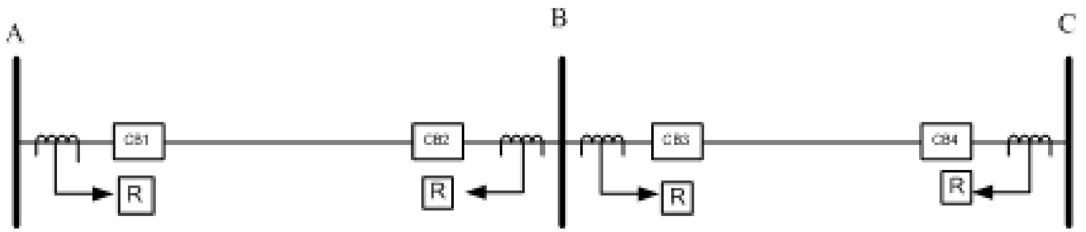
Ideal co-ordination of setting curves



Topic 1: Power System Protection

1.4-Prt 3: Directional Over Current Protection



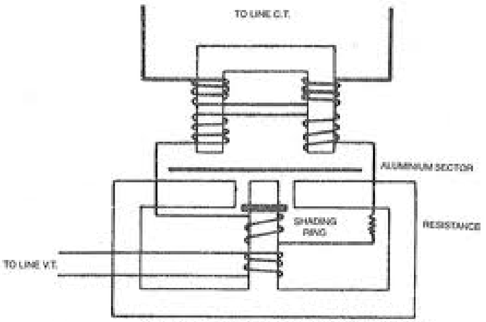


Directional Overcurrent Relays

◇ When fault current can flow in both directions through the relay location, it may be necessary to make the response of the relay directional by the introduction of a directional control facility. The facility is provided by use of additional voltage inputs to the relay.

◇ Directional over-current protection comprises over-current relay and power directional relay- in a single relay casing. The power directional relay does not measure the power but is arranged to respond to the direction of power flow.

◇ The directional relay recognizes the direction in which fault occurs, relative to the location of the relay. It is set such that it actuates for faults occurring in one direction only. It does not act for faults occurring in the other direction.



Directional Element: Electro-magnetic System.

Directional Overcurrent Relays

- When fault current can flow in both directions through the relay location, it may be necessary to make the response of the relay directional by the introduction of a directional control facility. The facility is provided by use of additional voltage inputs to the relay.
- Directional over-current protection comprises over-current relay and power directional relay- in a single relay casing. The power directional relay does not measure the power but is arranged to respond to the direction of power flow.
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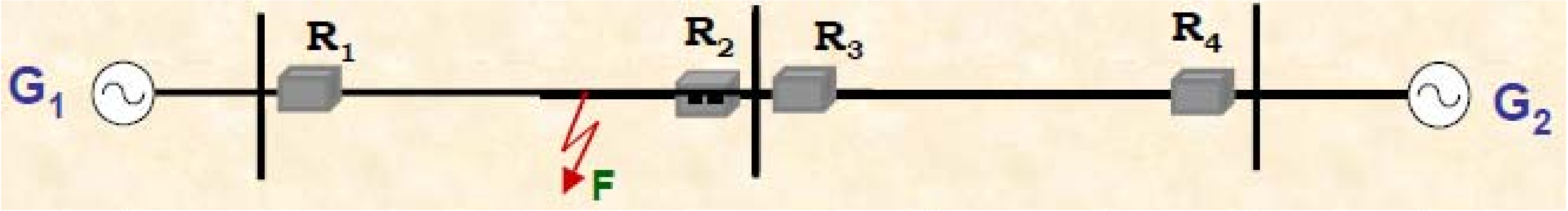


Directional Overcurrent Relays – Parallel Feeders

- If non-unit, non-directional relays are applied to **parallel feeders having a single generating source**, any faults that might occur on any one line will, regardless of the relay settings used, isolate both lines and completely disconnect the power supply.

- With this type of system configuration, it is necessary to apply **directional relays** at the **receiving end** and to grade them with the **nondirectional relays** at the **sending end**, to ensure correct discriminative operation of the relays during line faults.

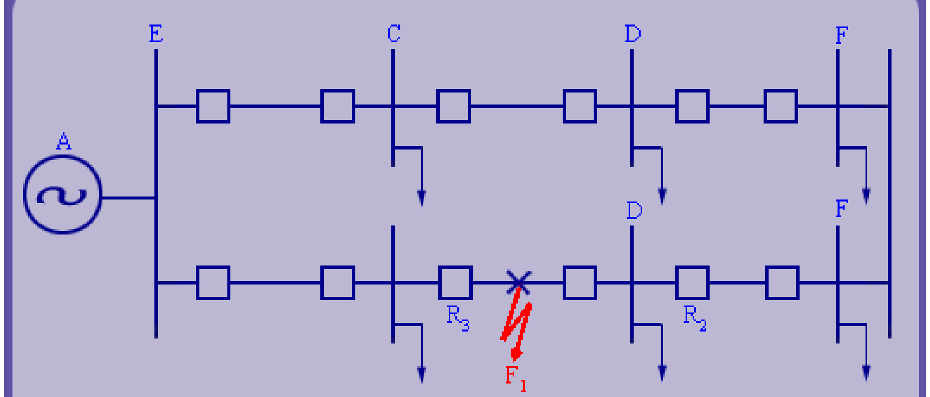
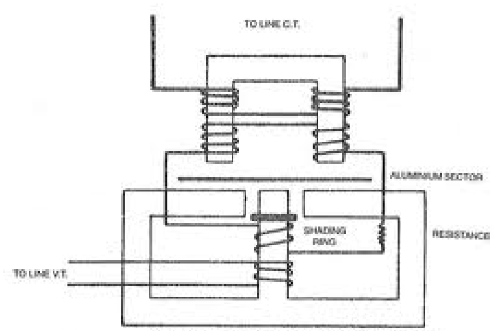


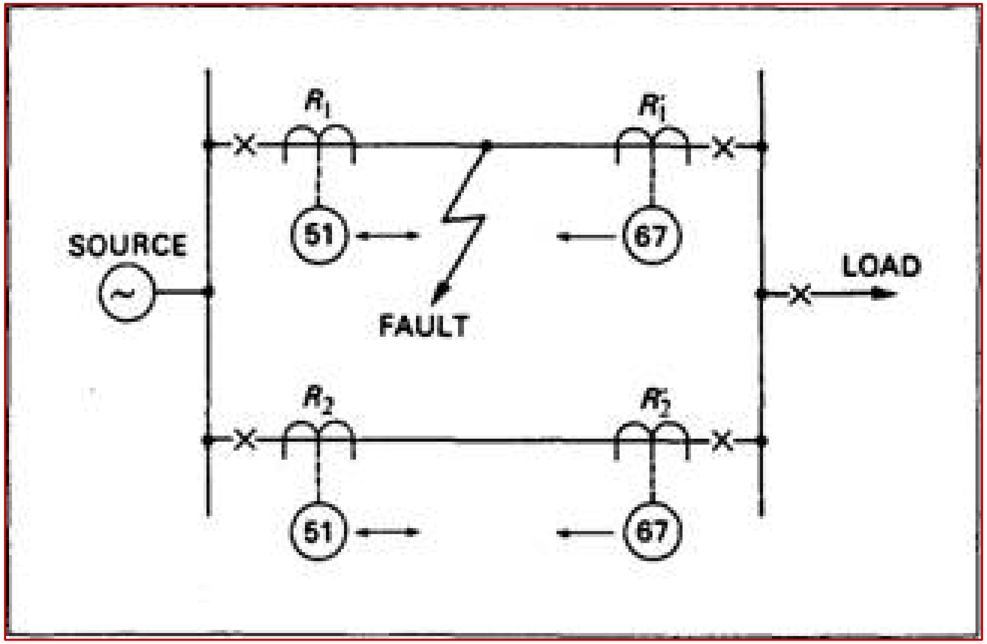
Fig 18.2 A Distribution System with Parallel Paths

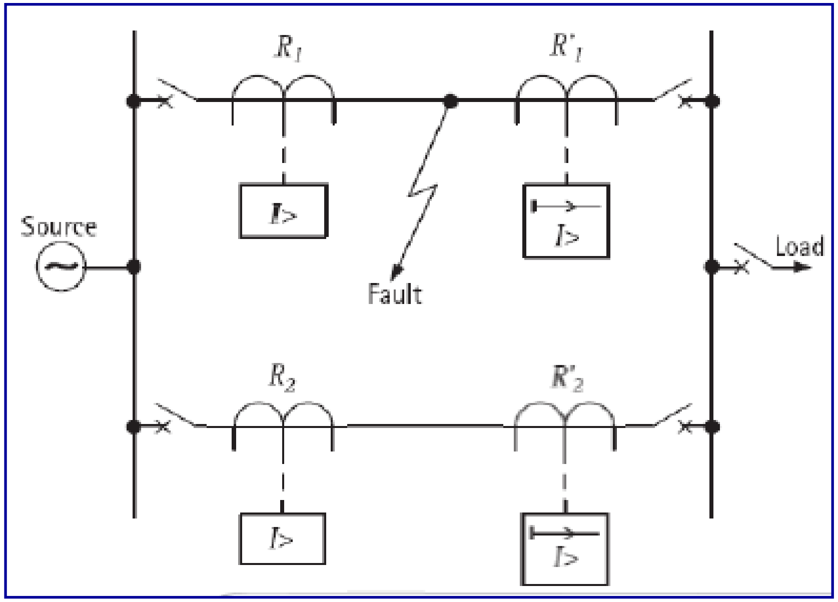


Directional Element: Electro-magnetic System.

Directional OC Relays-Parallel Feeders

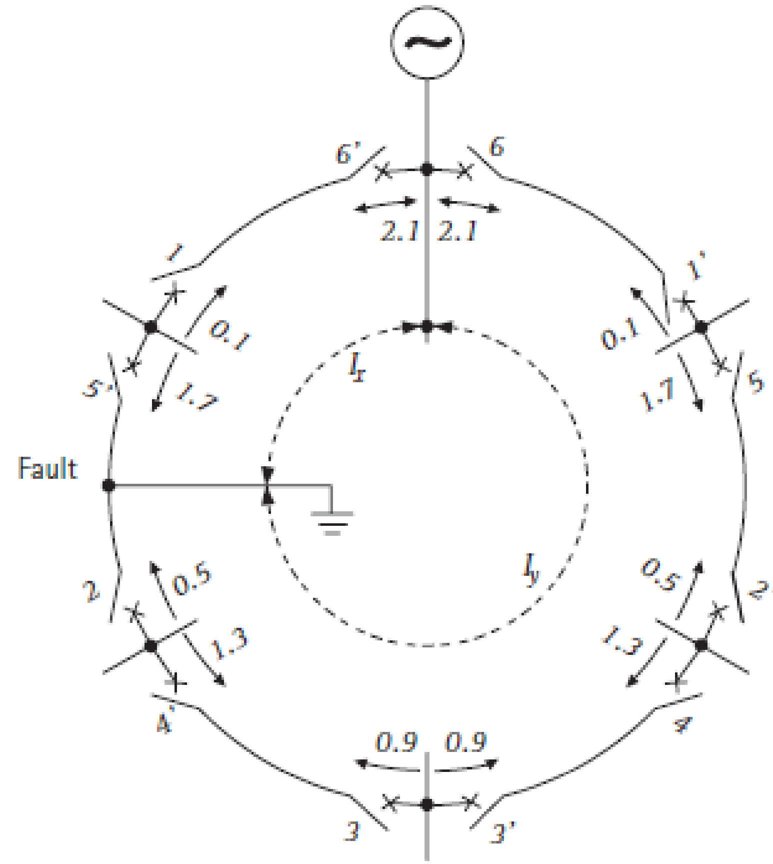
- This is done by setting the **directional relays *R_1'* and *R_2'*** in with their directional elements looking into the protected line, and giving them lower time and current settings than relays R_1 and R_2.
- The usual practice is to set relays ***R_1'* and *R_2'*** to **50% of the normal full load** of the protected circuit and **0.1TMS**.





Directional OC Relays- RING MAINS

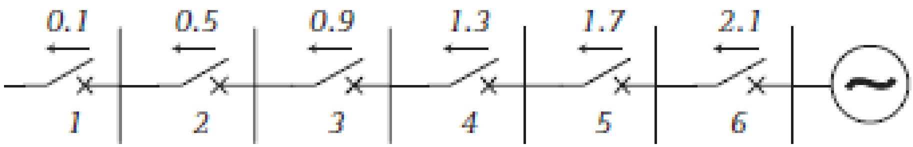
- A particularly common arrangement within distribution networks is the Ring Main. The primary reason for its use is to maintain supplies to consumers in case of fault conditions occurring on the interconnecting feeders.
- In a typical ring main with associated overcurrent protection, current may flow in either direction through the various relay locations, and therefore **directional overcurrent relays** are applied.
- With **modern numerical relays**, a directional facility is often available for little or no extra cost, so that it may be simpler in practice to apply **directional relays** at all locations



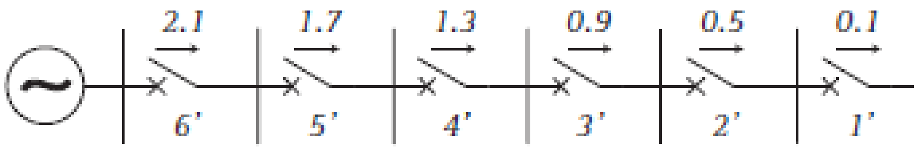
Grading of Ring Mains

- The usual grading procedure for relays in a ring main circuit is to open the ring at the supply point and to grade the relays first clockwise and then anti-clockwise.

- The relays looking in a **clockwise direction** around the ring are arranged to operate in the sequence 1-2-3-4-5-6



- The relays looking in the **anti-clockwise direction** are arranged to operate in the sequence 1’-2’-3’-4’-5’-6.

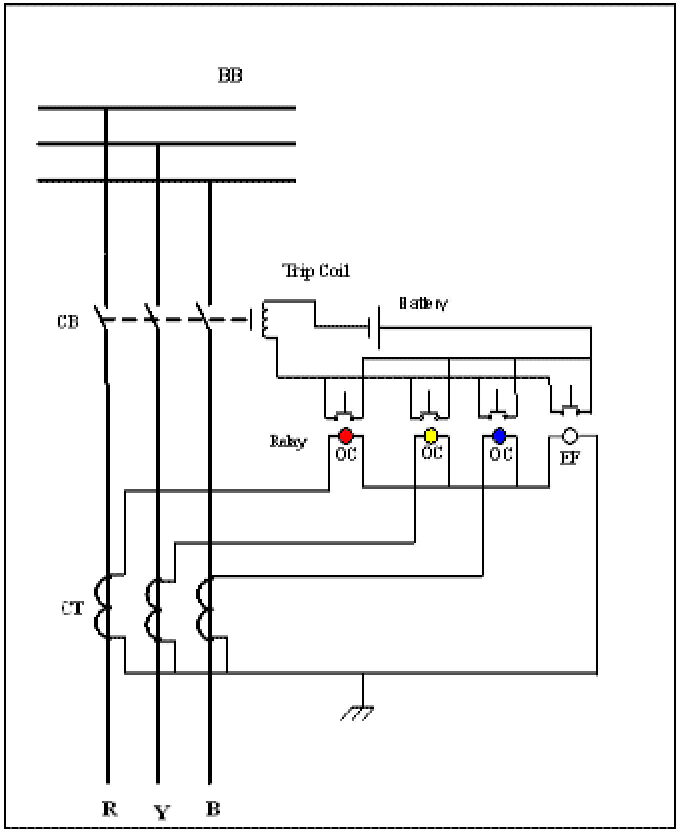


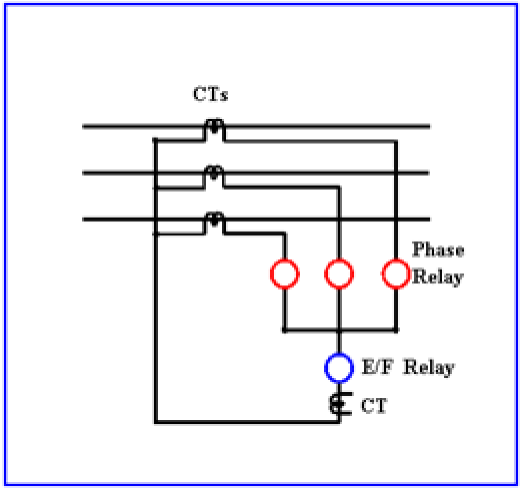
Grading of Ring Mains

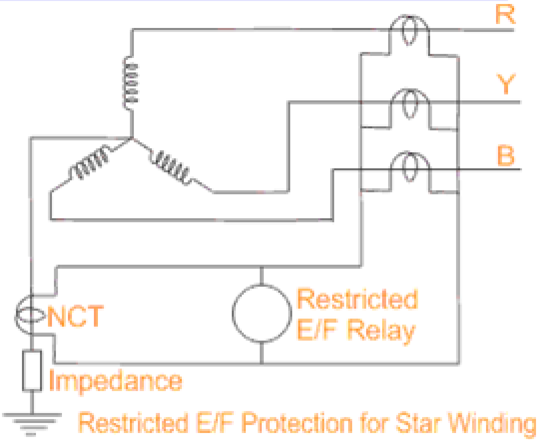
- The arrows associated with the relaying points indicate the direction of current flow that will cause the relay to operate.
- A double-headed arrow is used to indicate a **non-directional relay**, such as those **at the supply point** where the **power can flow only in one direction**.
- A single-headed arrow is used to indicate a directional relay, such as those **at intermediate substations** around the ring where the **power can flow in either direction**.
- **The directional relays** are set in accordance with the invariable rule, applicable to all forms of directional protection, that the **current in the system must flow from the substation busbars into the protected line** in order that the relays may operate.

Topic 1: Power System Protection

1.4-Prt 4: Earth-Fault Protection





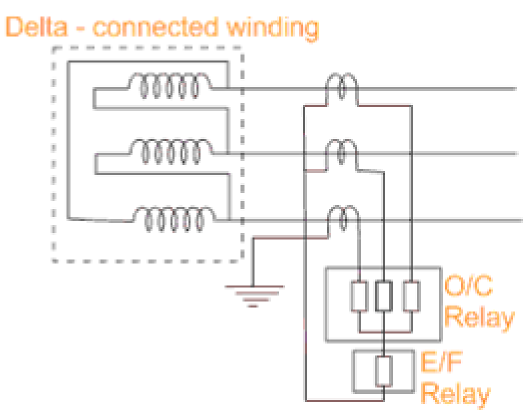


Earth-Fault Protection

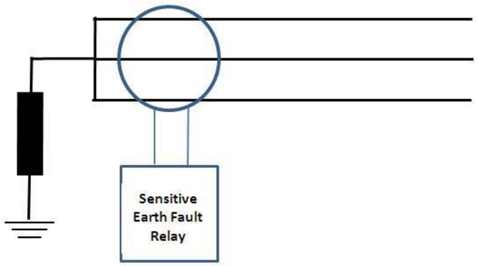
- When the fault current flows through earth return path, the fault is called ***Earth Fault***.
- Other faults which do not involve earth are called phase faults.
- Since earth faults are relatively frequent, earth fault protection is necessary in most cases.
- When separate earth fault protection is not economical, the phase relays sense the earth fault currents. However such protection lacks sensitivity. Hence separate earth fault protection is generally provided.
- Earth fault protection senses earth fault current. Following are the method of earth fault protection.

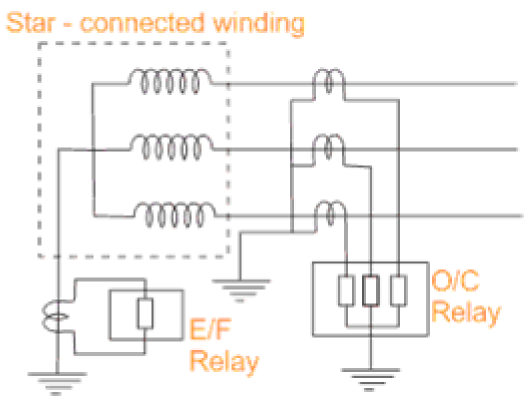
Methods of Earth-Fault Protection

1. Residually connected relay.
2. Relay connected in neutral-to-ground circuit.
3. Core-balance-scheme.
4. Distance relays arranged for detecting earth faults on lines.
5. Circulating current differential protection.



Backup O/C & E/F Protection Scheme





O/C & Unrestricted E/F Protection Scheme

Connections of CT's for Earth-Fault Protection

1. Residually connected Earth-fault Relay

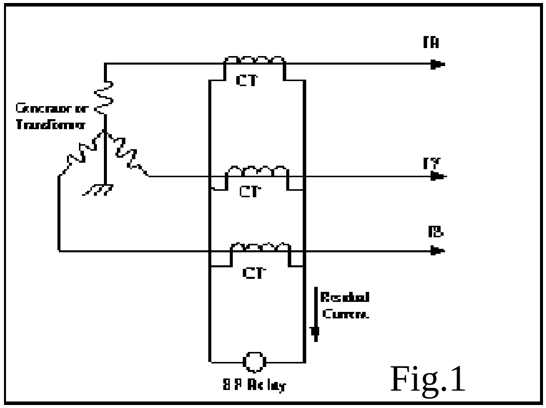


Fig.1

- More sensitive protection against earth faults can be obtained by using a relay that responds only to the residual current of the system, since a residual component exists only when fault current flows to earth.
- In absence of earth-fault the vector sum of three line currents is zero. Hence the vector sum of three secondary currents is also zero.

$$\mathbf{I_R+I_Y+I_B=0}$$

The sum $(\mathbf{I_R+I_Y+I_B})$ is called residual current

- The earth-fault relay is connected such that the residual current flows through it (Fig.1 and Fig. 2).
- In the absence of earth-fault, therefore, the residually connected earth-fault relay does not operate.

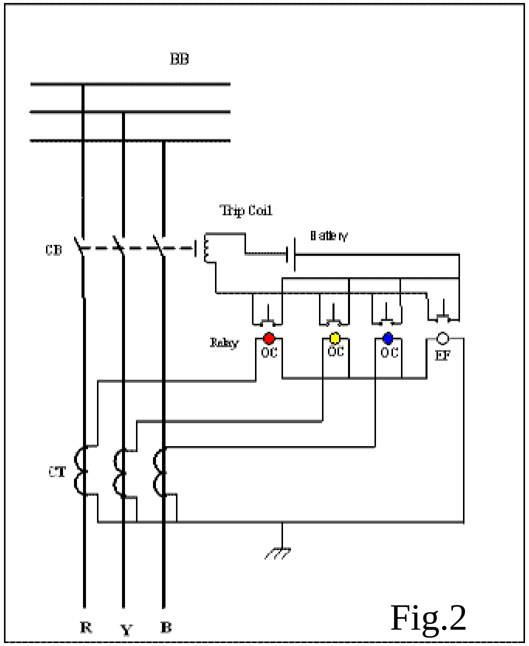


Fig.2

Connections of CT's for Earth-Fault Protection

1. Residually connected Earth-fault Relay

- More sensitive protection against earth faults can be obtained by using a relay that responds only to the residual current of the system, since a residual component exists only when fault current flows to earth.
- In absence of earth-fault the vector sum of three line currents is zero. Hence the vector sum of three secondary currents is also zero. $\mathbf{I_R+I_Y+I_B=0}$
- The sum $\mathbf{(I_R+I_Y+I_B)}$ is called residual current
- The earth-fault relay is connected such that the residual current flows through it (Fig.1 and Fig. 2).
- In the absence of earth-fault, therefore, the residually connected earth-fault relay does not operate.

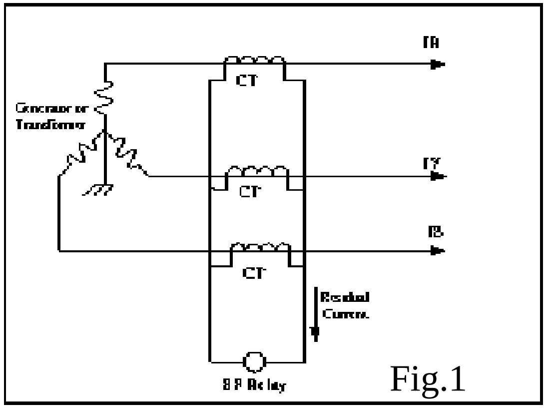


Fig.1

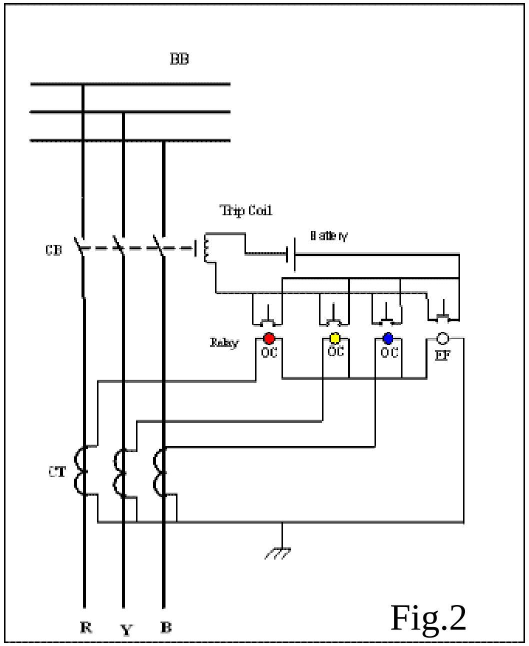


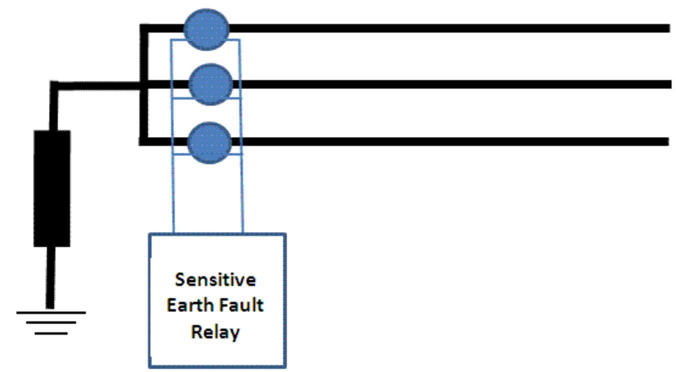
Fig.2

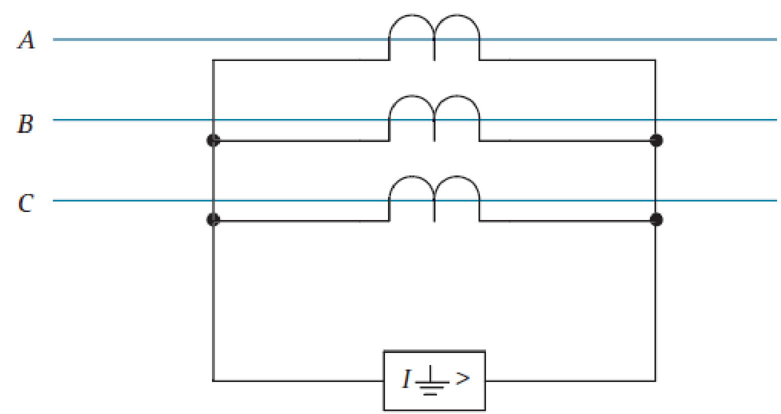
1. Residually connected Earth-fault Relay

- The residual component is extracted by connecting the line current transformers in parallel.

- However, in presence of earth fault the conditions is disturbed and $(I_R+I_Y+I_B)$ is no more zero. Hence flows through the earth-fault relay. If the residual current is above the pick-up value, the earth-fault relay operates. (20%-40% of Full-Load Current)

- In the Residually Connection, the earth-fault at any location near or away from the location of CT's can cause the residual current flow. Hence the protected zone is not definite. Such protection is called **Unrestricted Earth-Fault Protection**





(a)

2. Earth-fault Relay connected in Neutral to Earth Circuit

- Another method of connecting an earth-fault relay is illustrated in Fig 3. The relay is connected to secondary of a CT whose primary is connected in neutral to earth connection.
- Such protection can be provided at various voltage levels by connecting earth-fault relay in the neutral-to-earth connection of that voltage level.
- The fault current finds the return path through the earth and then flows through the neutral-to-earth connection.

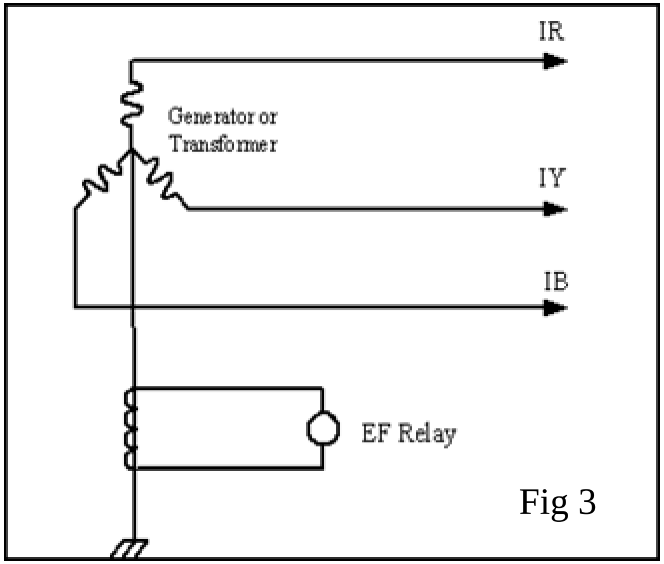


Fig 3

- The magnitude of earth fault current depends on type of earthing (resistance, reactance or solid) and location of fault. In this type of protection, the zone of protection cannot be accurately defined. The protected area is not restricted to the transformer/generator winding alone. The relay senses the earth faults beyond the transformer/generator winding hence such protection is called **Unrestricted Earth-Fault Protection.**

3. Combined Earth-Fault and Phase-Fault Protection

◇ It is convenient to incorporate phase-fault relays and earth-fault relay in a combined phase-fault and earth-fault protection. (Fig. 4)

◇ The increase in current of phase causes corresponding increase in respective secondary currents. The secondary current flows through respective relay-units Very often only two-phase relays are provided instead of three, because in case of phase faults current in any at least two phases must increase. Hence two relay-units are enough.

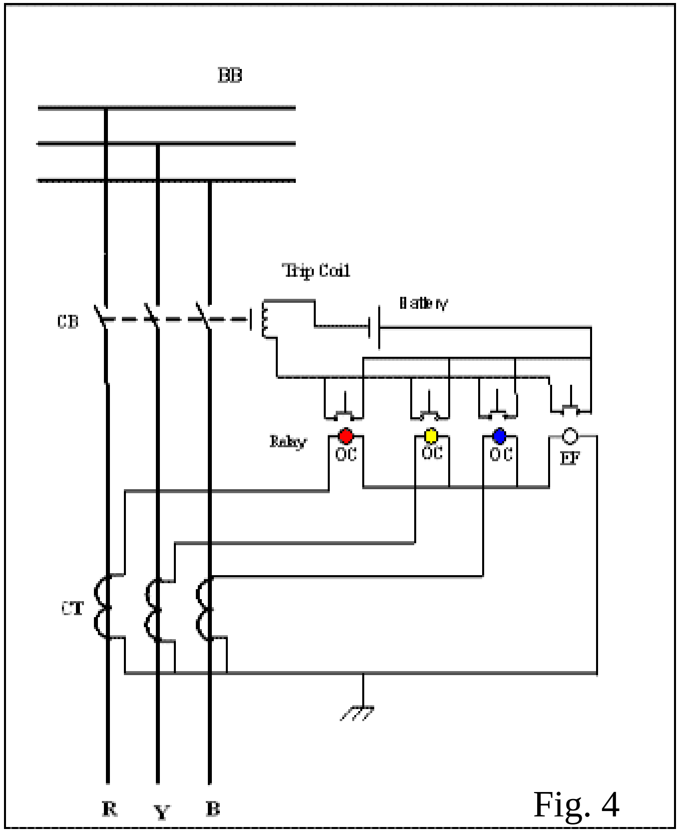
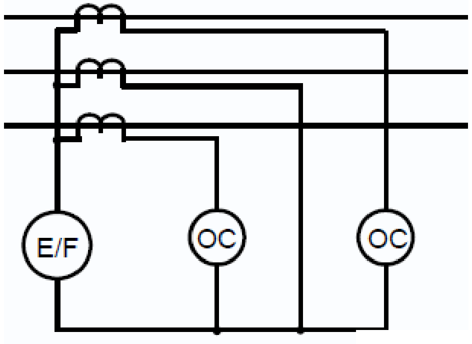
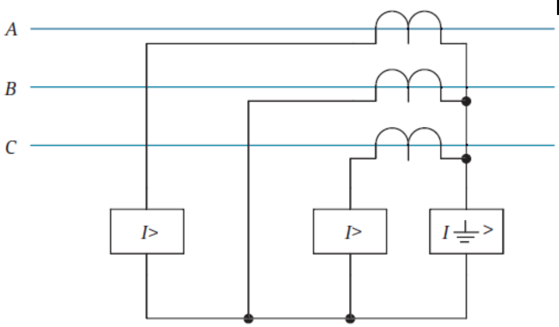
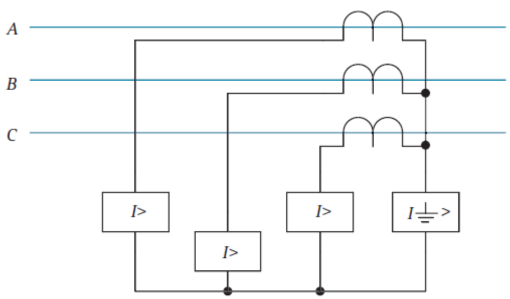


Fig. 4







(b)

Economize using 2x OC relays

4. Earth-fault Protection with Core Balance Current Transformers. Sensitive Earth-Fault Protection

- In this type of protection (Fig. 5) a single ring shaped core of magnetic material, encircles the conductors of all the three phases. A secondary coil is connected to a relay unit.
- The cross-section of ring-core is ample, so that saturation is not a problem.
- During no-earth-fault condition, the components of fluxes due to the fields of three conductors are balanced and the secondary current is negligible.
- During earth faults, such a balance is disturbed and current is induced in the secondary.
- Core-balance protection can be conveniently used for protection of low-voltage and medium voltage systems.

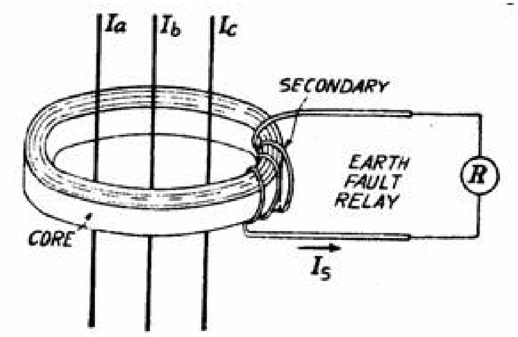
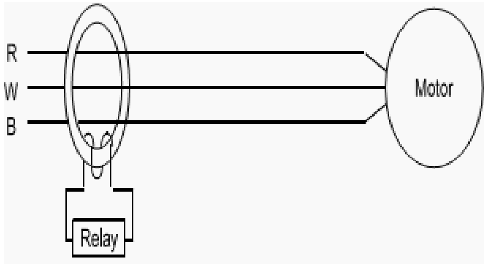


Fig. 5



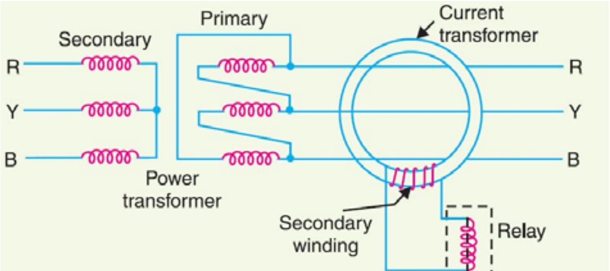
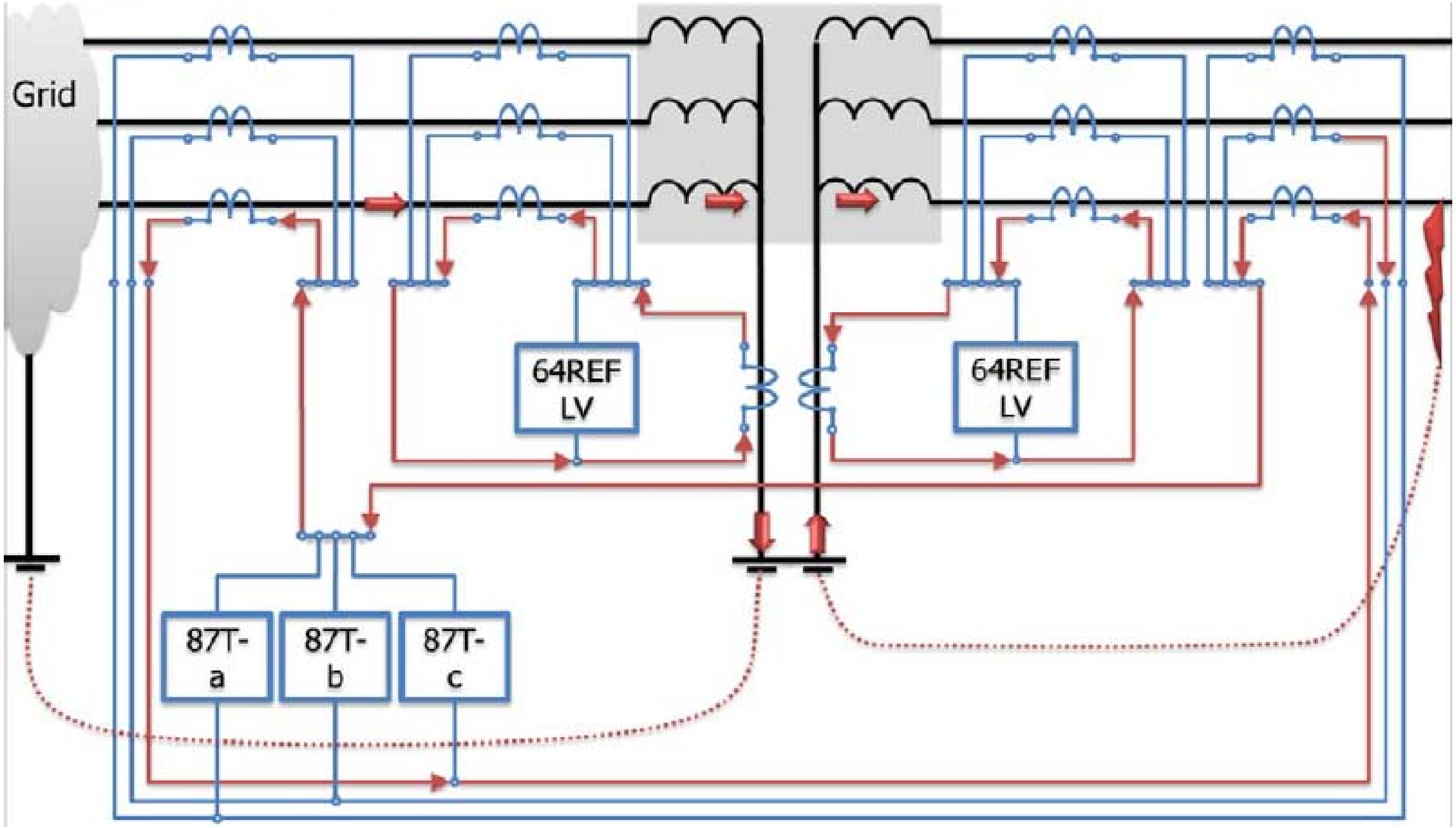


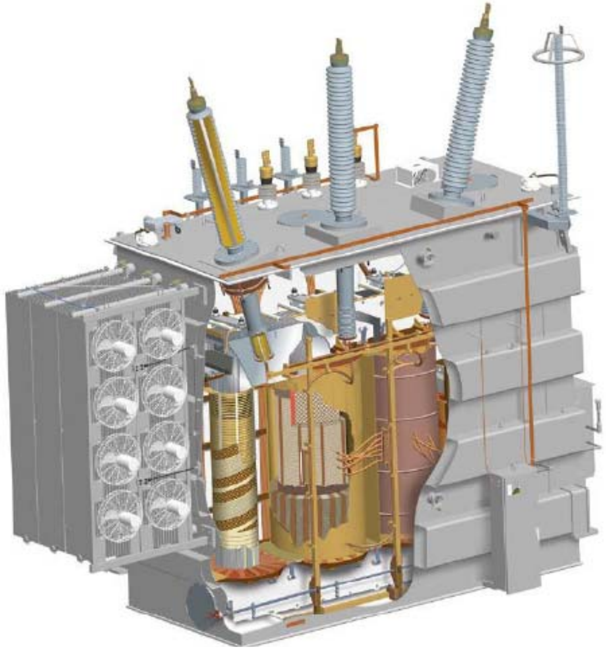
Fig: Transformer's Earth-fault or Leakage Protection

Topic 1: Power System Protection

1.5 Transformer Protection

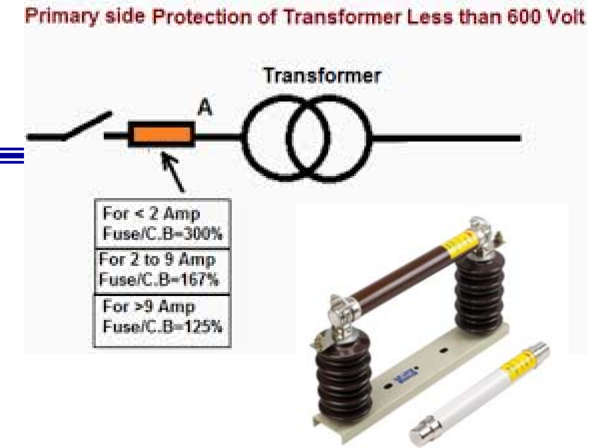


Transformer Protection

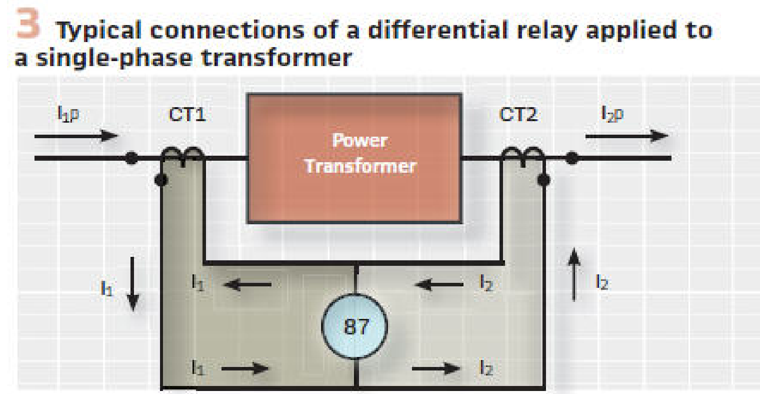


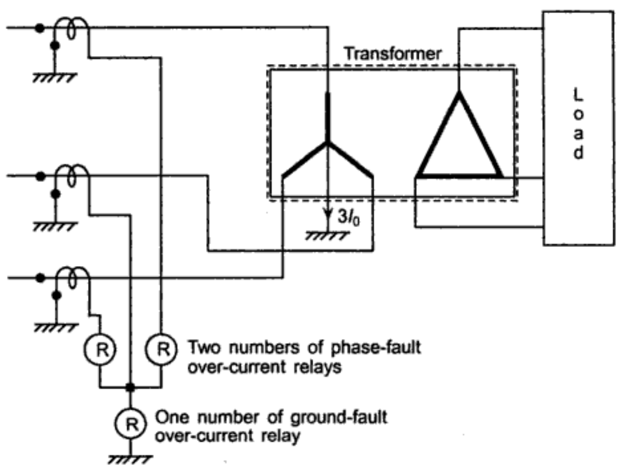
- Power transformers are expensive.
  - ➔Protection must be effective
- What can go wrong?
  - Winding-to-winding faults
  - Winding-to-ground faults
  - Bushing faults

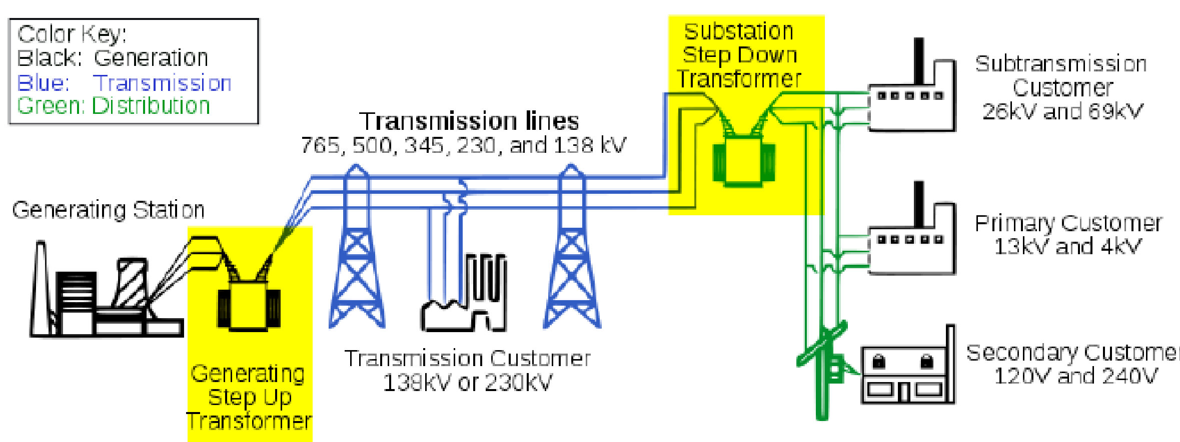
Transformer Protection



- Protection Methods
  - ➔ Protection must be effective
- What can go wrong?
  - ➢ Fuse Protection
  - ➢ Overcurrent Protection
  - ➢ Differential Protection



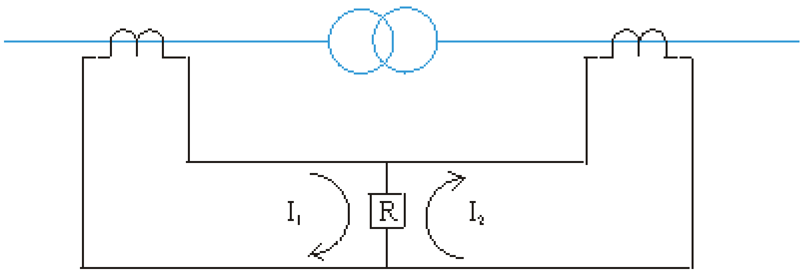




Protective Relay Systems

- Basic function of protection is to detect faults and to clear them as soon as possible.
- Minimum number of items of equipment should be disconnected.
  - Called **SELECTIVITY**.
- **Speed** and **Selectivity** are the most desirable features of Protection
- But Cost also Decides the selection

Unit Protection ⇒



Protection of Transformers

- Transformers are expensive and important.
- IDMTL relays are not for Overload.
- Recommended protection
  - **Differential protection (optional)**
  - **HV and LV restricted earth fault.**
  - **Buchholz gas and surge relay.**
  - **Oil and winding temperature.**

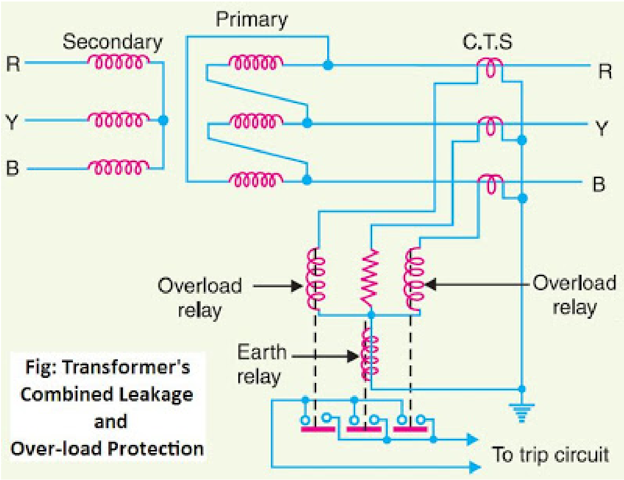
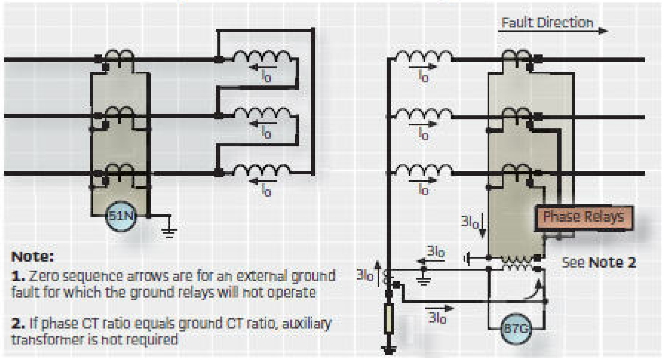


Fig: Transformer's Combined Leakage and Over-load Protection

8 Ground fault protection for a delta-wye transformer



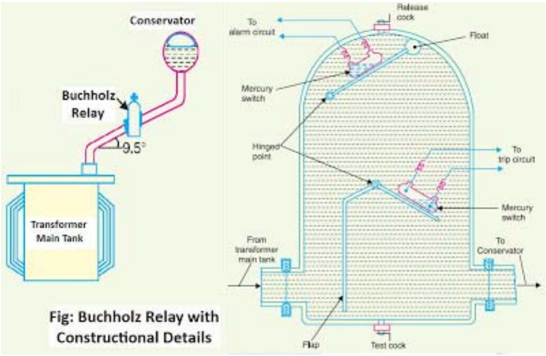


Fig: Buchholz Relay with Constructional Details

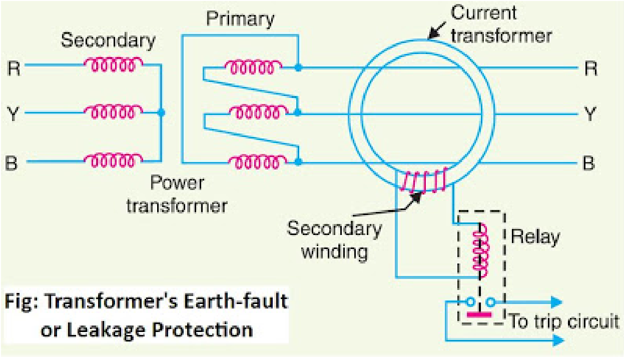
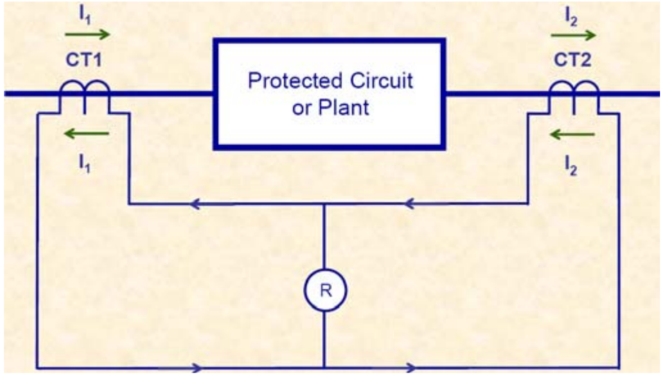


Fig: Transformer's Earth-fault or Leakage Protection

Applications of Unit Protection

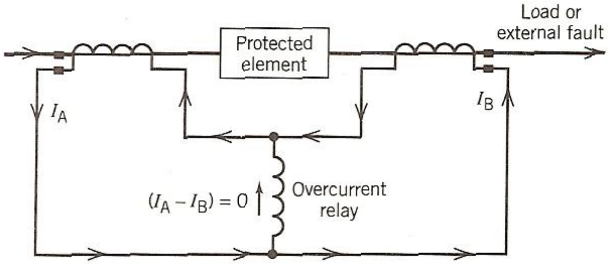
➔ **Circulating current** systems generally used for

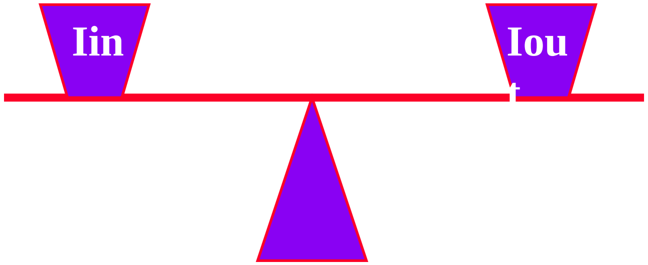
- Transformer,
- Generator,
- Busbars



➔ CT's are situated in same sub-station with common relay

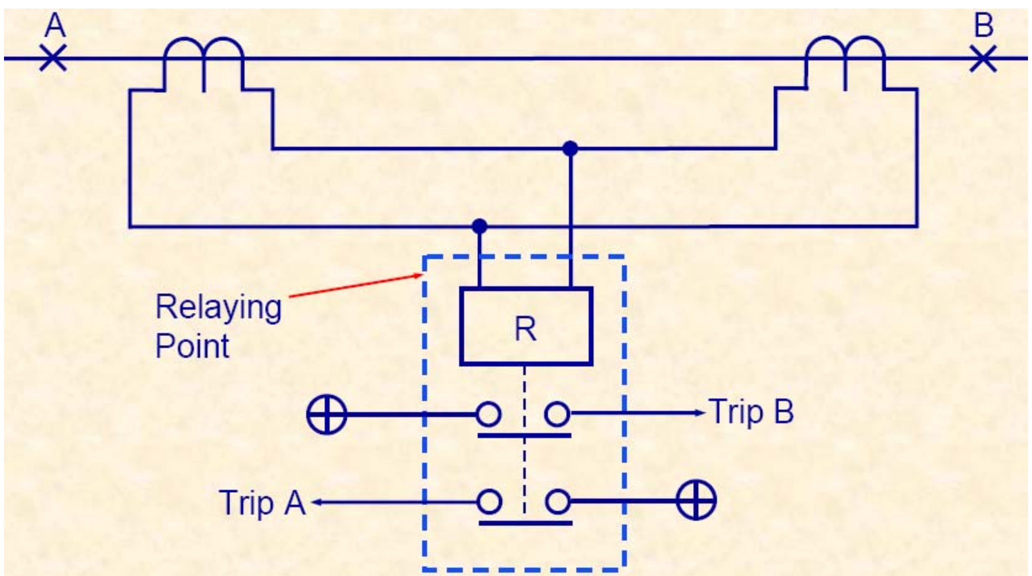
- Compares currents flowing into and leaving a protected zone
- Operates when a set value of differential (difference) currents is reached
- Analog is a balancing beam





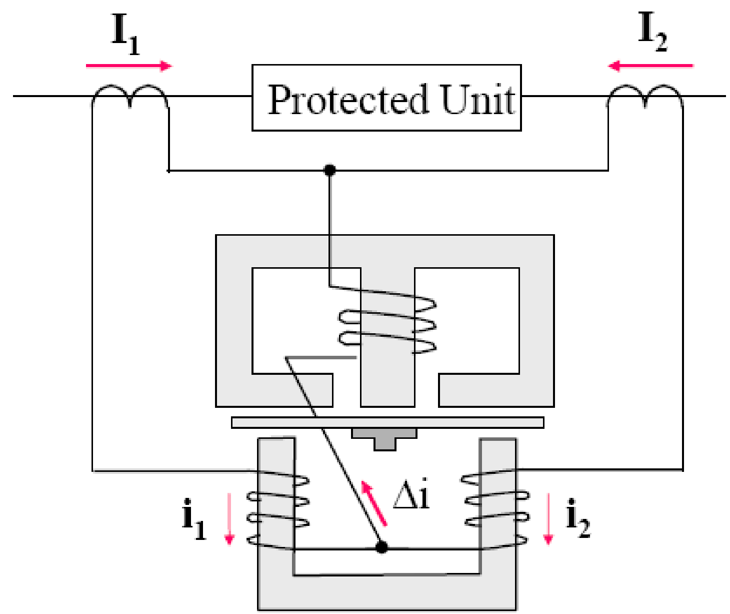
Balanced Circulating current

- Compares currents flowing into and leaving a protected zone
- Use Two sets of CTs at two ends with relay in between
- Require Matching CT's at both the ends



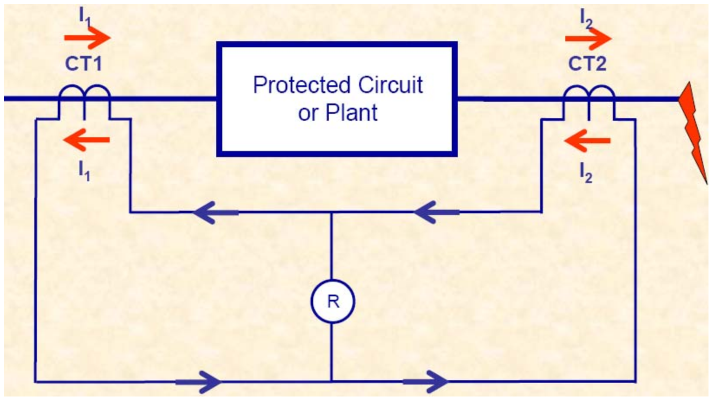
Current balance

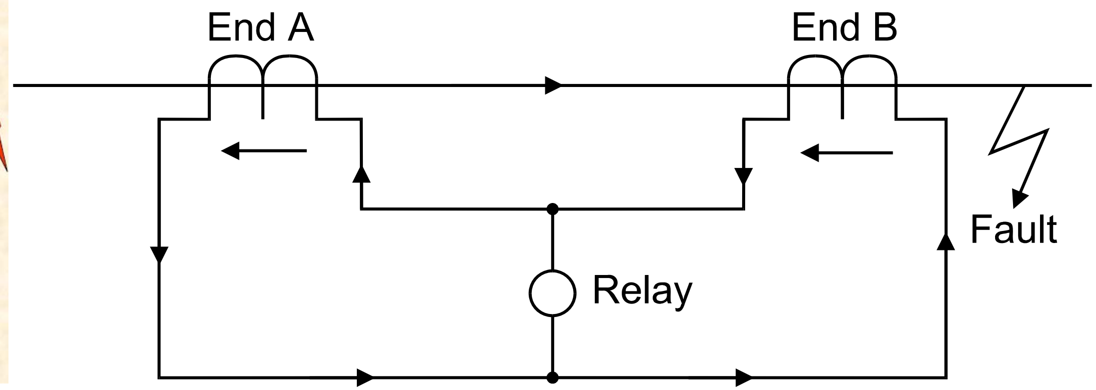
circulating current scheme

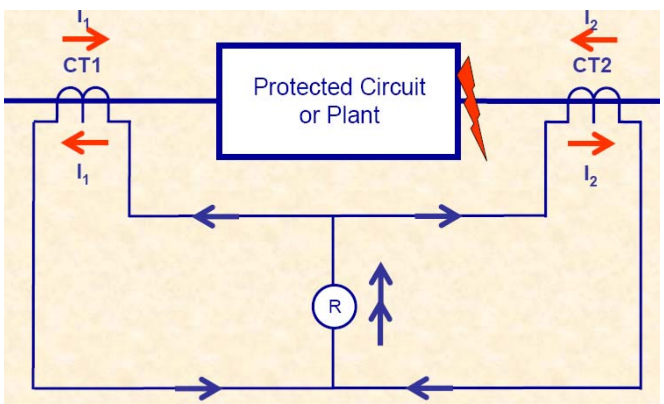


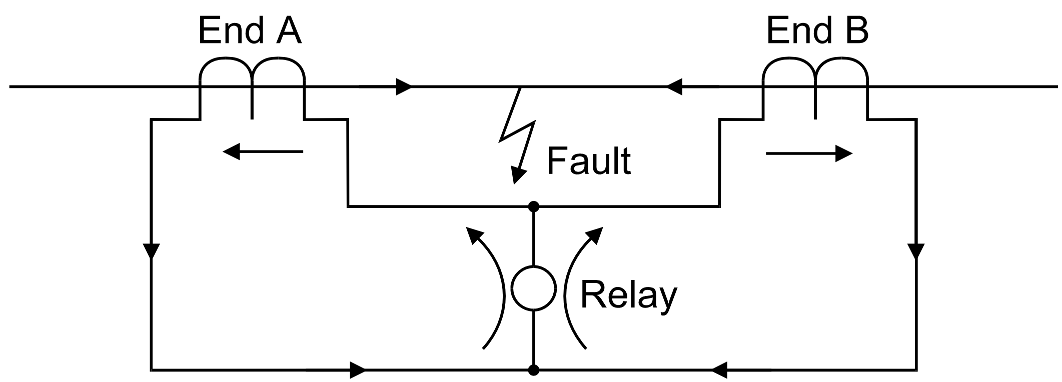
Balanced Circulating current

- **External Faults - Stable**





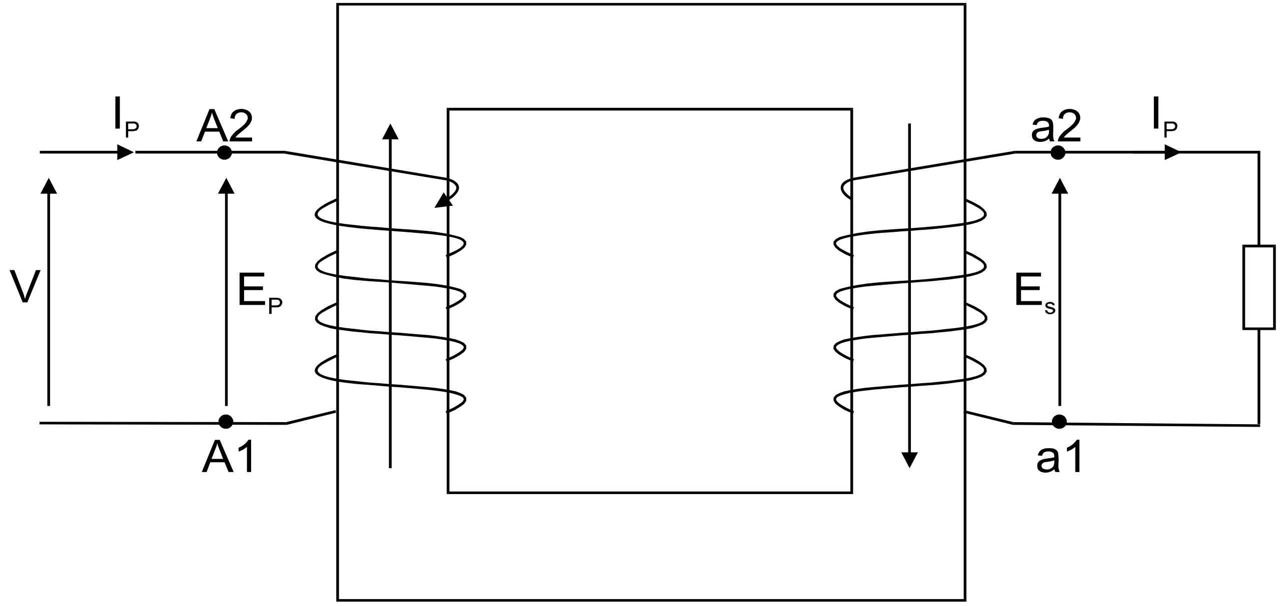




- **Internal Faults - Operates**

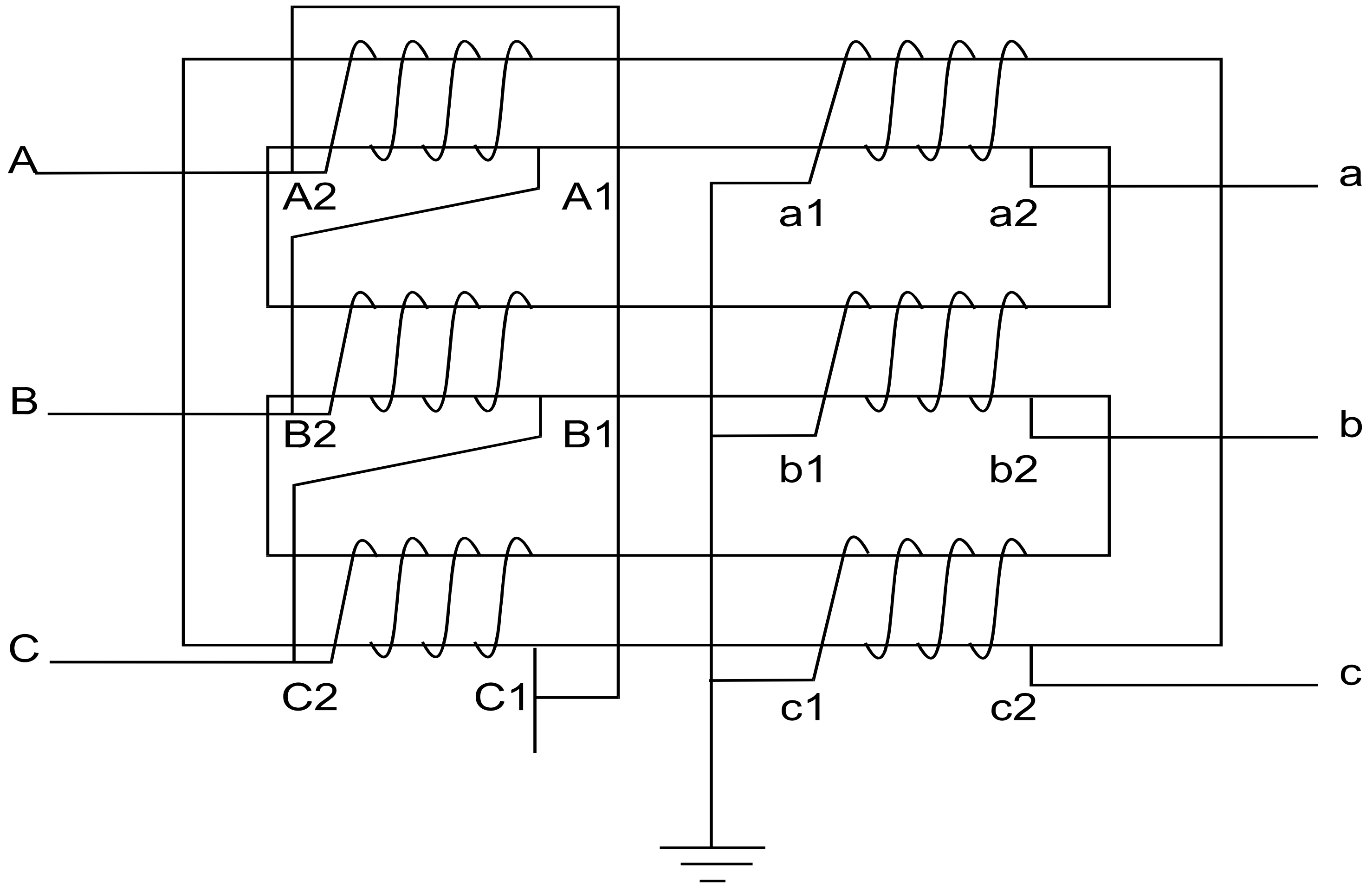
Winding Polarity

- International standards define polarity
- Current in towards **A2** on primary - Then current out from **a2** on secondary because E_s is from **a1** to **a2**

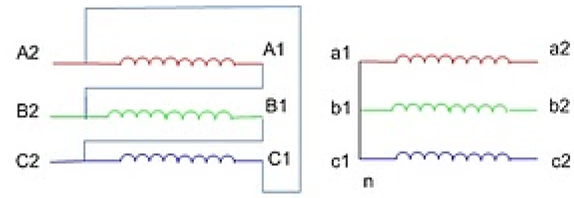


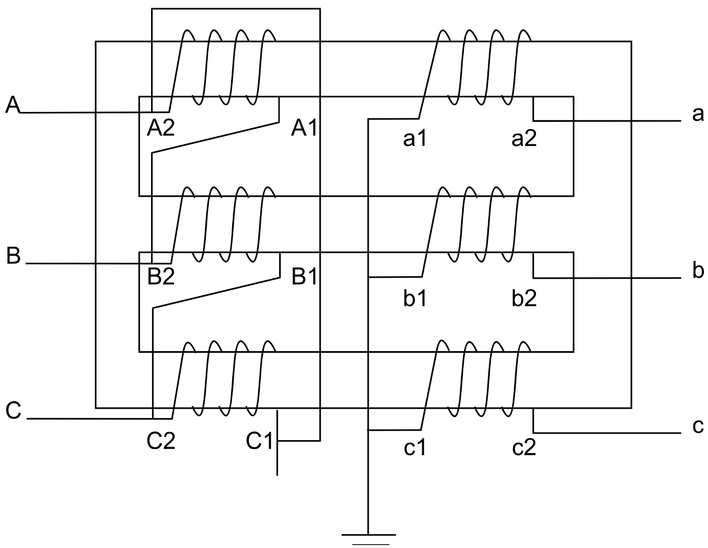
Transformer Connections

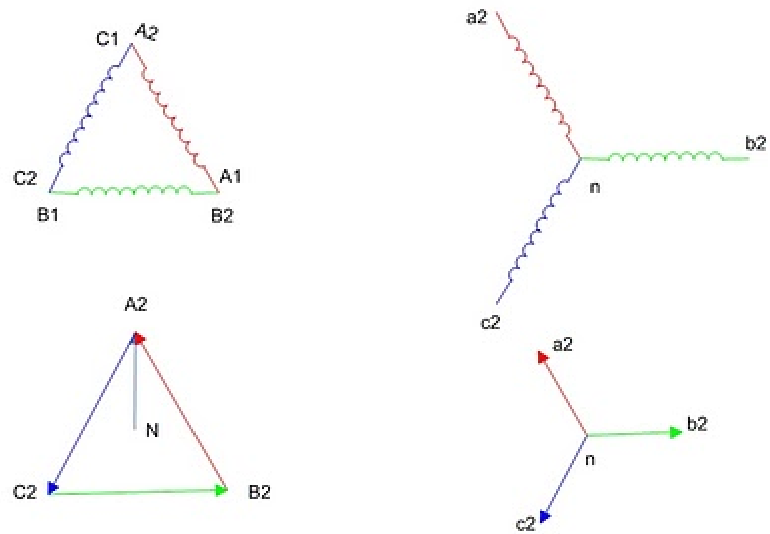
3 Phase -Typical Delta Star (Δ-Y) Connection

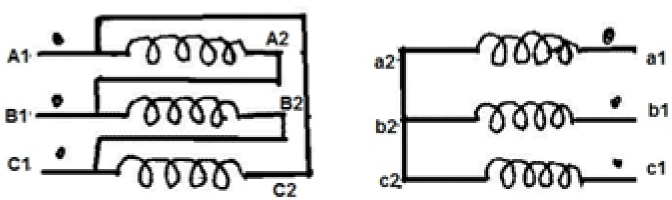


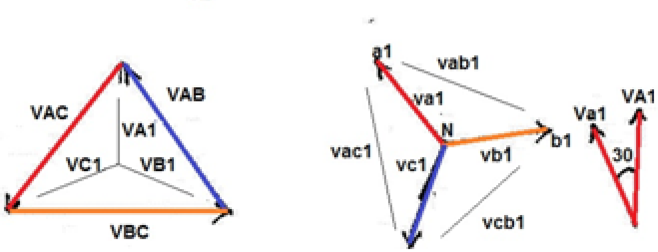
Vector Representation











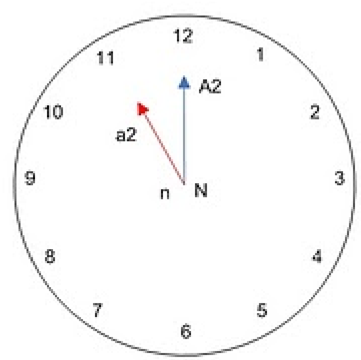
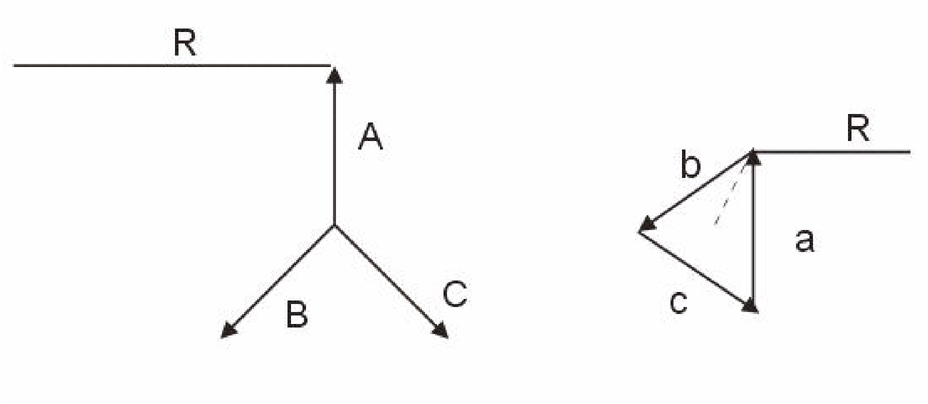


Fig D: Delta - Wye connection for Dy11 vector group

Phase shift

- Depending on how the windings in the transformer are arranged - the secondary voltages may be phase - shifted from the primary voltages e.g. Ynd1



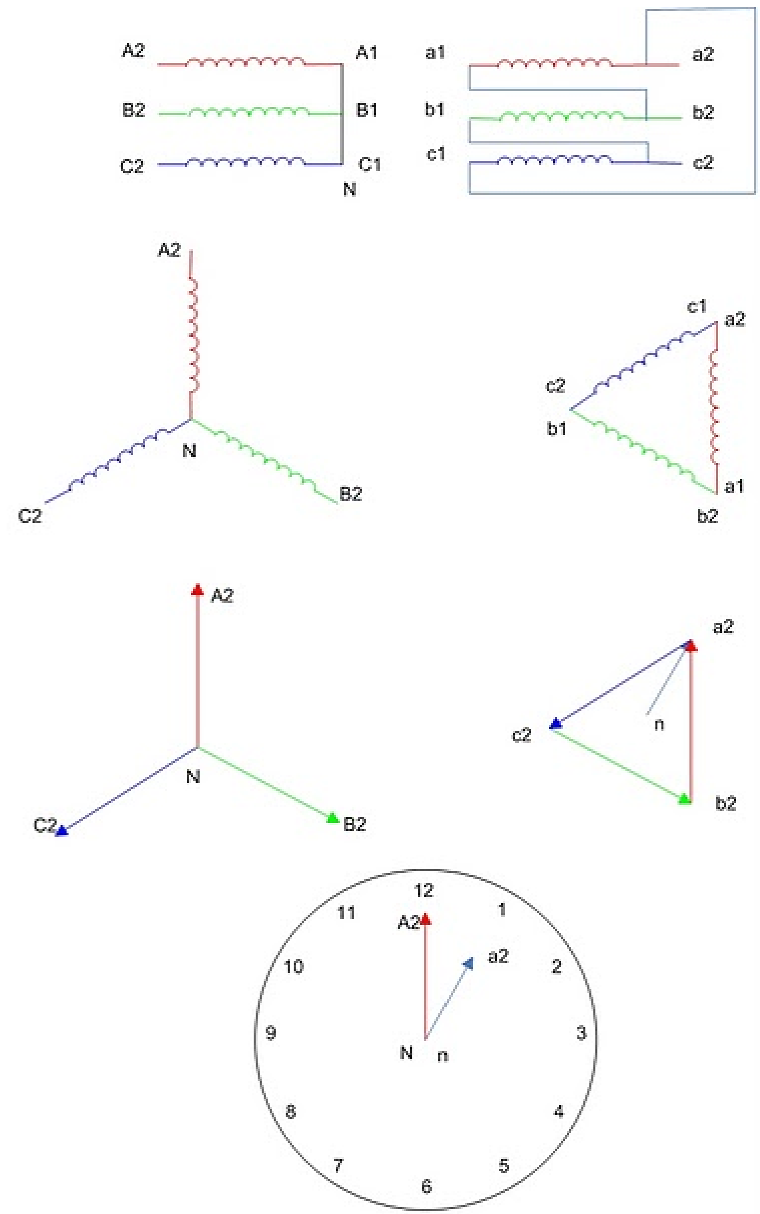


Fig B: Wye - Delta connection for Yd1 vector group

Transformer Vector Group Representation

- Phase Shift Represented by 12 hour Clock positions
- Each 30 degree corresponds to 1 hour shift
- Knowledge of Vector Group and Polarities MUST for Correct Protection

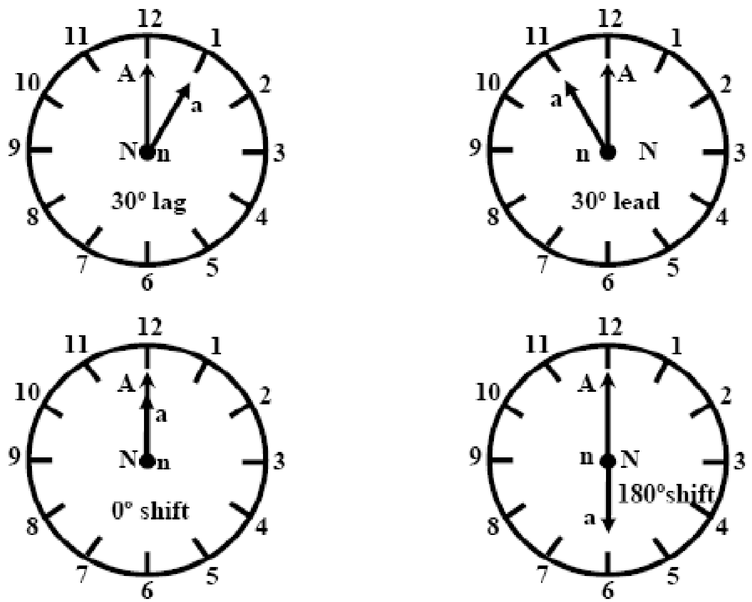


Figure2: Clock convention representing vector groups.

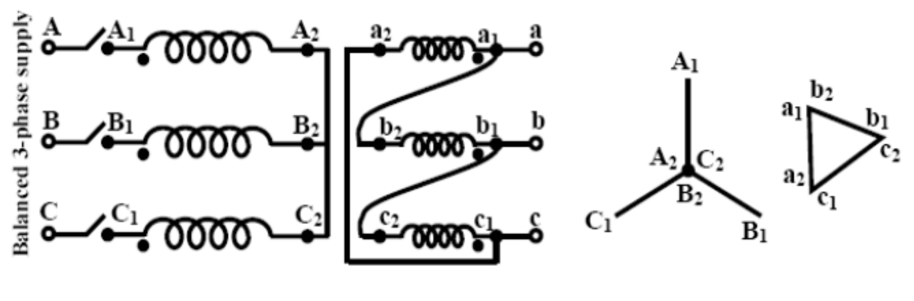


Figure 3: Connection and phasor diagram for $Y\,d_{11}$.

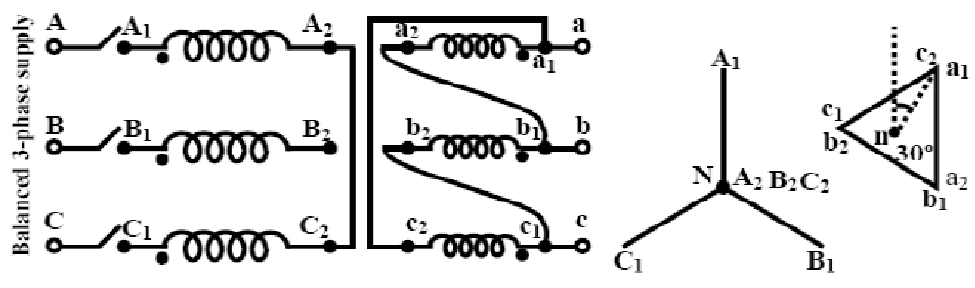
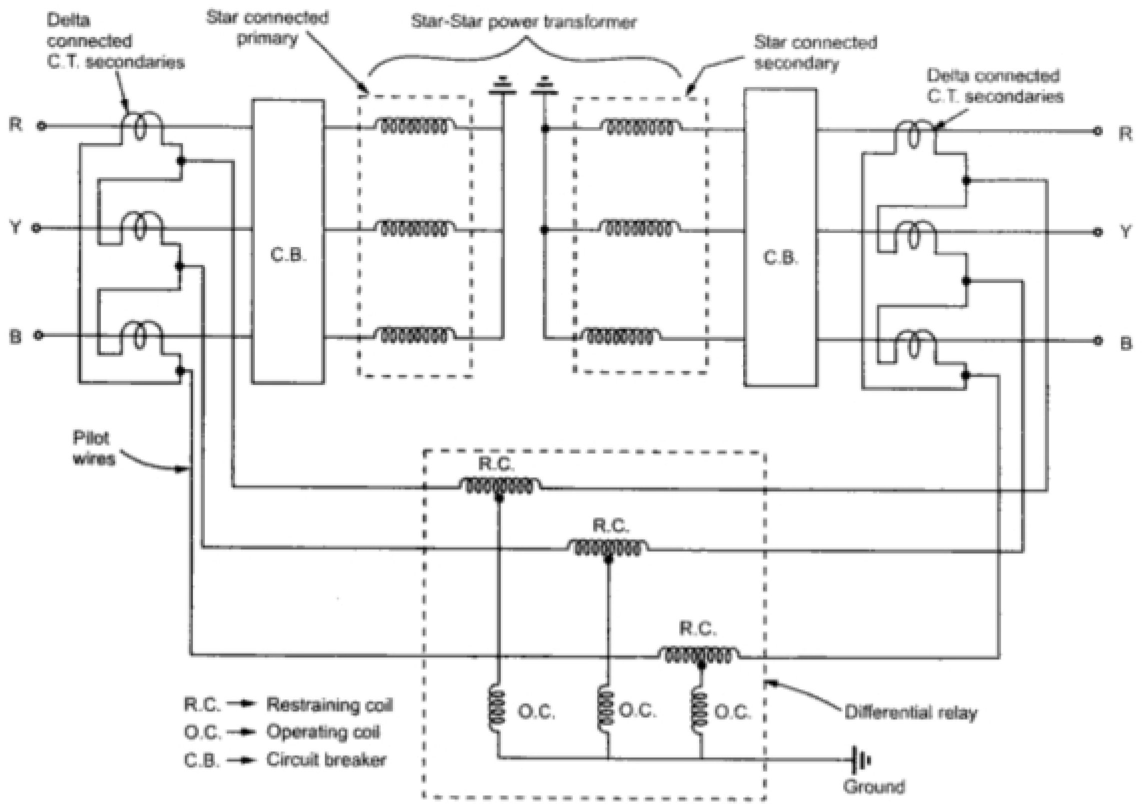
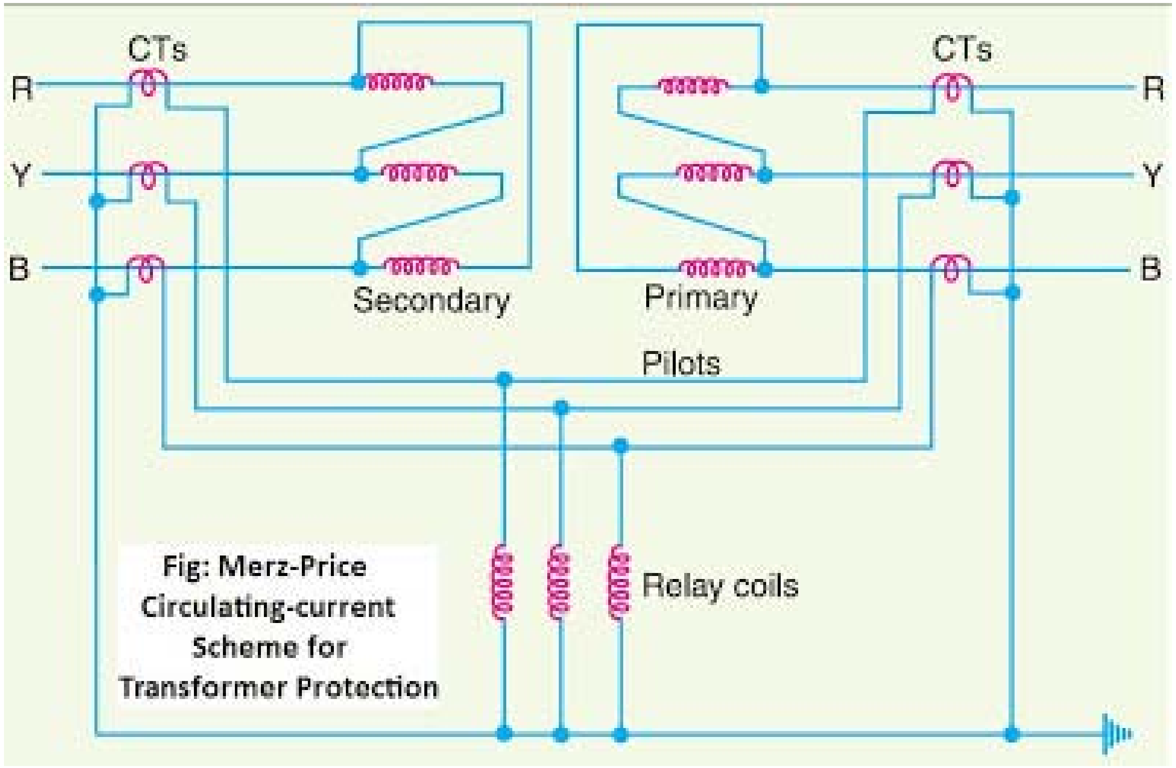


Figure 4: Connection and phasor diagram for $Y\,d_1$.

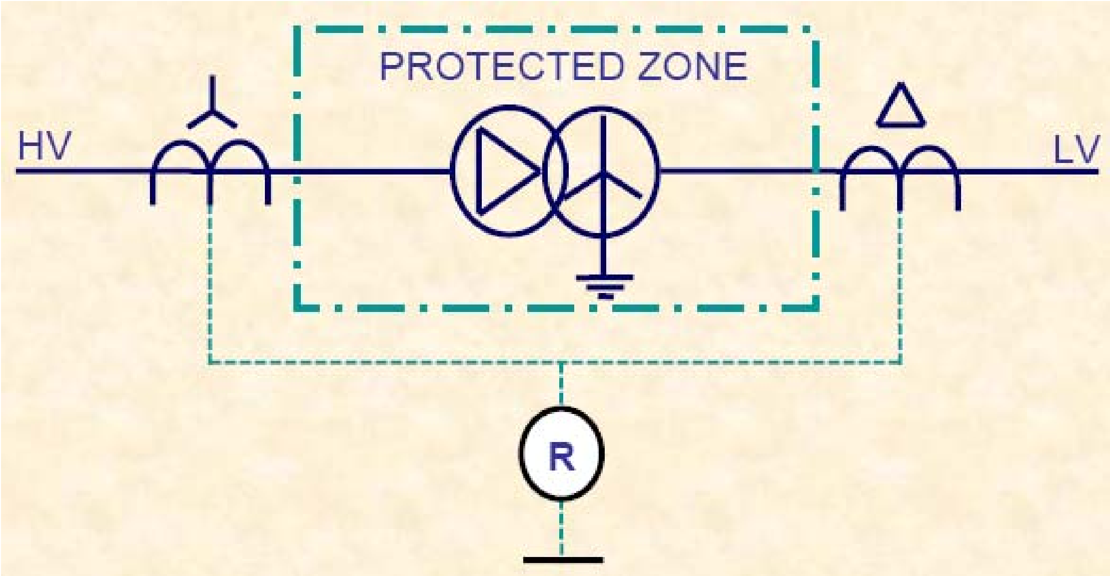
Star-Star transformer Differential Connection



Delta-Delta transformer Differential Connection



Delta-Star transformer Differential Connection



Correct application of differential protection requires CT ratio and winding connections to match those of transformer.

CT secondary circuit should be a “replica”of primary system.

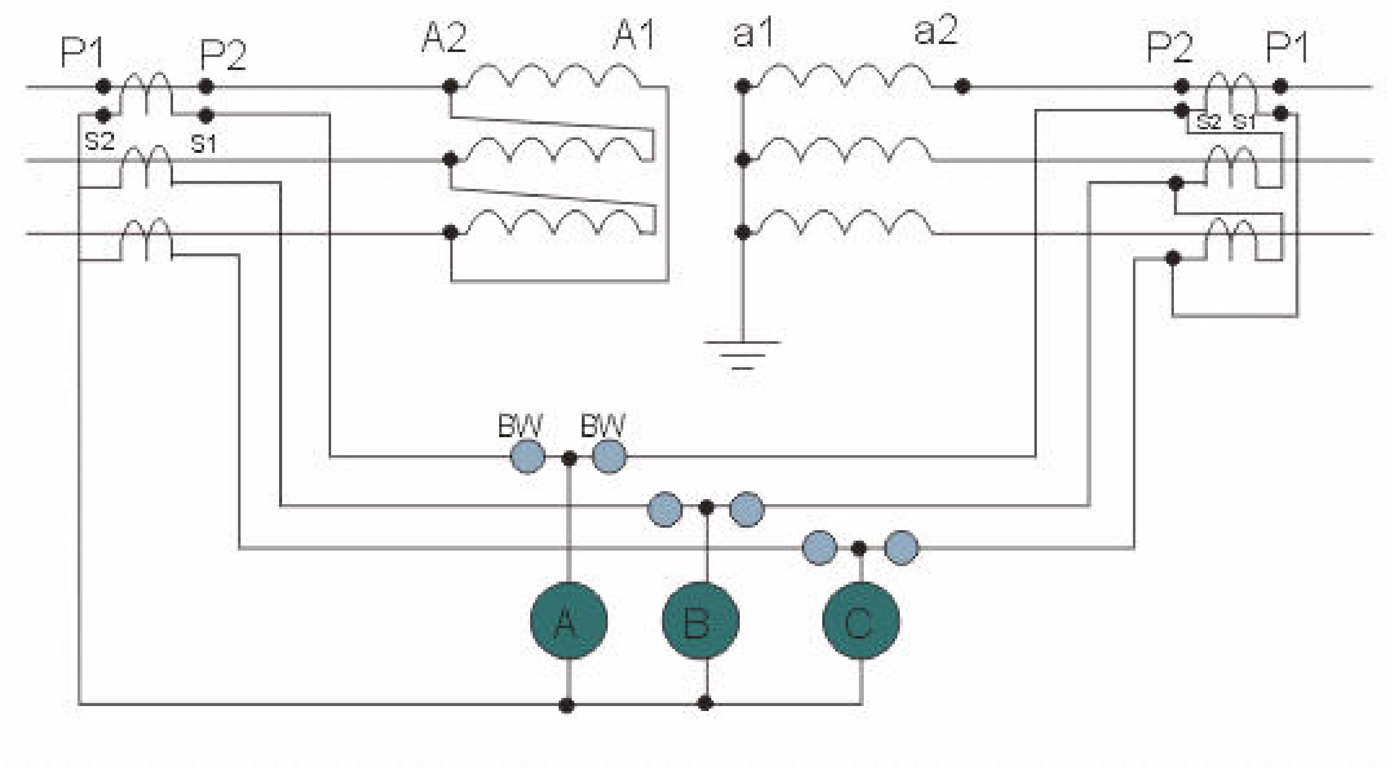
Consider:

(1)Difference in current magnitude

(2)Phase shift

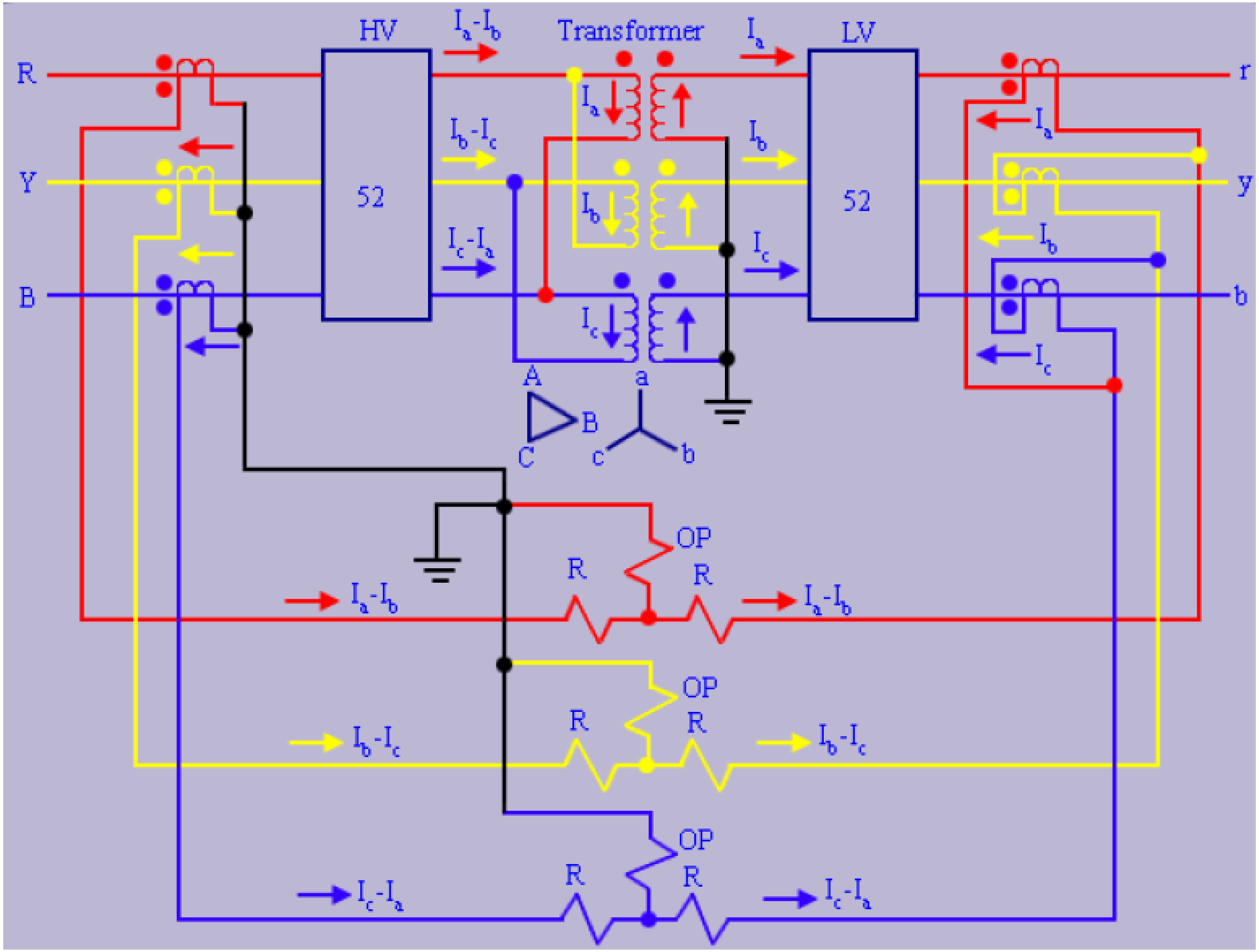
(3)Zero sequence currents

Delta-Star transformer Differential Connection

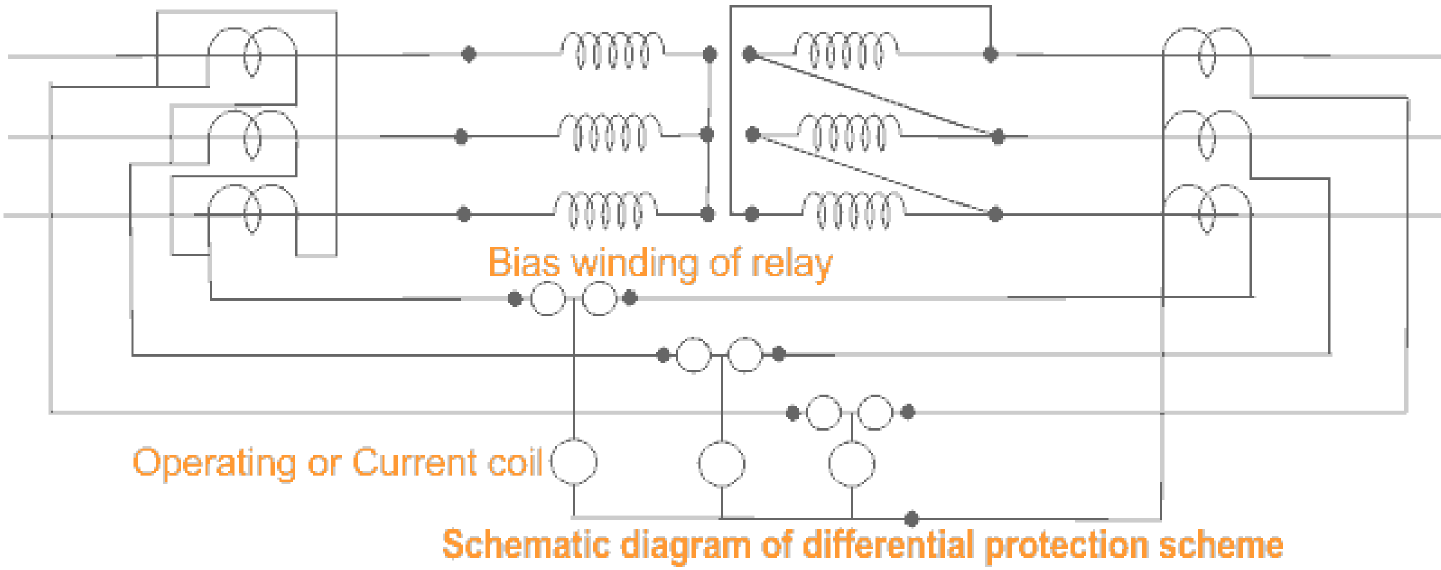


Connect HV and LV CT's in Star delta opposite to the vector group connections of the primary windings

Delta-Star transformer Differential Connection



Star- Delta transformer Differential Connection

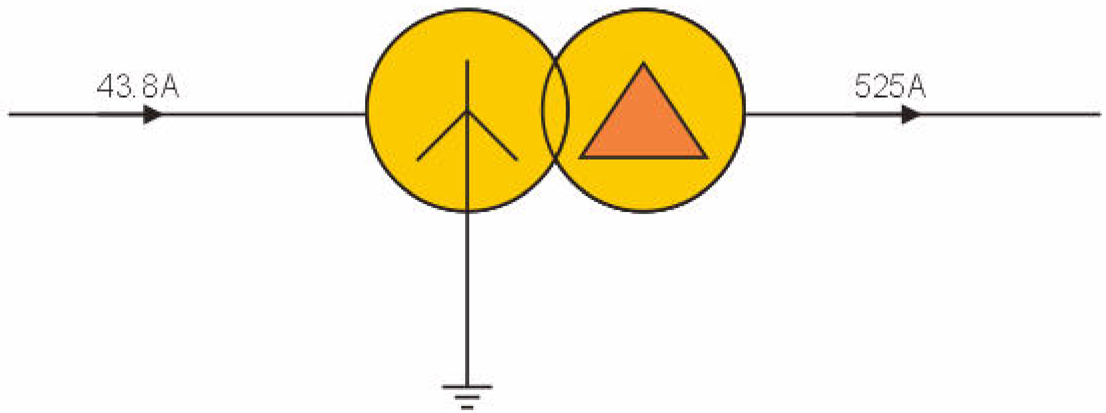


Connect HV and LV CT's in delta Star opposite to the vector group connections of the primary windings

Mis-match of CT's

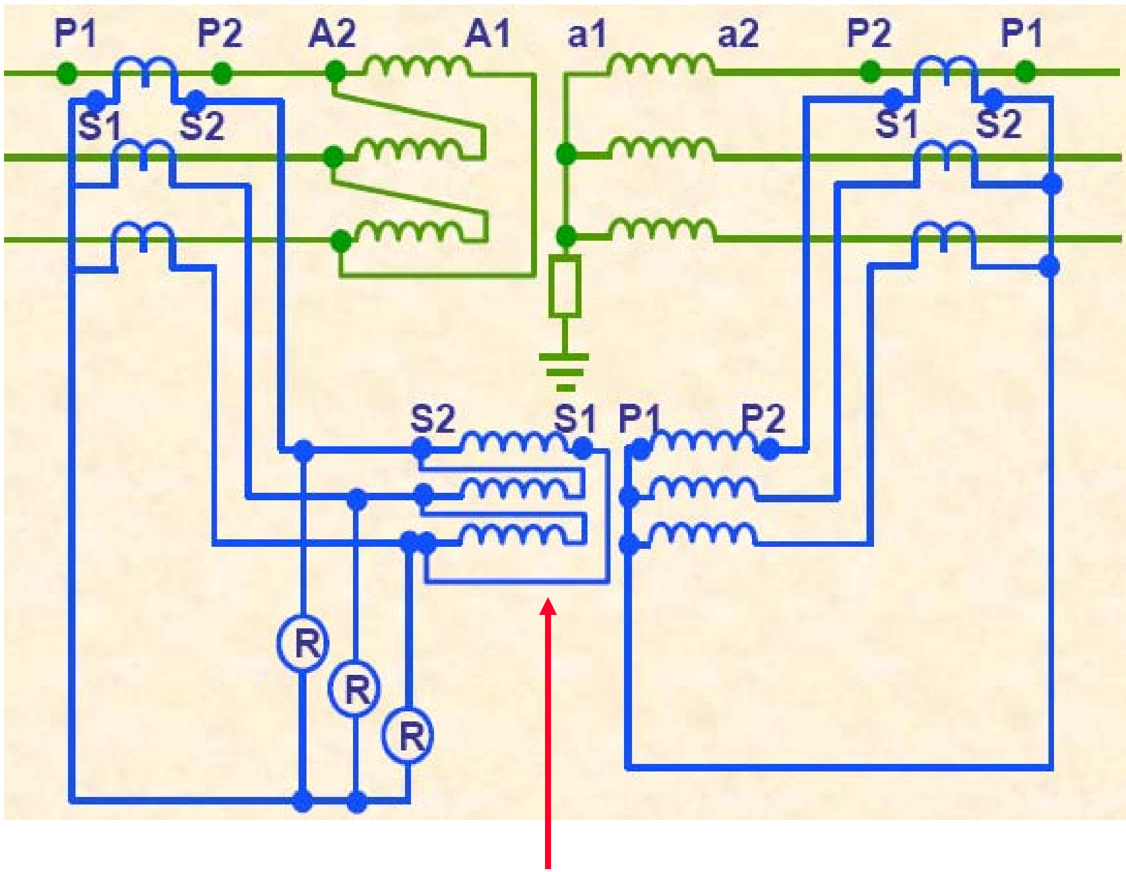
132/11 kV

10 MVA



- **CT's Required on primary and secondary for Protection**
- **The closest ratio available to 43.8 amps is 50/1 ➔ 0.876 secondary Amps**
- **the closest ratio to 525 Amps is probably 500/1 ➔ 1.05 secondary Amps**
- **Also, the CT's could be from different manufacturers**
- **Auxilairy or Matching Transformer is required**

Transformer Differential Protection Relay



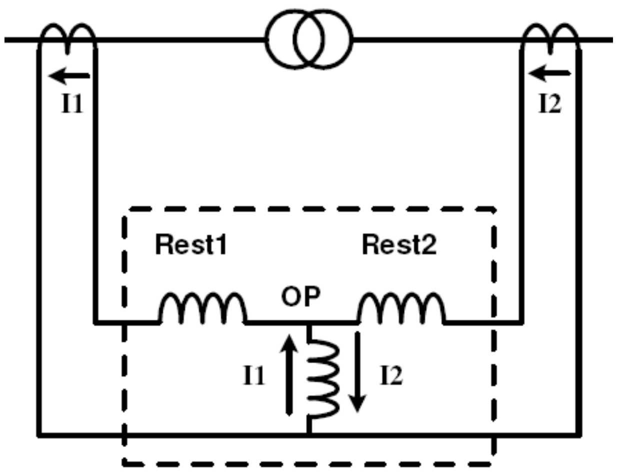
Matching Transformer

Interposing CT provides :

- Vector correction
- Ratio correction
- Zero sequence compensation

Biased Differential Relay

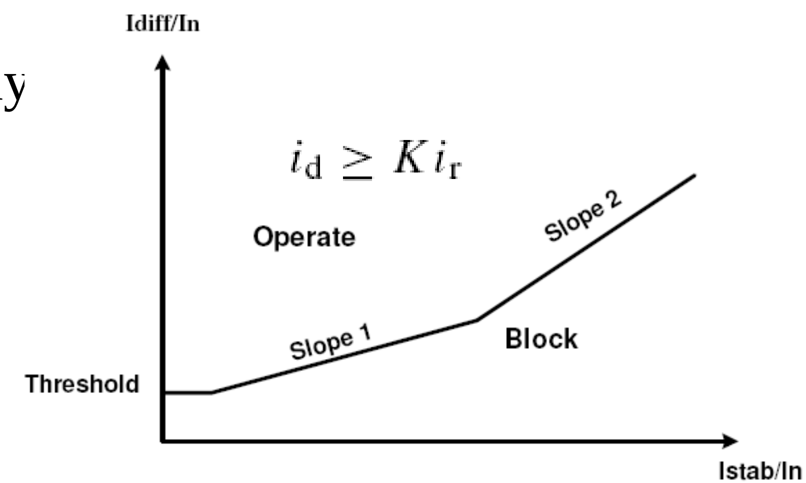
- Large external fault may cause false operation of simple differential relay (because of CT Saturation) .
- To make the differential relay more stable to external faults and improve relay quality, restraining coils were inserted.
- Two restraining (Biasing) coils and one operating are used as shown. Restraining coils will opposite the operation of operating coil. The relay will operate only when the operating force is higher than restraining force.



Biased Differential Relay

Measurement

- $I_{bias} = (|I_1| + |I_2|)/2$
- $I_{diff} = |I_1 + I_2|$

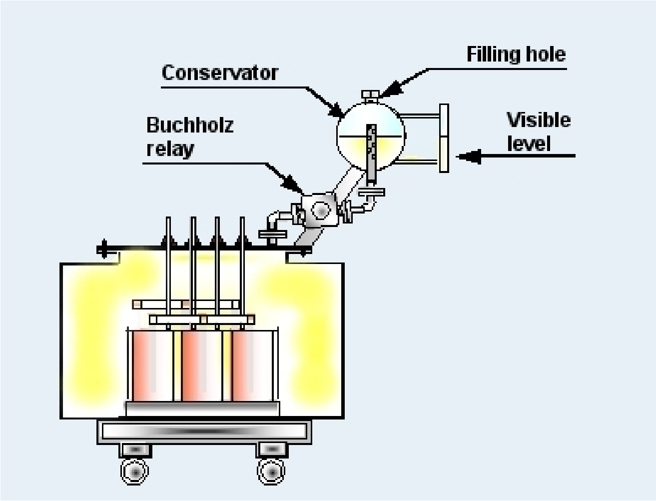


Stability provided by Biasing

Tripping Characteristics of Biased Differential Relay

Buchholz Protection

- Failure of the winding insulation will result in some form of arcing which can decompose the oil into Hydrogen, acetylene and methane.
- Localized heating can precipitate a breakdown in the oil into gas.
- Severe arcing will cause a rapid release of a large volume of gas as well as oil vapor. The action can be so violent that the build-up of pressure can cause an oil surge from the tank to the conservator.
- Buchholz relay can detect both gas and oil surges as it is mounted in the pipe to the conservator.



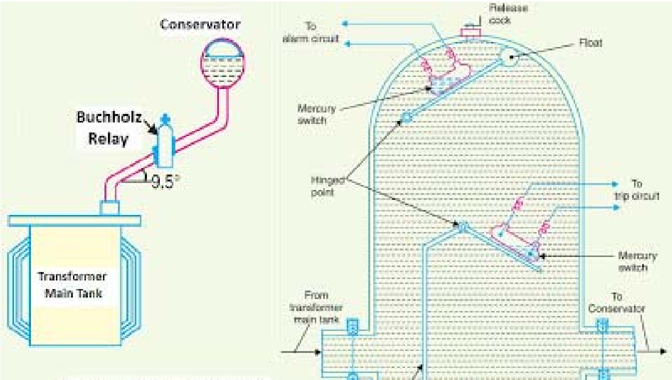
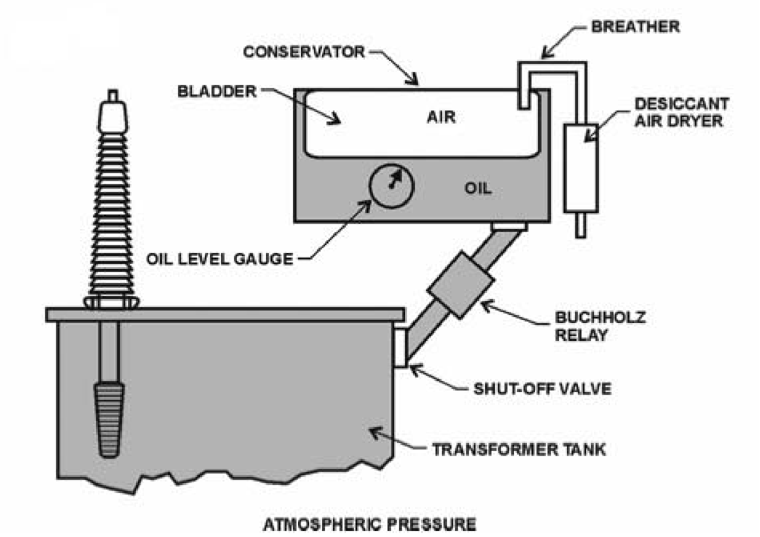
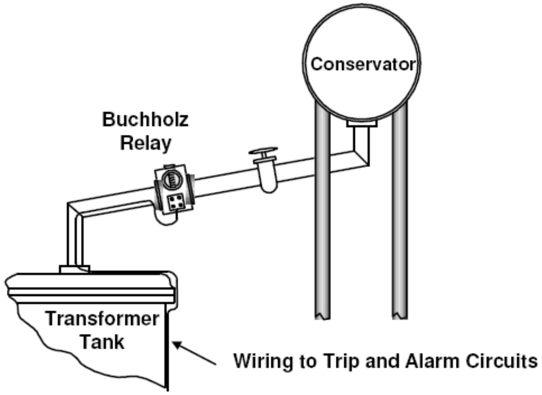


Fig: Buchholz Relay with Constructional Details

Buchholz Protection



A **Buchholz relay, is a safety device mounted on some oil-filled power** <u>transformers</u> equipped with an external overhead oil reservoir called a *conservator*.



The Buchholz Relay is used as a protective device sensitive to the effects of <u>dielectric</u> failure inside the equipment.

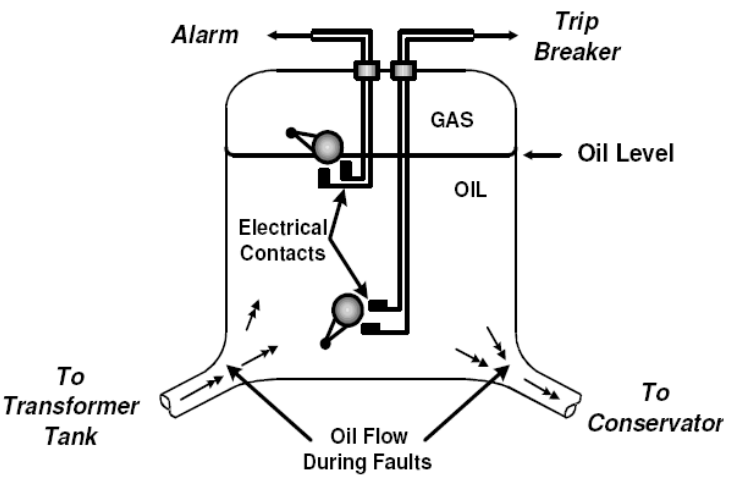
Buchholz Protection

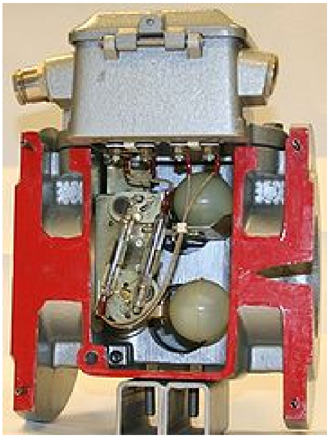
- **Gas discharge: The Buchholz relay detects gas bubbles**
- **Excess tank pressure: The Buchholz relay detects a rapid flow of dielectric fluid from the transformer's tank to the expansion tank**

In Buchholz relays:
A **first mercury contact** detects gas discharge and **initiate an alarm signal**.

A **second mercury contact** detects rapid flow of dielectric fluid from the transformer to the expansion tank and **initiate a trip signal**.

Transformer manufacturers usually mount Buchholz relays as standard equipment on expansion-tank transformers.





Transformer Overloading

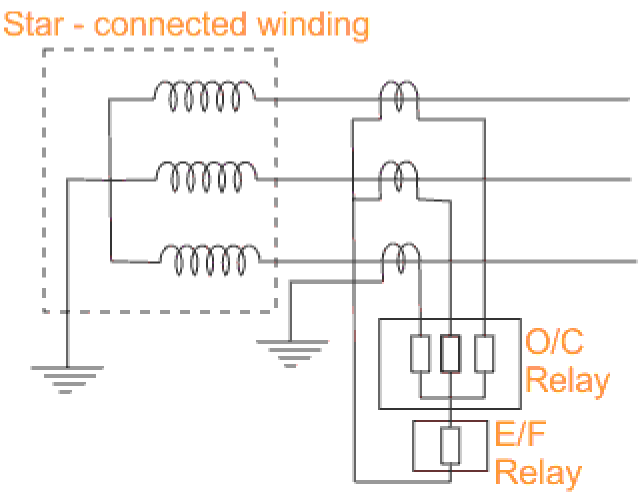
- Sustained overloading reduces transformer life
- Operating Temperatures also decide the transformer oil life

Operating Temperature	Oil Life
• 60 deg C	20 years
• 70 deg C	10 years
• 80 deg C	5 years
• 90 deg C	2.5 years
• 100 deg C	13 months
• 110 deg C	7 months

Over Current and Earth Fault Protection of Transformer

- **Backup protection of electrical transformer** is simple **Over Current and Earth Fault protection** applied against external short circuit and excessive over loads.
- These overcurrent and earth Fault relays may be of Inverse Definite Minimum Time (IDMT) or Definite Time type relays.
- Generally IDMT relays are connected to the in-feed side of the transformer .

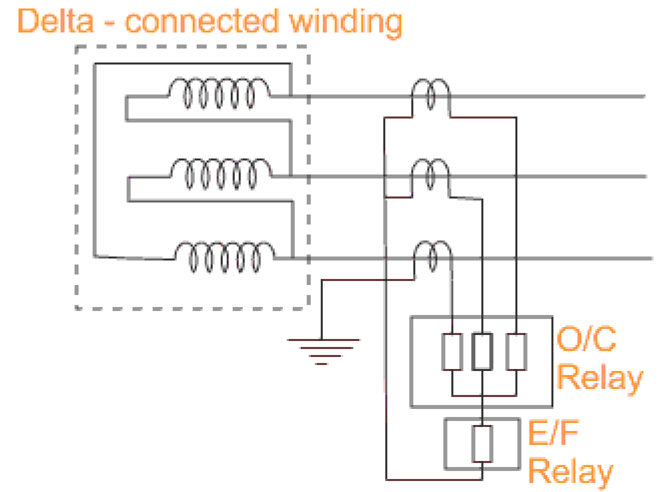
Over Current and Earth Fault protection relays may be also provided in load side of the transformer too, but it should not interrupt the primary side Circuit Breaker



Backup O/C & E/F Protection Scheme

Over Current and Earth Fault Protection of Transformer

- **Backup protection of transformer** has four elements, three **OC relays** connected each in each phase and one **EF relay** connected to the common point of three **OC relays**.
- The normal range of current settings available on IDMT **OC relays** is 50% to 200% and on **EF relay** 20 to 80%.

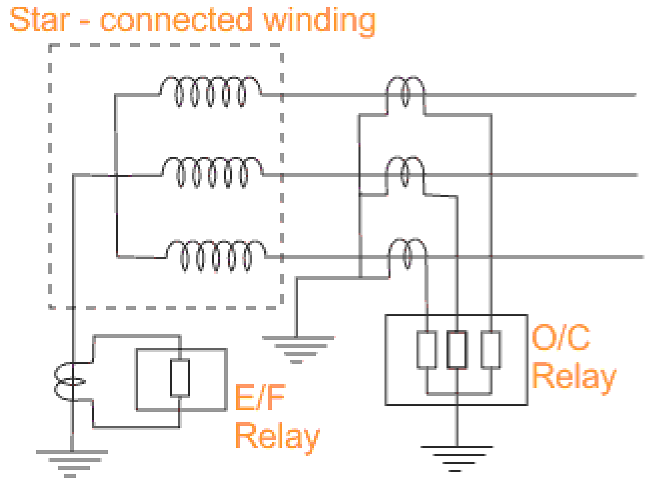


Backup O/C & E/F Protection Scheme

Over Current and Earth Fault Protection of Transformer

- In the case of transformer winding with **neutral earthed**, **unrestricted earth fault protection** is obtained by connecting an ordinary earth fault relay across a neutral current transformer.

- The unrestricted **OC relays** and **EF relay** should have proper time lag to coordinate with the protective relays of other circuit to avoid



O/C & Unrestricted E/F Protection Scheme

Over Current and Earth Fault Protection of Transformer

- In the case of transformer winding with **neutral earthed**, **unrestricted earth fault protection** is obtained by connecting an ordinary earth fault relay across a neutral current transformer.

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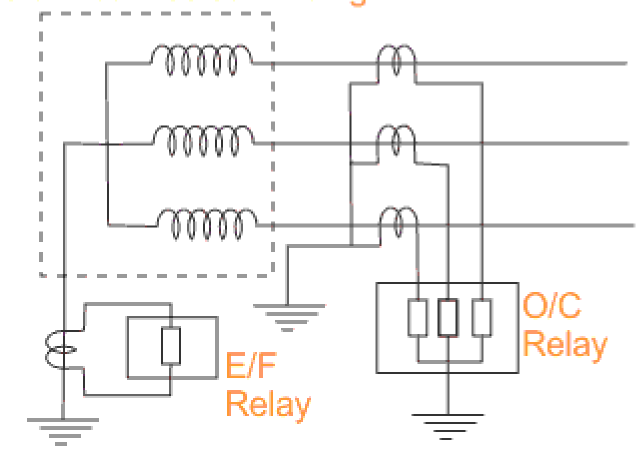
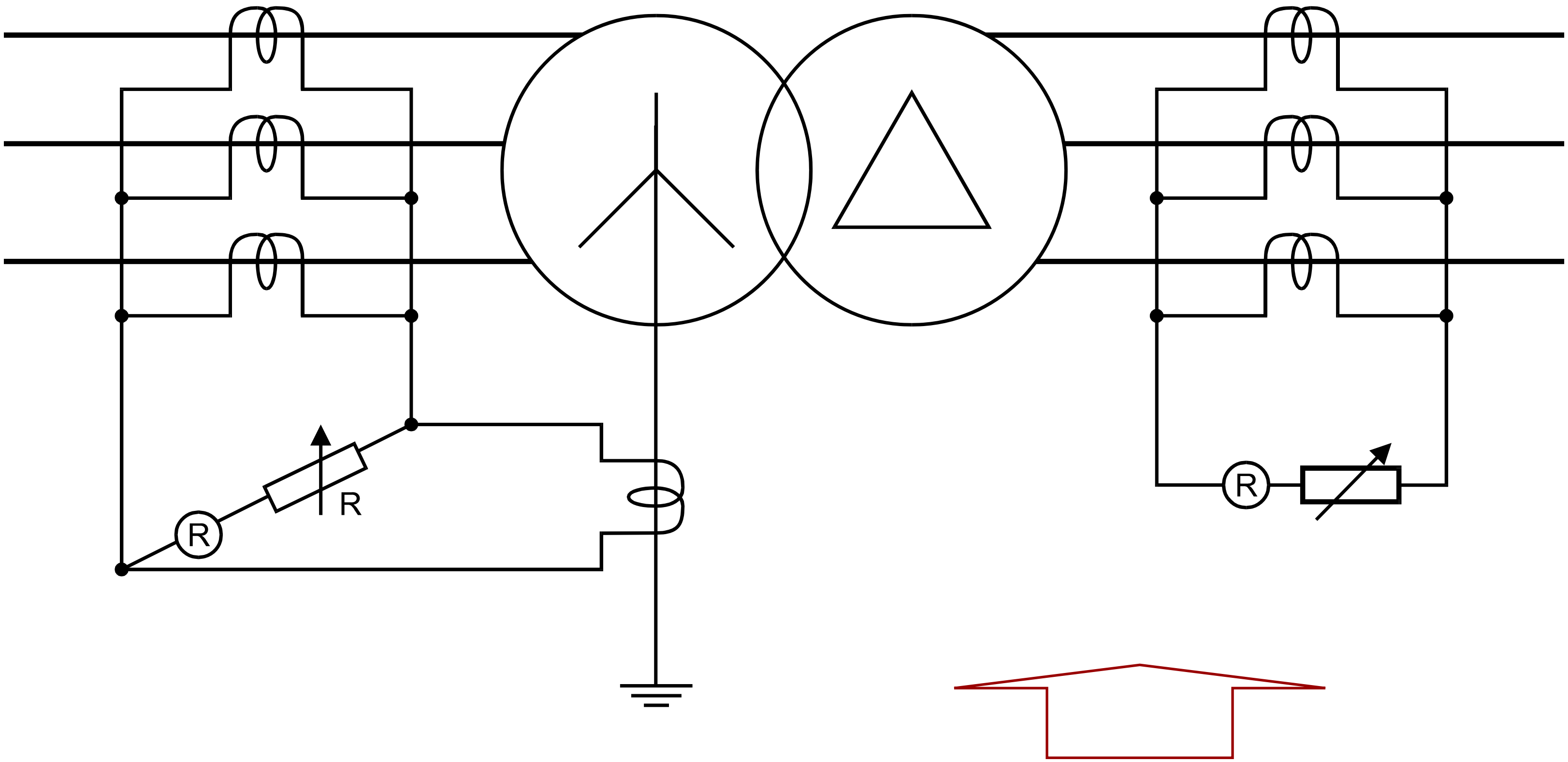


O/C & Unrestricted E/F Protection Scheme

A Restricted Earth Fault System



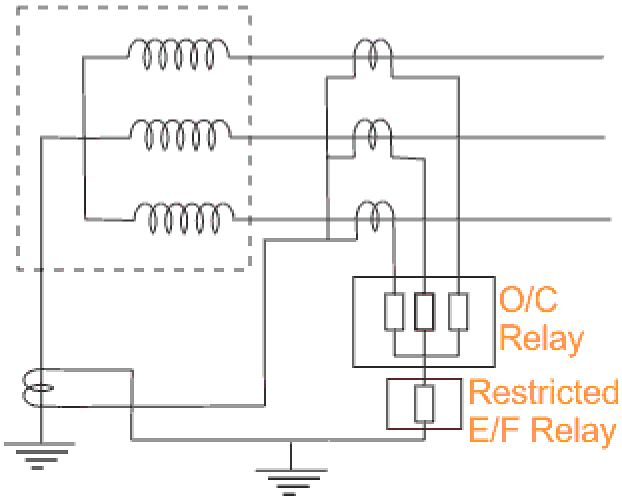
CT currents balance - no operating voltage to relay

Any residual current will cause relay to operate for E/F in zone

Restricted Earth Fault (REF) Protection

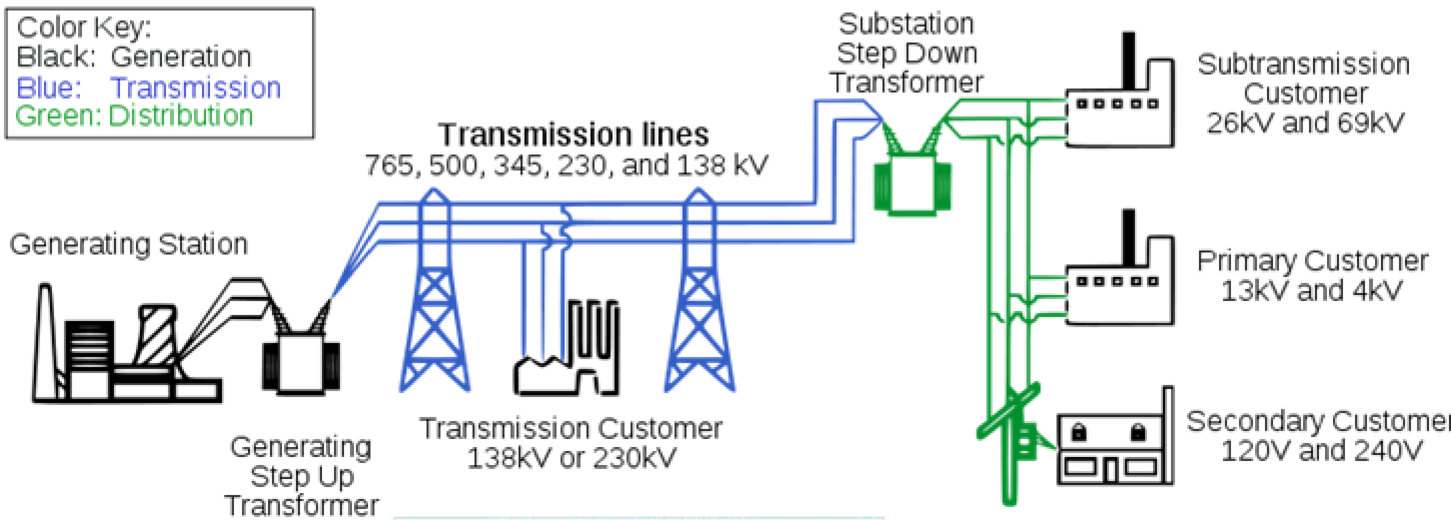
- On the HV side, the **residual current** of the 3 line CT's is balanced against the output current of the CT in the **neutral conductor**.
- The **REF relay** will not be actuated for **external earth fault**. But during **internal fault** the neutral current transformer only carries the **unbalance fault** **current** and operation of **REF Relay** takes place.
- Both windings of the transformer can thus be protected separately with restricted **REF Relay** .
- Provide high speed protection against earth faults over the whole of the transformer windings.
- Relay used is an instantaneous type.

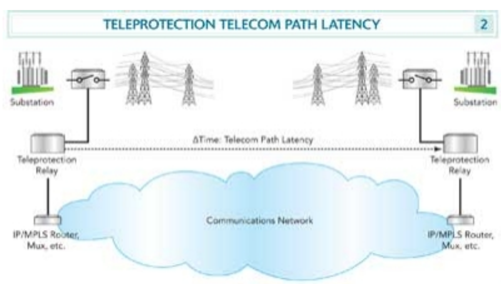
Restricted E/F Protection Scheme with O/C

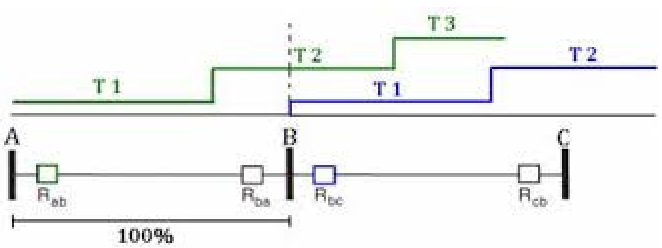


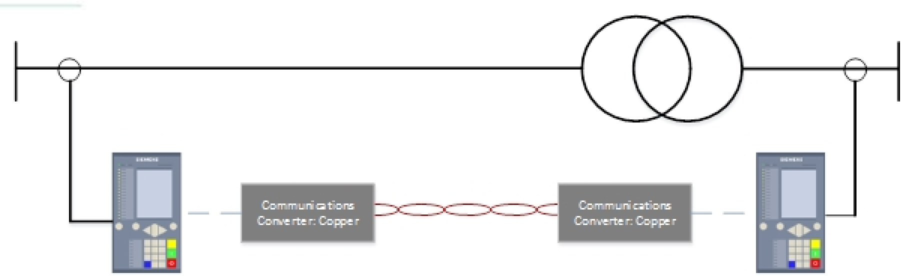
Topic 1: Power System Protection

1.6: Distance Protection



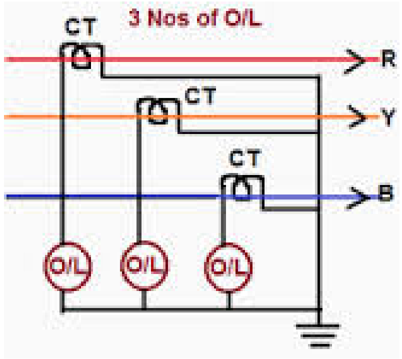




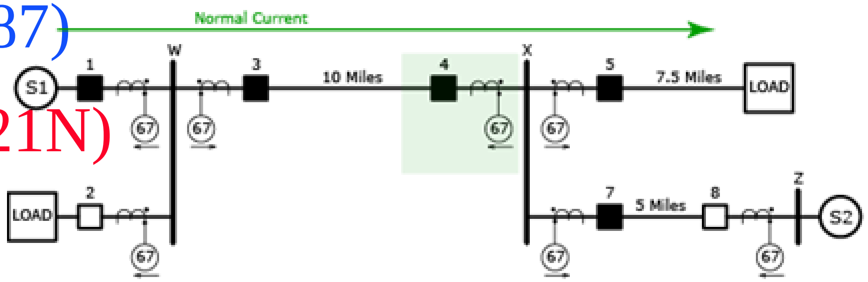


Transmission Line Protection

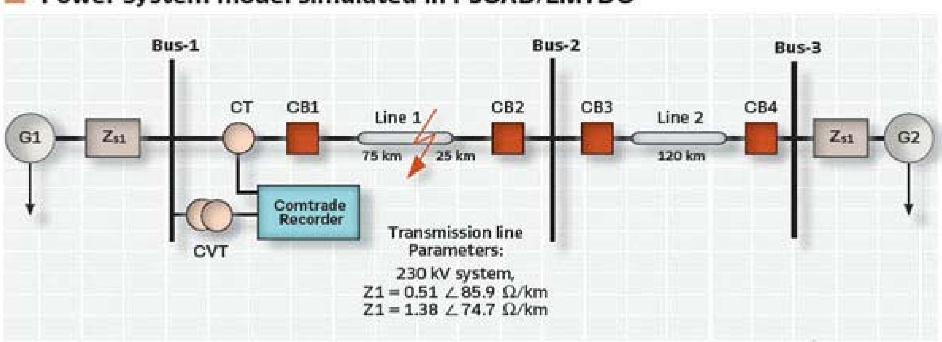
How Do We Protect Transmission Lines?

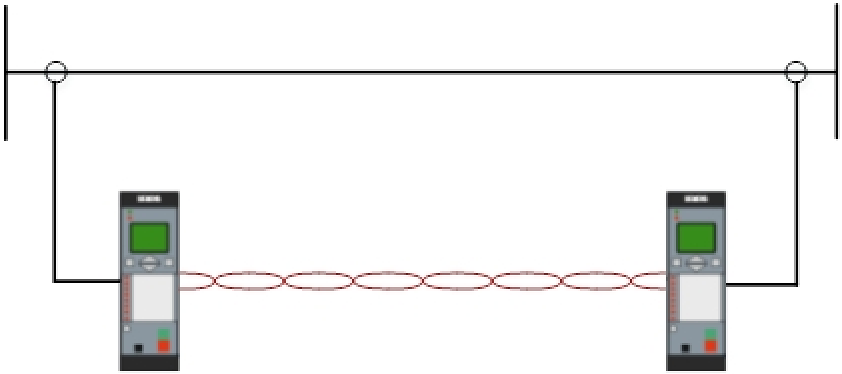


- ➔ **Overcurrent** (50, 51, 50N, 51N)
- ➔ **Directional Overcurrent** (67, 67N)
- ➔ **Pilot Wire Protection**
- ➔ **Line Current Differential** (87)
- ➔ **Distance (Impedance)** (21, 21N)



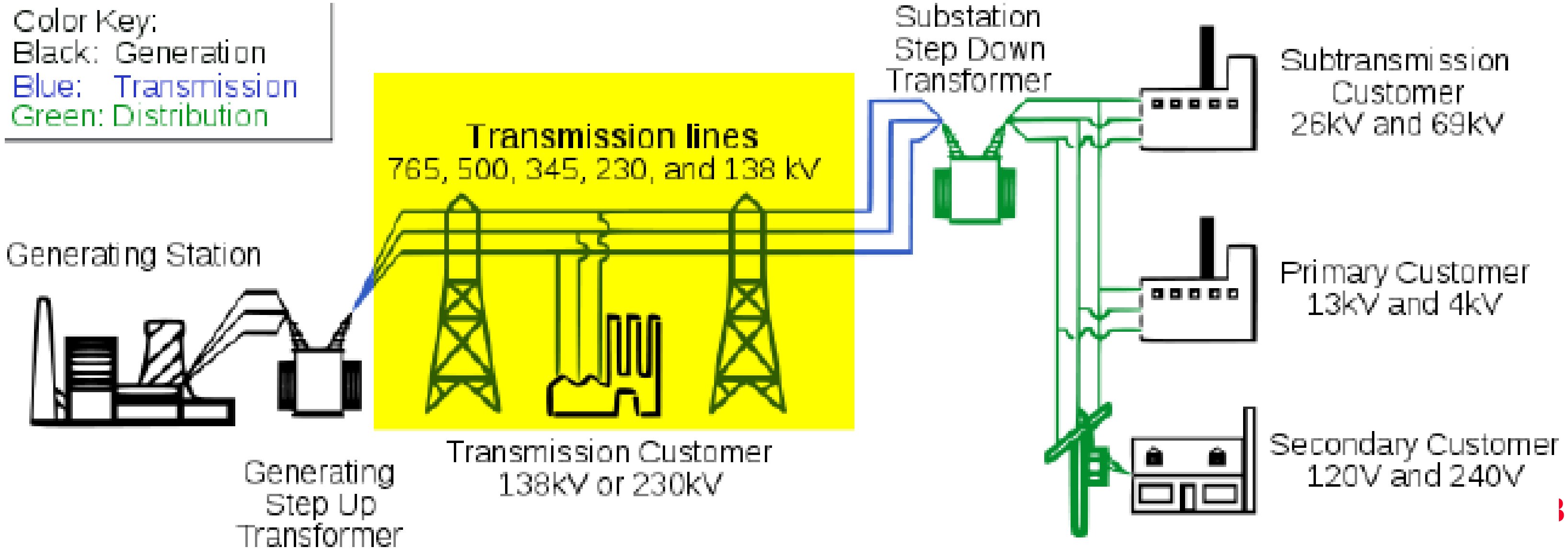
2 Power system model simulated in PSCAD/EMTDC





Transmission Line Protection

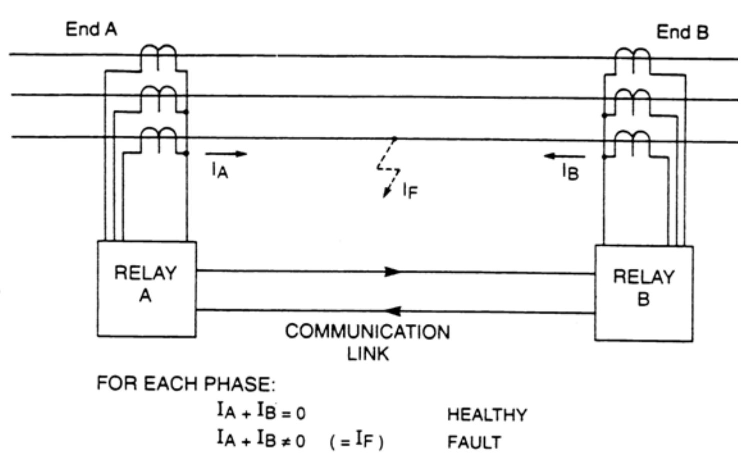
- **Overcurrent Protection**
  - Non-Directional
  - Relay responds to overcurrent condition
- **Instantaneous (IOC) device #50**
  - No intentional time delay
- **Time Overcurrent (TOC) device #51**

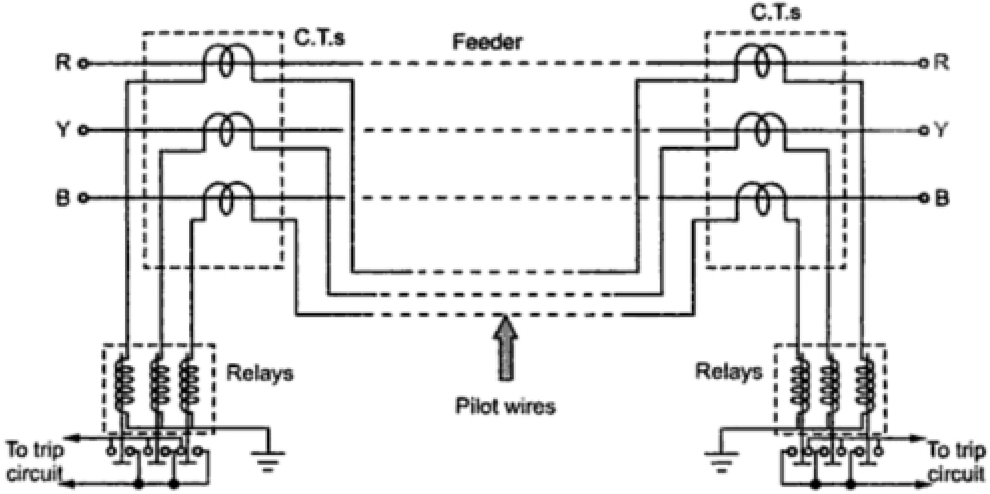


Pilot wire differential Protection

- **Protection of Cables and Short Lines**
- **Pilot protection schemes** use communication channels to send information from the local relay terminal to the remote relay terminal, thereby allowing high-speed tripping for faults occurring within 100% of the protected line.

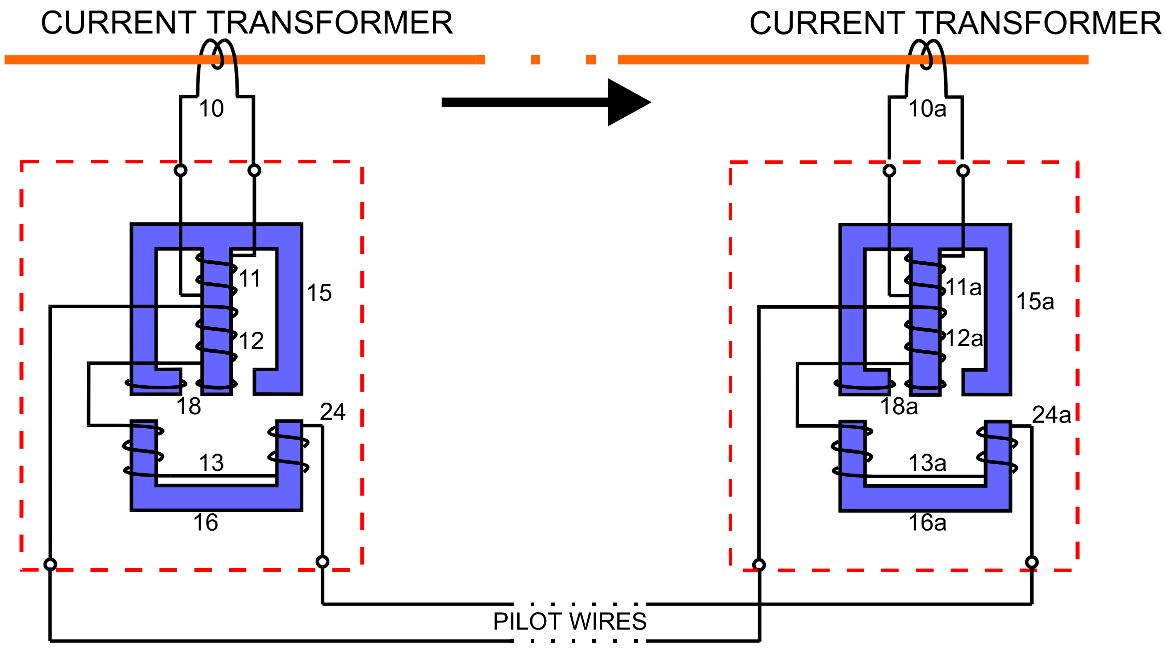
- **Pilot wire protection**: Pilot protection in which a metallic circuit is used for the communications channel between relays at the circuit terminals.

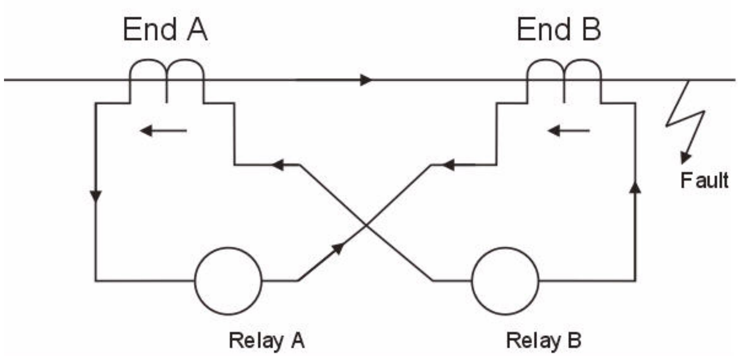




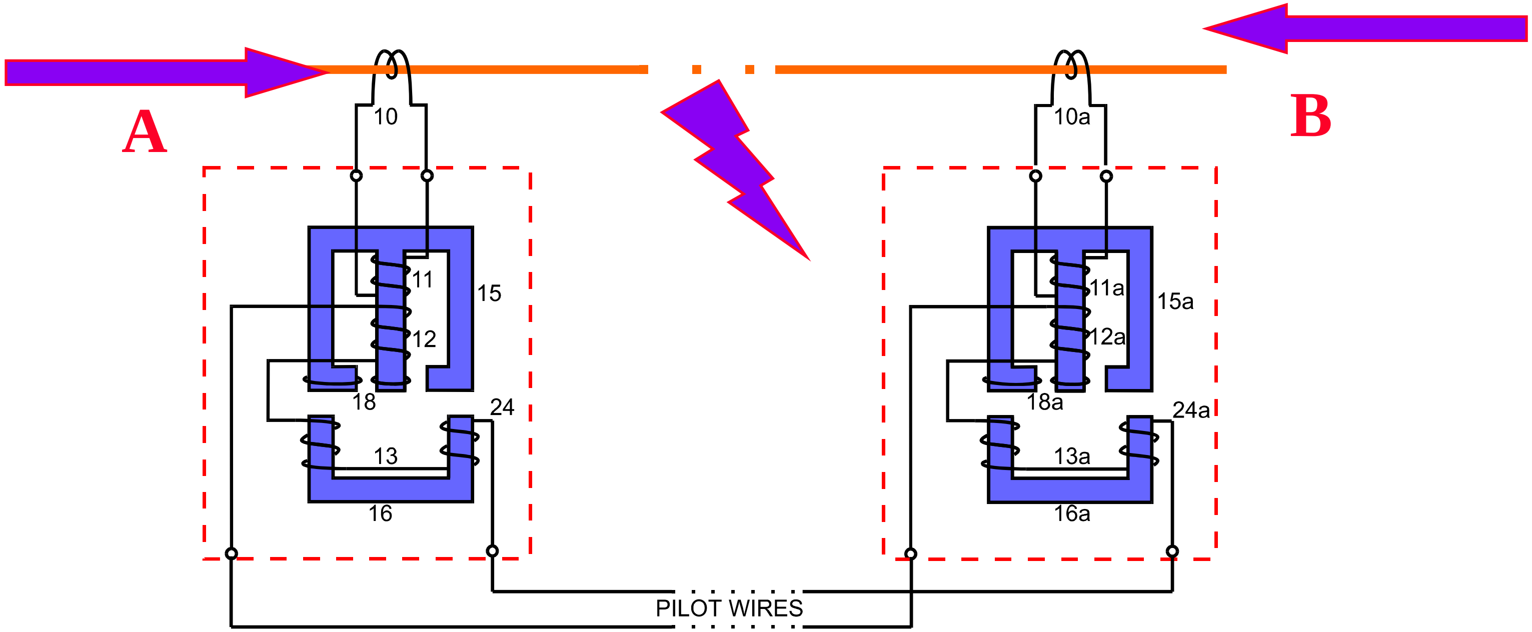
Applications of Unit Protection

➔ **Balanced Voltage** Systems are used on feeder systems where CT's are away from one another with independent relays at both ends





Fault fed from Both ends

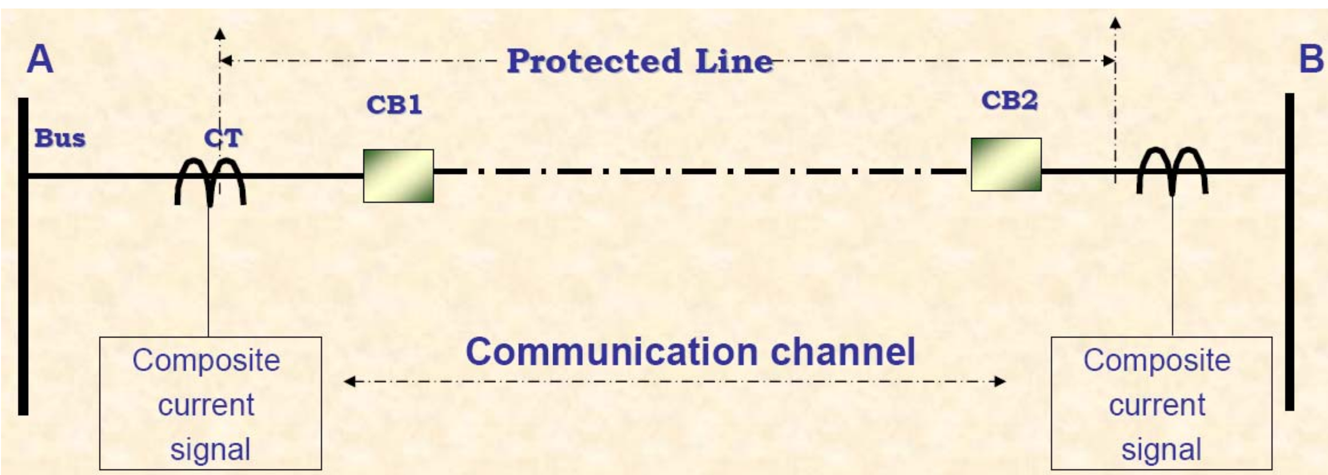


• An internal fault fed from A and B increases the current in primary winding (11) and (11a) with a corresponding current reversal in (11a)

• This results in the induced voltage in (12a) adding to that in (12) - producing an operating torque in both discs - tripping both ends

Differential Protection of Transmission Lines

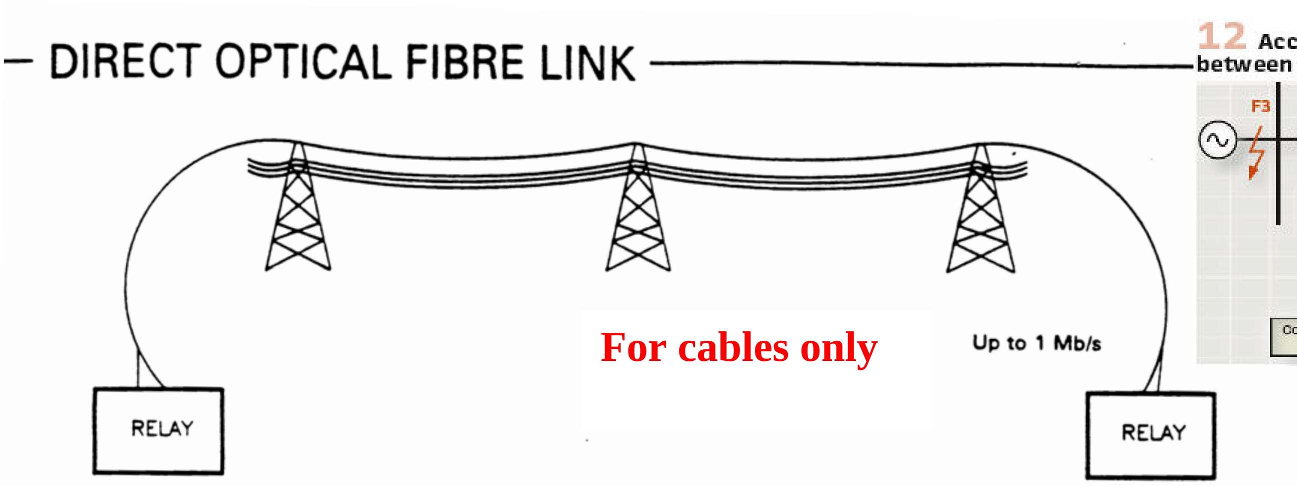
- The ideal way of protecting any piece of power system equipment is to compare the current entering that piece of equipment, with the current leaving it.
- Under normal healthy conditions the two are equal. If the two currents are not equal, then a fault must exist. This is the principle of "Differential Protection" , which is commonly used in TX Protection.
  - The current differential relay is a unit protection intended for overhead lines and underground cables



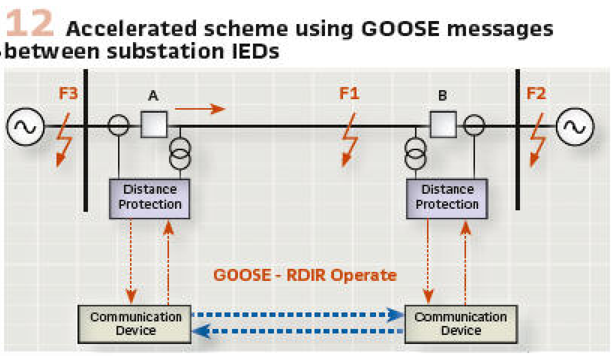
Channel of Communication

- The data is digitised before being sent over an optical fibre.
- The comparison is on a per phase basis.

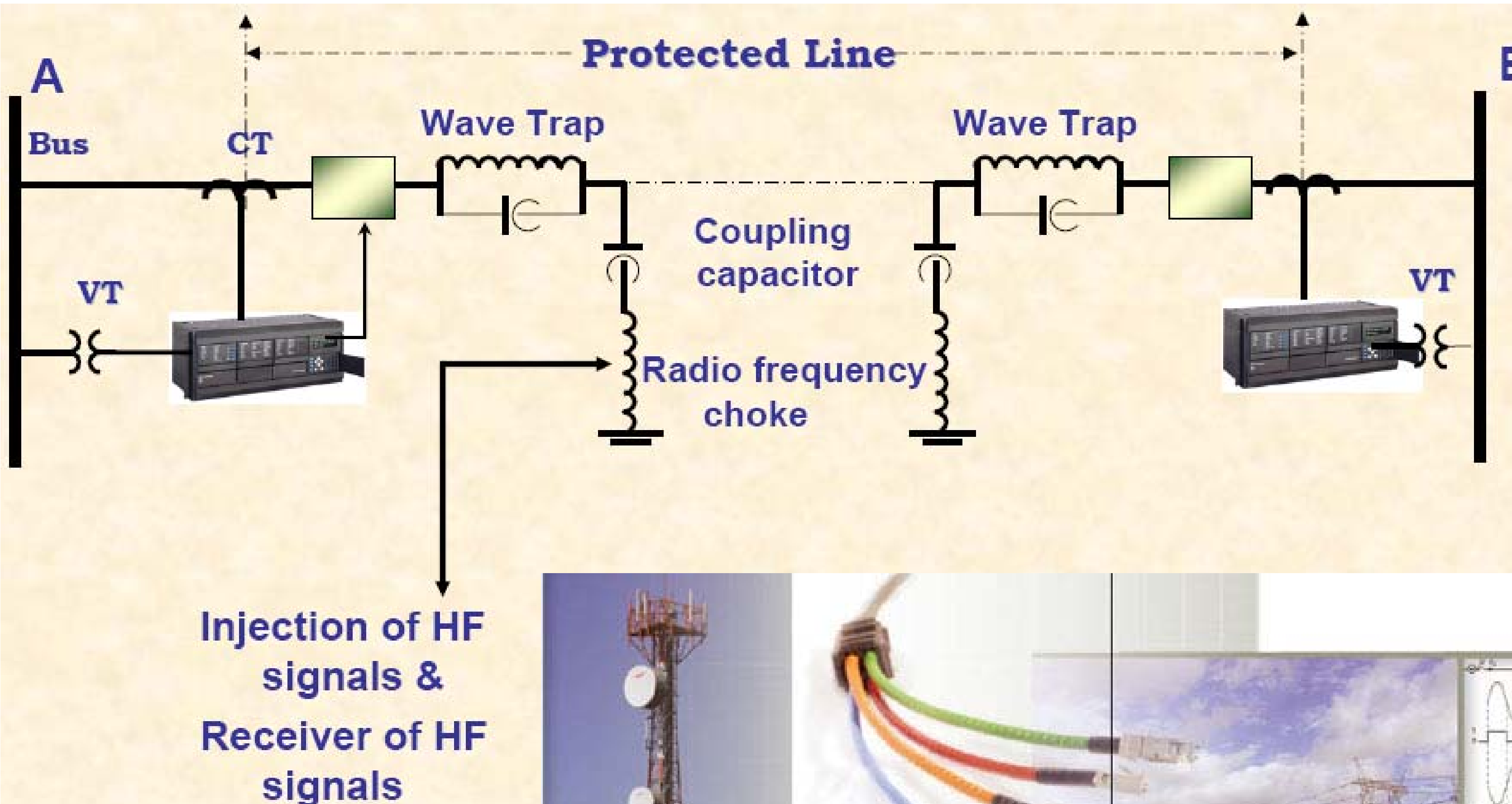
Direct optical fibre link – up to about 2 – 20 km, depending on the type of optical fibre. Used mainly for cables



For cables only

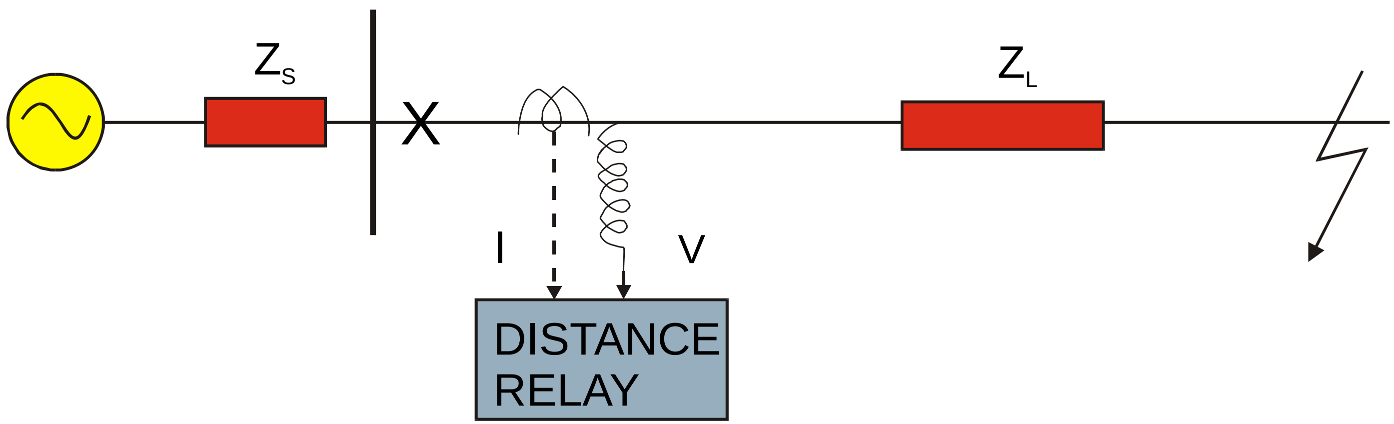


Power Line Carrier



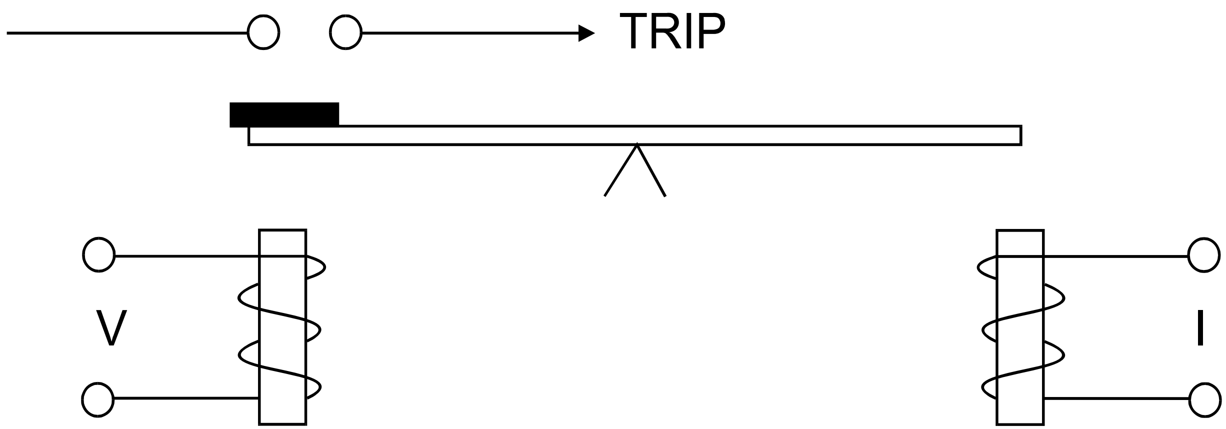
Distance Protection

- A distance relay has the ability to detect a fault within a pre-set distance along a transmission line or power cable from its location.
- Total impedance of power cable is a function of its length.
- A distance relay looks at current and voltage and compares these two quantities on the basis of Ohms Law.

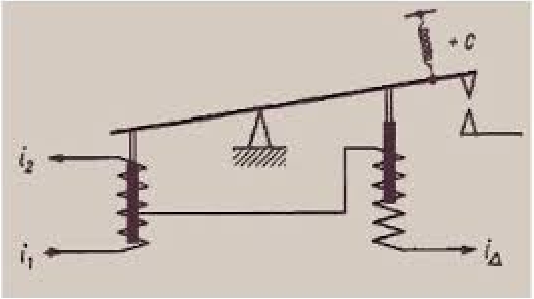


$$Z = \frac{V}{I}$$

Pioneer-Type Balanced Beam Relay



14 Percentage differential relay, McColl, 1917



- Voltage is fed into one coil to provide restraining torque.
- Current is fed into the other coil to provide the operating torque.

Pioneer-Type Balanced Beam Relay

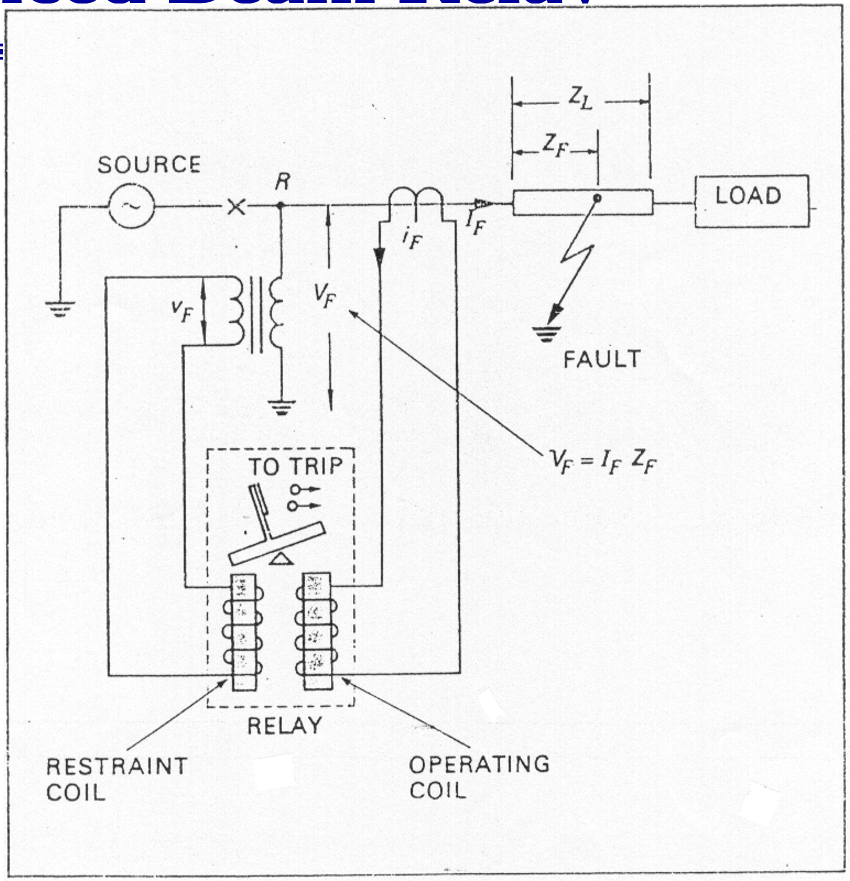
- Under healthy conditions the voltage will be high (at full rated level) whilst the current will be low (at normal load level) thereby balancing the beam so that the contacts remain open

- Under fault conditions, the voltage collapses and the current increase dramatically causing the beam to unbalance and close the contacts.

- By changing the **ampere-turns** relationship of the **current coil** to the voltage coil the ohmic reach of the relay can be adjusted.

Pioneer-Type Balanced Beam Relay

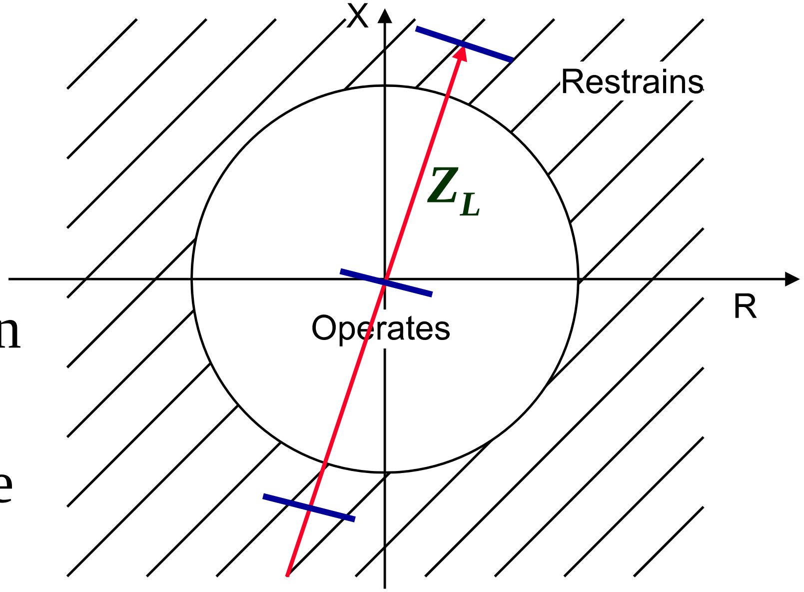
- This relay uses voltage and current inputs to measure "electrical distance" or impedance from relay to fault.
- Typically, the relay is set to 80% of the line impedance, $Z_S = 0.80 \times Z_L$. Z_s is the relay setting and Z_L -the line impedance.
- If the fault impedance, $Z_F < Z_S$, then the fault is within the relay setting and the relay trips instantaneously.
- If $Z_F > Z_S$, then the fault is outside the relay setting.



Z_F	=	V_F/I_F	
Z_F	<	Z_L	INTERNAL LINE FAULT
Z_F	>	Z_L	EXTERNAL LINE FAULT

Plain Impedance Relay

- Transmission line has resistance and reactance proportional to length.
- Represented on an R-X diagram as shown below.



- Plot the relay's operating boundary on an R-X diagram.
- Its impedance characteristic is a circle with its centre at the origin of the co-ordinates.
- The radius will be the setting in ohms.
- The relay will operate for all values less than its setting … i.e. for all points within the circle.

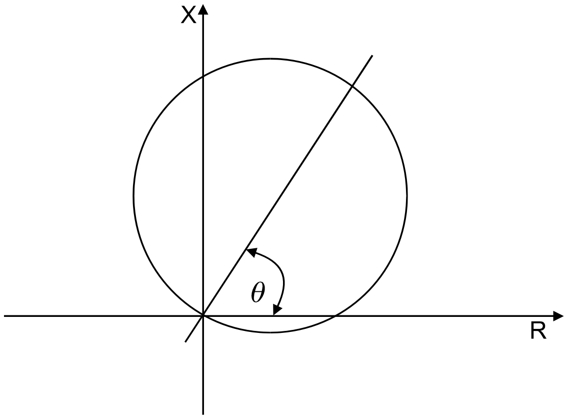
$$Z_L = \sqrt{R^2 + X^2}$$

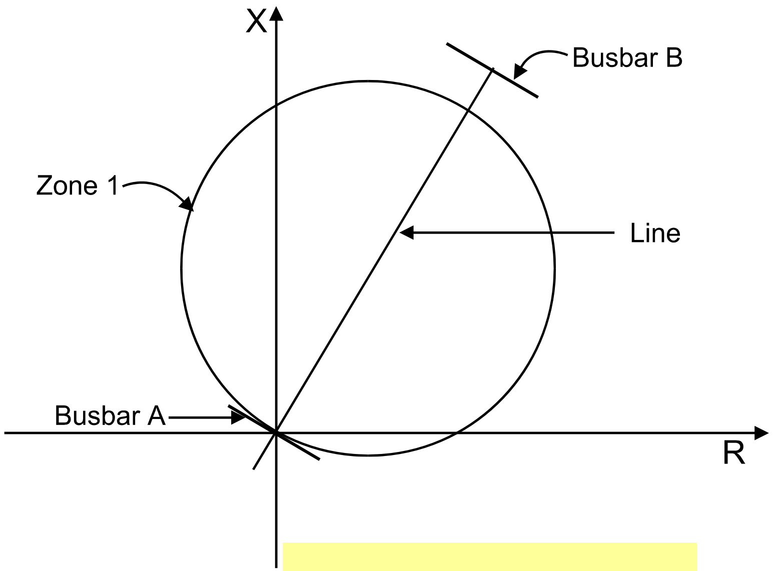
$$Z_r = 0.8 Z_L$$

Non-directional….it can operate for faults behind the relaying point

SMHO Relay Characteristic

- Additional voltages are fed into the comparator in order to compare the relative phase angles of voltage and current; so providing a directional feature.
- This moves the circle so that the circumference of the circle passes through the origin.
- Angle θ is called characteristic angle.
- Called MHO relay.





$$Z_{r1} = 0.8Z_L$$

Distance Protection

2 Power system model simulated in PSCAD/EMTDC

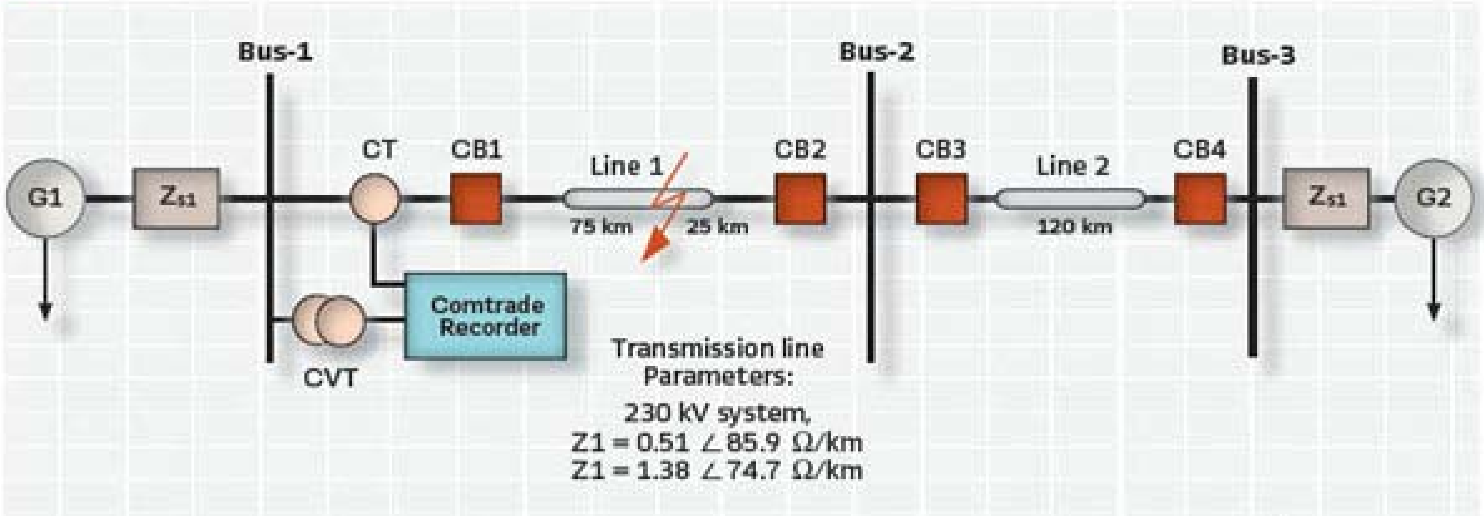
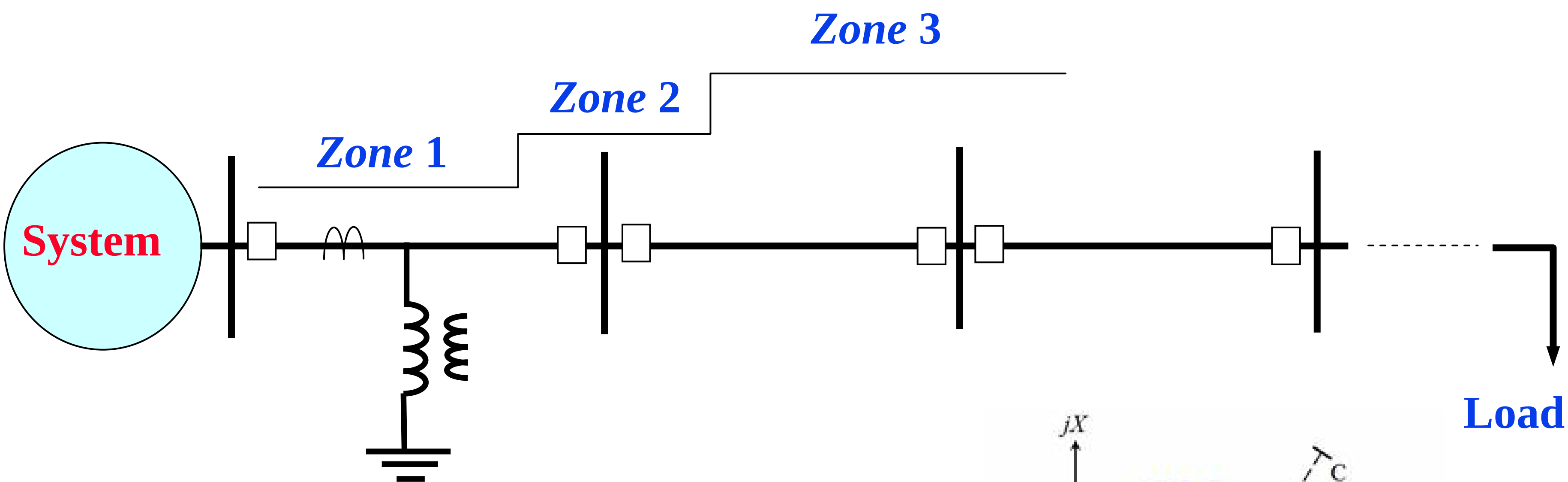


table 1 Distance relay zone settings

Serial Number	Zones of Protection	Zone Coverage	Time Delay (msec)
1	Zone - 1	80% of Line - 1	0
2	Zone - 2	100% of Line - 1 + 20% of Line - 2	300
3	Zone - 3	100% of Line - 1 + 100% of Line - 2	1,000

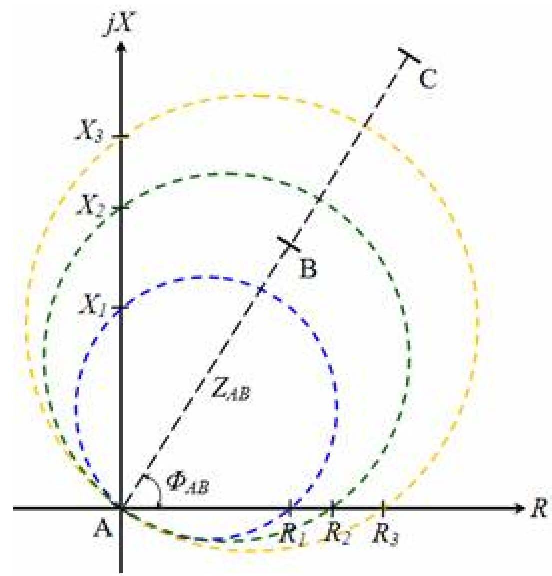
Conventional Distance Protection



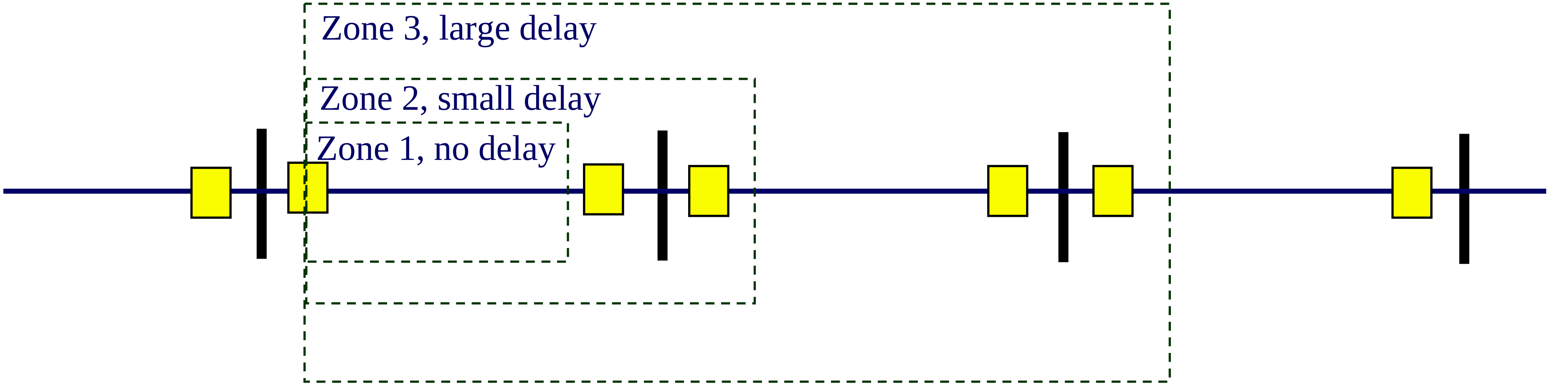
Zone 1: 0-80% of line length

Zone 2: 80% - 120% of line length

Zone 3: 120%- 220% of line length



Conventional Distance Protection



- Relays sense V/I and trip if it is too low; good approach because fault conditions are low voltage, high current.

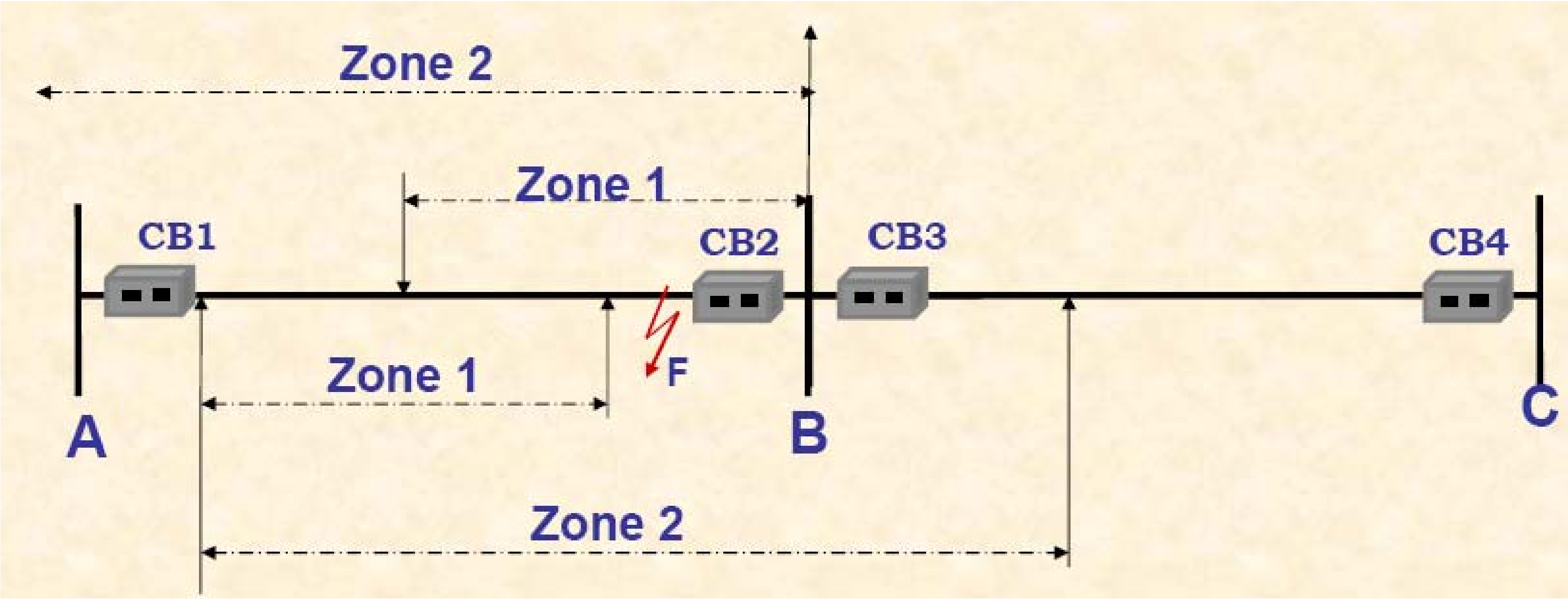
- Relays are directional; trip only for faults “looking” in one direction.

- Zone 1 trips instantly; trip zone for primary protection

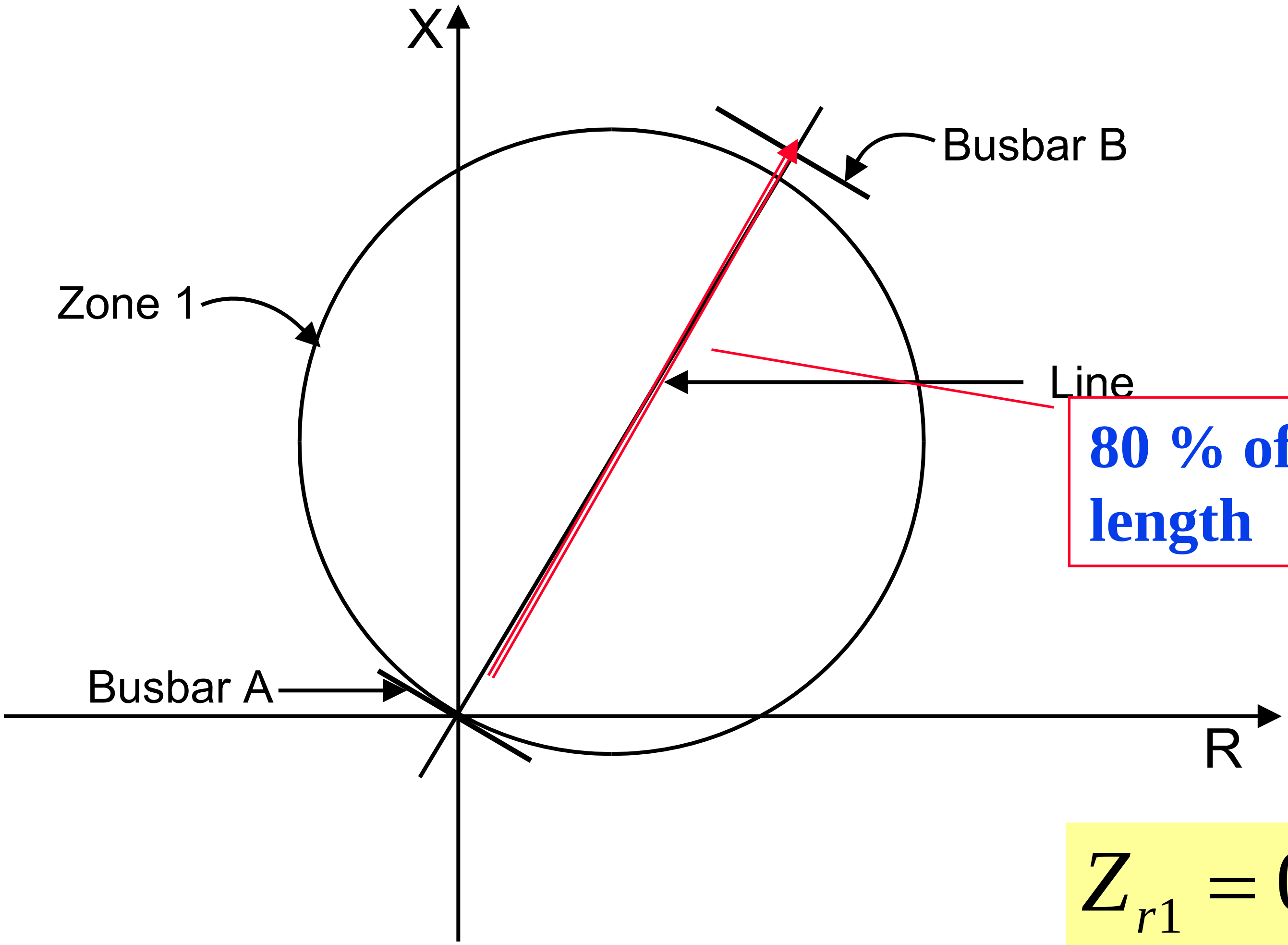
- Zone 2 has small delay. Zone 3 has large delay; these are trip zones for “backup” protection

Zone 1

- Relay characteristic has been added.
- **Reach** of the measuring element is approximately **80% of the line length**.
- **Under reach** setting purposely chosen to avoid **over-reaching** into the next line section to ensure sound **selectivity**.



Zone 1 - under-reach



80 % of line length

$$Z_{r1} = 0.8Z_L$$

Reasons for Under-reach setting

- Not practical to measure the impedance of transmission line 100% accurately.
- Errors are present in voltage and current transformers.
- Manufacturing tolerances on the relay's ability to measure accurately.
- Known as Zone 1 of the distance relay. **Instantaneous operation**

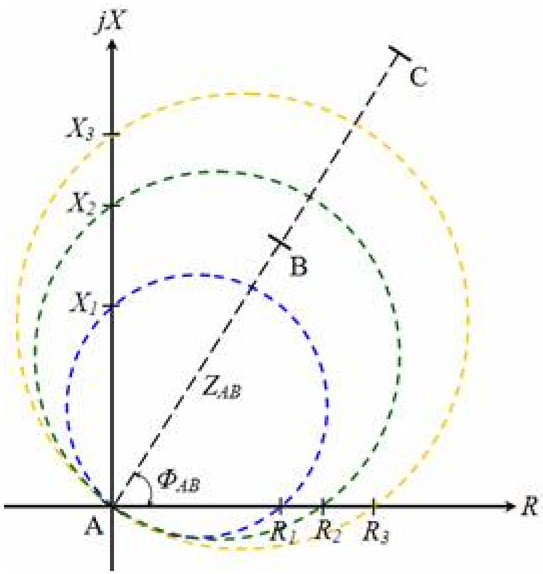
Zone 2

- A **second measuring element** is fitted to cover the **remaining 20% of the line length**. (normally set to measure 120% from source bus)
- Time delayed by 0.5 secs to provide the necessary co-ordination with the downstream relay (as this Zone actually **over-reaches** the next Breaker and provides **back-up**)
- Measuring element called Zone 2.

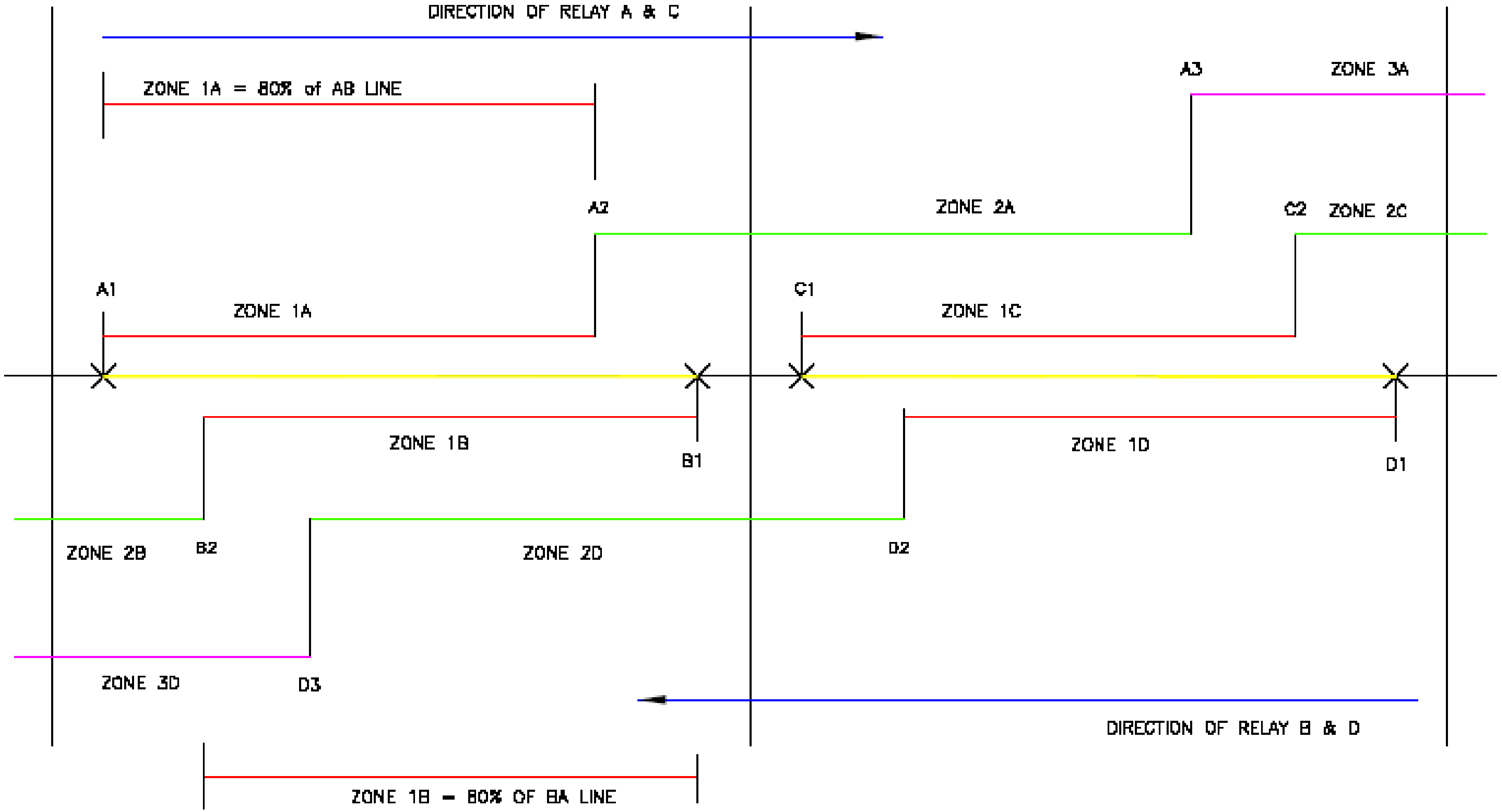
Zone 3- MHO Characteristic

- Third Zone is added as a starter element.
- Takes form of **offset mho** characteristic.
- This offset provides a closing-onto-fault feature.

  (If voltage from VT is zero when closing onto fault - faulted phase mho zone 1 may not work! **Close onto fault feature will trip breaker instantly**)
- As a starter it can be used to switch the Zone 1 element to a Zone 2 reach after 0.5 secs.

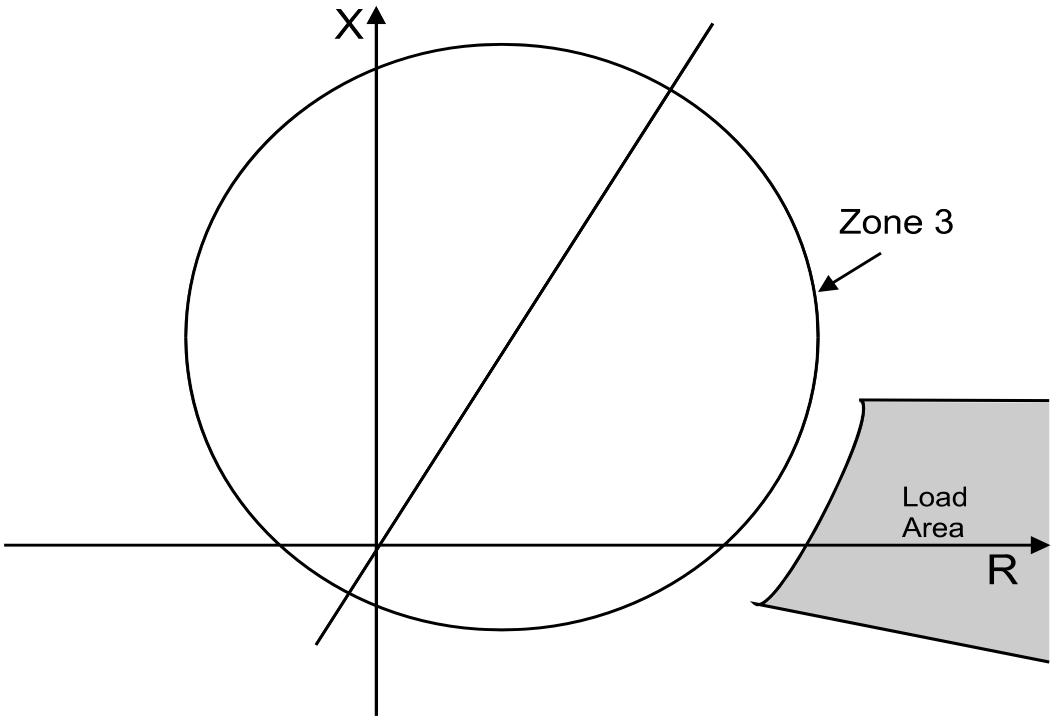


Distance Zone 1-Zone 3



Effect *of Load Current*

- Load Current can be expressed as an impedance.
- When setting a distance relay (esp. Zone 3), ensure that characteristic does not encroach on the load area as unnecessary tripping will occur.



Effect of Arc Resistance

- Resistance of the fault arc can also have an impact on performance of a distance relay.

- ***R*** of fault arc takes the fault impedance outside the relay's tripping characteristic.

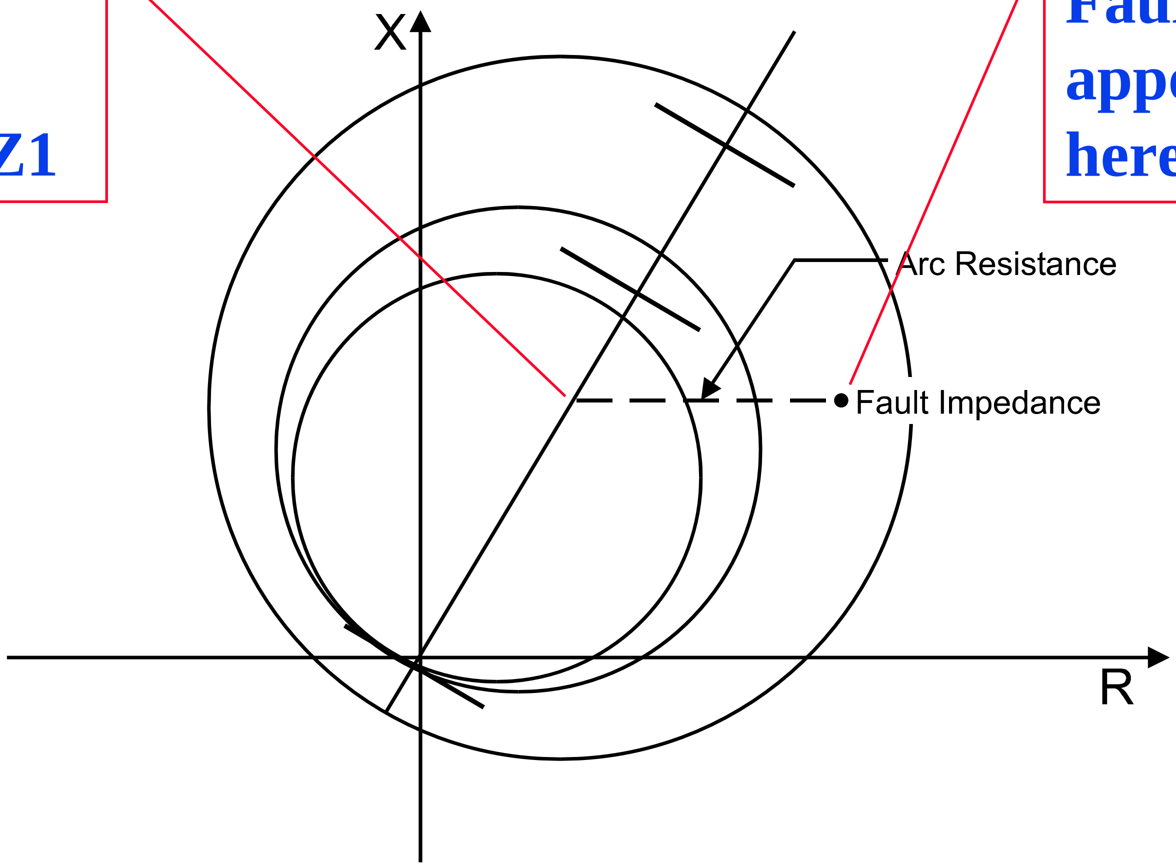
- Effect of arc resistance is most significant on short lines where the reach of the relay setting is small.

- Can be a problem for faults at end of the reach.

Arc resistance can cause "Under-reaching"

Fault is actually here in Z1

Fault appears here



Effect of Arc Resistance

- High Fault Arc resistances tend to occur during mid-span flashovers to ground during a bush fire

OR

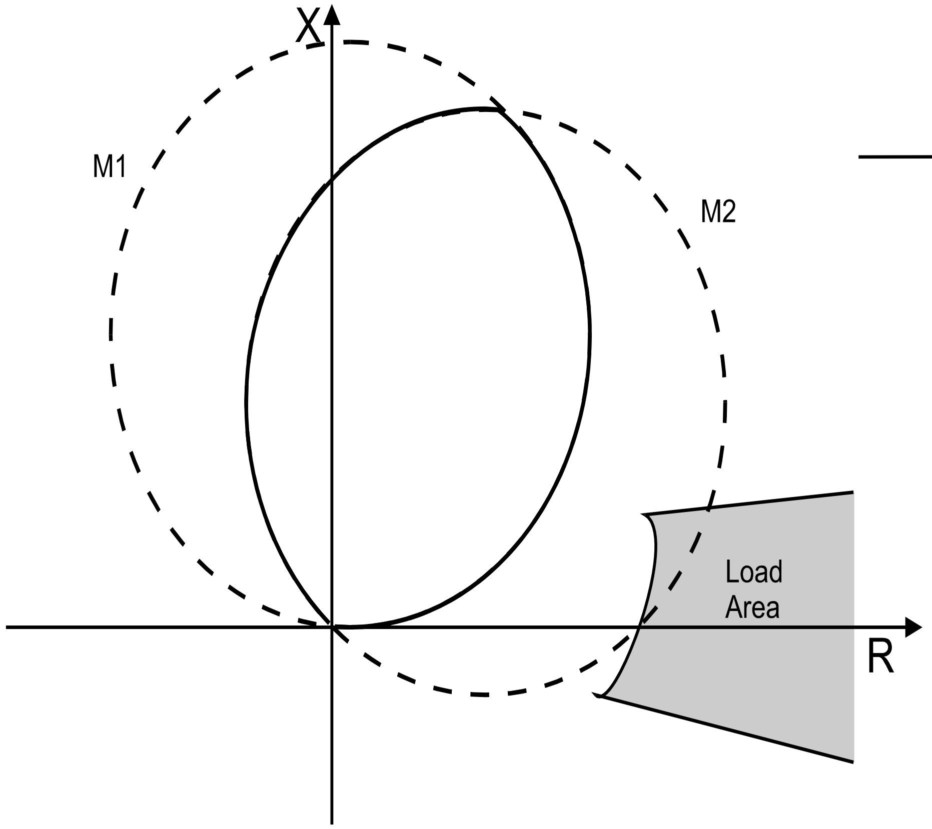
- On transmission lines carried on wood poles without earth wires.

- Overcome these problems as discussed next using different characteristic relays...

Different Shaped Characteristics

- To overcome the problems of **load encroachment** and **arc resistance**…
- Distance relays have been developed:
  - **Circular**
  - **Lenticular**
  - **Figure of Eight**
  - **Trapezoidal**
- **Digital Relays** help to **get any characteristics** as required.

Lenticular



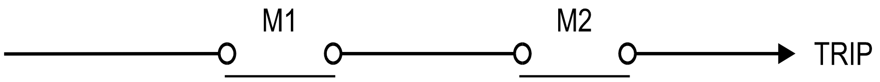
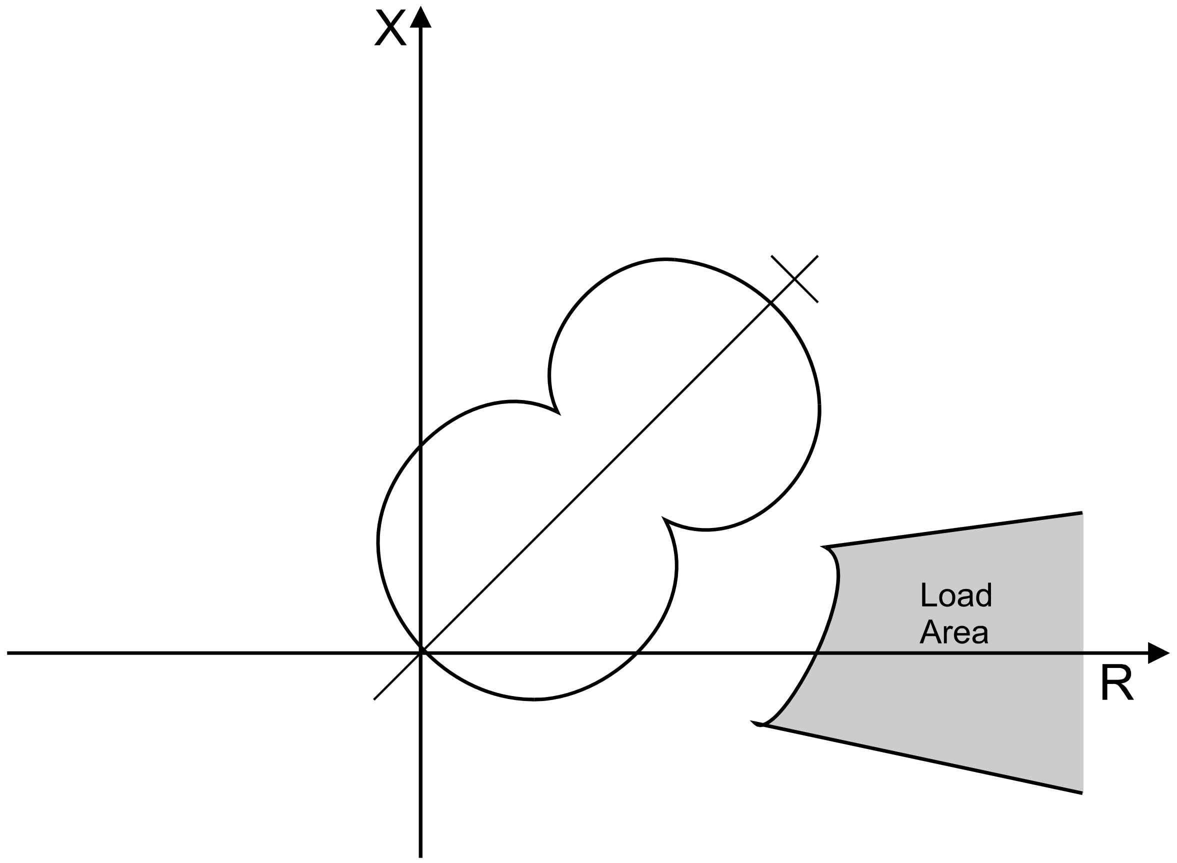
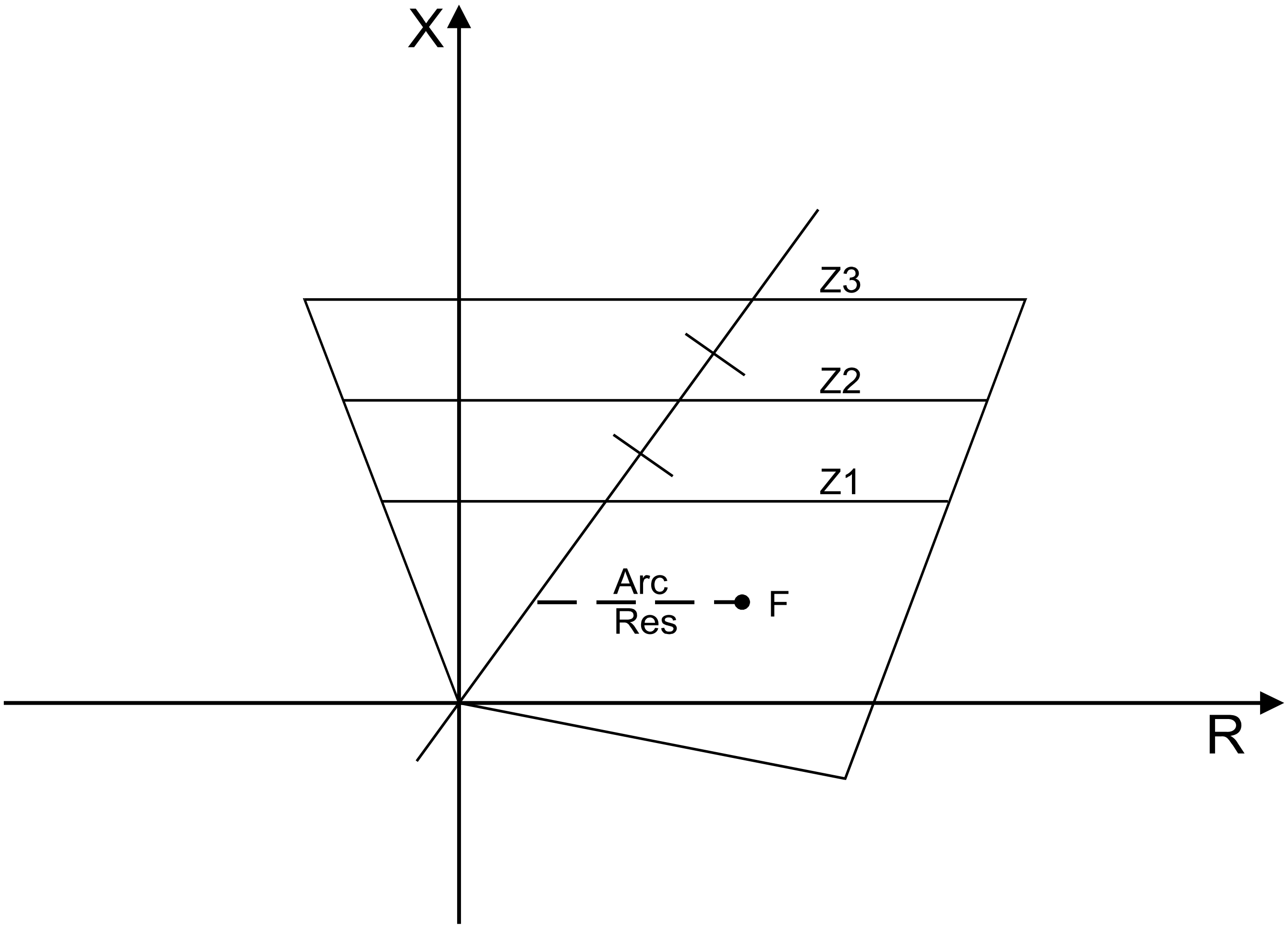


Figure of Eight

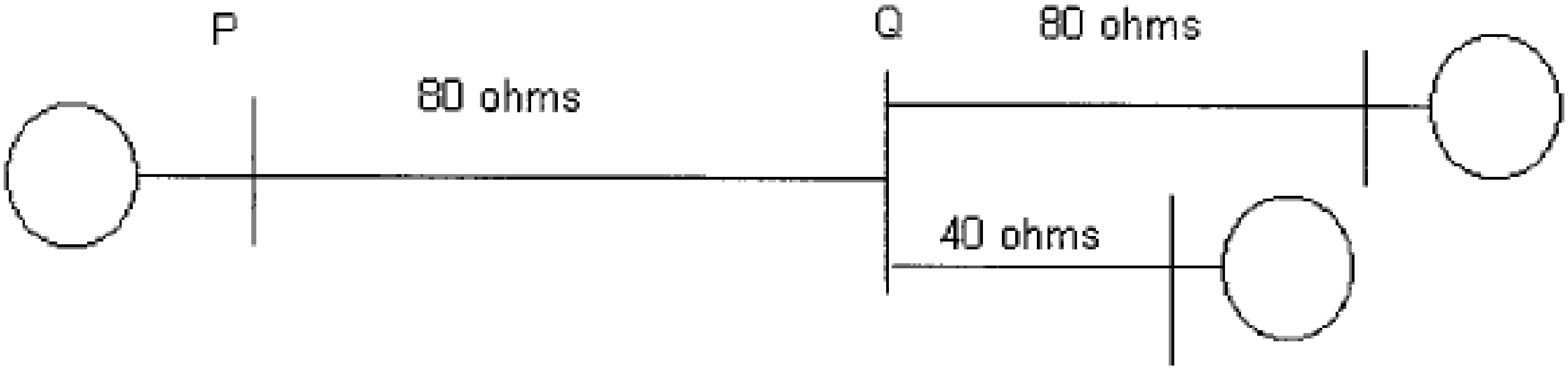


Trapezoidal
(increases arc resistance coverage)



Example

Consider the settings for line **PQ** at bus **P**.The impedance angle for all lines is **75°.** The line length is **80 Ω** . The distance relay at bus **P** is fed by current transformers rated at 2000 A:5 A and voltage transformers rated at 345 kV/200 kV Y:120 V/69 V Y. Set Zone 1 for 85% of this value (85%–90%) settings are typical for phase distance, slightly lower for ground distance):



Example

Zone 1 setting = 0.85 .80 Ω= 68 Ω ,primary ohm setting

CT ratio = 2000/5 = 400

VT ratio = 200,000/69 = 2900

Relay setting = primary setting (Ω) .CT ratio/VT ratio

= 68 .(400)/(2900)

= 9.38 relay ohms

Zone 2 setting = length×115% (minimum)

=line length+0.5 × length of shortest next adjacent line (preferred)

Example

The two next adjacent lines are 40 Ω and 80 Ω ,respectively.
The shortest of these is 40 Ω. Half of that is 20 Ω.
The setting 80+20Ω =100 Ω is greater than the minimum setting of 92 Ω (which guarantees seeing the entire line).

The relay setting is then

Zone 2 setting = 100 Ω (primary)

= 100400/2900 = 13.8 relay ohms

Topic 2: Stability

- **The ability of the power system to remain in synchronism and maintain the state of equilibrium following a disturbing force**
  - Steady-state stability: analysis of small and slow disturbances
    - gradual power changes
  - Transient stability: analysis of large and sudden disturbances
    - faults, outage of a line, sudden application or removal of load

Generator Dynamic Model

- **Under normal conditions, the relative position of the rotor axis and the stator magnetic field axis is fixed**
  - the angle between the two is the power angle or torque angle, δ
  - during a disturbance, the rotor will accelerate or decelerate w.r.t. the rotating stator field
  - acceleration or deceleration causes a change in the power angle

$$T_e = \frac{P_e}{\omega_e} = \frac{P_e}{2\pi(60Hz)} \qquad \frac{P_m}{\omega_{rotor}} = T_m$$

$$T_{accelation} = \Delta T = T_m - T_e$$

$$J\frac{d^2\theta_m}{dt^2} = \Delta T = T_m - T_e \qquad \theta_m = \omega_{ms}t + \delta_m \qquad \frac{\omega_{rotor}}{\omega_{ms}} = \frac{poles}{2}$$

Generator Dynamic Model

$$\omega_m = \frac{d\theta_m}{dt} = \omega_{ms} + \frac{d\delta_m}{dt} \qquad \alpha_m = \frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2}$$

$$J\frac{d^2\theta_m}{dt^2} = J\frac{d^2\delta_m}{dt^2} = T_m - T_e$$

$$J\omega_m\frac{d^2\delta_m}{dt^2} = \omega_m T_m - \omega_m T_e = P_m - P_e$$

$$W_{KE} = \tfrac{1}{2}J\omega_m^2 = \tfrac{1}{2}M\omega_m \qquad M = \frac{2W_{KE}}{\omega_m} = J\omega_m$$

$$\omega_m \approx \omega_{ms} \rightarrow M \approx \frac{2W_{KE}}{\omega_{ms}} = J\omega_{ms}$$

Generator Dynamic Model

$$M\frac{d^2\delta_m}{dt^2} = P_m - P_e$$

$$\delta = \delta_e = \frac{poles}{2}\delta_m \quad \rightarrow \quad \frac{p}{2}M\frac{d^2\delta}{dt^2} = P_m - P_e$$

$$\frac{p}{2}M\frac{d^2\delta}{dt^2} \quad = \quad \frac{p}{2}\frac{2W_{KE}}{\omega_{ms}}\frac{d^2\delta}{dt^2} \quad = \quad \frac{2W_{KE}}{\omega_s}\frac{d^2\delta}{dt^2}$$

$$\frac{2W_{KE}}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e \quad \rightarrow \quad \frac{2W_{KE}}{\omega_s S_B}\frac{d^2\delta}{dt^2} = \frac{P_m}{S_B} - \frac{P_e}{S_B}$$

Generator Dynamic Model

$$\frac{2\,W_{KE}}{\omega_s\;S_B}\,\frac{d^2\delta}{dt^2} = P_{m(pu)} - P_{e(pu)}$$

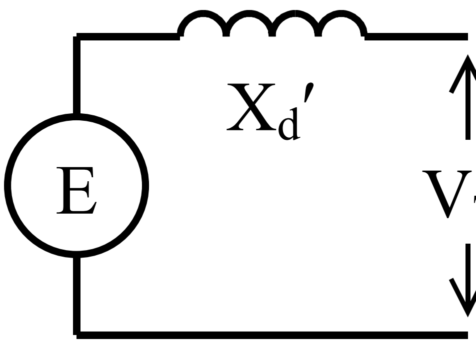
$$\frac{W_{KE}}{S_B} = \frac{\text{kinetic energy in MJ at rated speed}}{\text{machine power rating in MVA}} = H$$

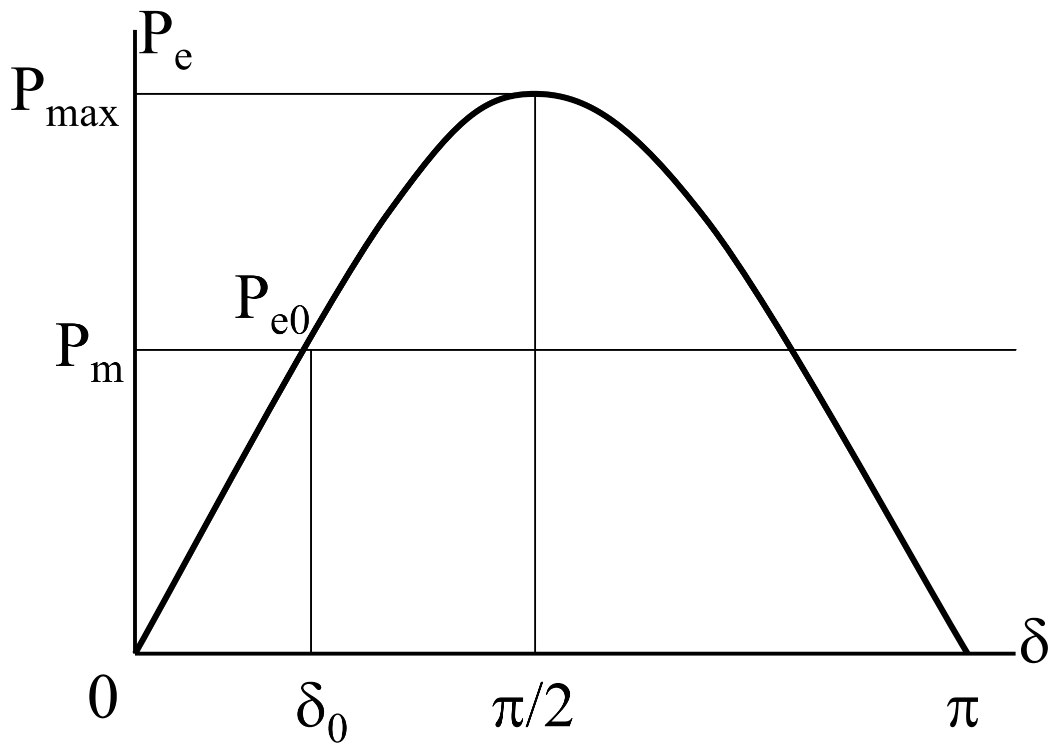
$$\frac{2\,H}{\omega_s}\,\frac{d^2\delta}{dt^2} = P_{m(pu)} - P_{e(pu)}$$

$$\rightarrow \quad \frac{H}{\pi\,f}\,\frac{d^2\delta}{dt^2} = P_{m(pu)} - P_{e(pu)} \qquad (radians)$$

$$\rightarrow \quad \frac{H}{180\,f}\,\frac{d^2\delta}{dt^2} = P_{m(pu)} - P_{e(pu)} \qquad (degrees)$$

Synchronous Machine Model





power angle curve

Round Rotor Machine Model

$$E' = |E'|\angle\delta$$

$$V_G = |V_G|\angle 0°$$

$$B = \frac{1}{X_d'}$$

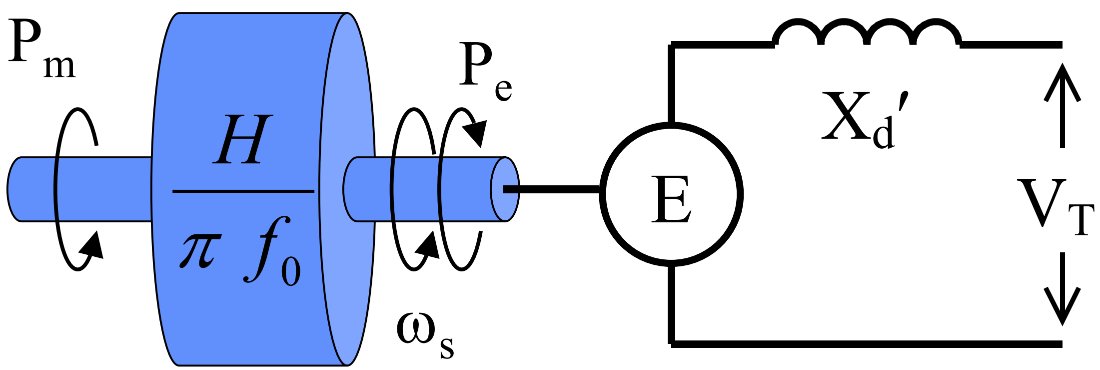
$$P_e = |E'||V_G||B|\cos(\delta - 90°) = \frac{|E'||V_G|}{X_d'}\sin\delta = P_{max}\sin\delta$$

The Swing Equation

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_e$$ Dynamic Generator Model

$$P_e = P_{\max}\sin\delta$$ Synchronous Machine Model

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_{\max}\sin\delta$$ Forming the Swing Equation



Transient Stability

- **The ability of the power system to remain in synchronism when subject to large disturbances**
  - Large power and voltage angle oscillations do not permit linearization of the generator swing equations
- **Lyapunov energy functions**
  - simplified energy method: the Equal Area Criterion
- **Time-domain methods**
  - numerical integration of the swing equations
  - Runga-Kutta numerical integration techniques

Equal Area Criterion

- **Quickly predicts the stability after a major disturbance**
  - graphical interpretation of the energy stored in the rotating masses
  - method only applicable to a few special cases:
    - one machine connected to an infinite bus
    - two machines connected together
- **Method provides physical insight to the dynamic behavior of machines**
  - relates the power angle with the acceleration power

Equal Area Criterion

- **For a synchronous machine connected to an infinite bus**

$$\frac{H}{\pi\, f_0}\frac{d^2\delta}{dt^2} = P_m - P_e = P_{accel}$$

$$\frac{d^2\delta}{dt^2} = \frac{\pi\, f_0}{H}\left(P_m - P_e\right) = \frac{\pi\, f_0}{H} \times P_{accel}$$

- **The energy form of the swing equation is obtained by multiplying both sides by the system frequency (shaft rotational speed)**

$$\left(2\frac{d\delta}{dt}\right)\left(\frac{d^2\delta}{dt^2}\right) = \frac{\pi\, f_0}{H}\left(P_m - P_e\right)\left(2\frac{d\delta}{dt}\right)$$

Equal Area Criterion

$$2\left(\frac{d^2\delta}{dt^2}\right)\left(\frac{d\delta}{dt}\right)=\frac{\pi f_0}{H}\left(P_m-P_e\right)\left(2\frac{d\delta}{dt}\right)$$

- **The left hand side can be reworked as the derivative of the square of the system frequency (shaft speed)**

$$\frac{d}{dt}\left[\left(\frac{d\delta}{dt}\right)^2\right]=\frac{2\pi f_0}{H}\left(P_m-P_e\right)\frac{d\delta}{dt}$$

$$d\left[\left(\frac{d\delta}{dt}\right)^2\right]=\frac{2\pi f_0}{H}\left(P_m-P_e\right)d\delta$$

Equal Area Criterion

- **Integrating both sides with respect to time,**

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2\pi f_0}{H}\int_{\delta_0}^{\delta}(P_m - P_e)\,d\delta$$

$$\frac{d\delta}{dt} = \sqrt{\frac{2\pi f_0}{H}\int_{\delta_0}^{\delta}(P_m - P_e)\,d\delta}$$

- **The equation gives the relative speed of the machine. For stability, the speed must go to zero over time**

$$\left.\frac{d\delta}{dt} = 0\right|_{t\to\infty}$$

$$0 = \int_{\delta_0}^{\delta}(P_m - P_e)\,d\delta$$

Equal Area Criterion

- **Consider a machine operating at equilibrium**
  - the power angle, $\delta = \delta_0$
  - the electrical load, $P_{e0} = P_{m0}$
- **Consider a sudden increase in the mechanical power input**
  - $P_{m1} > P_{e0}$; the acceleration power is positive
  - excess energy is stored in the rotor and the power frequency increases, driving the relative power angle larger over time

$$U_{Potential} = \int_{\delta_0}^{\delta_1} (P_{m1} - P_e) d\delta > 0$$

$$\frac{d\delta}{dt} = \omega = \sqrt{\frac{2\pi f_0}{H} \int_{\delta_0}^{\delta} (P_m - P_e) d\delta} > 0$$

Equal Area Criterion

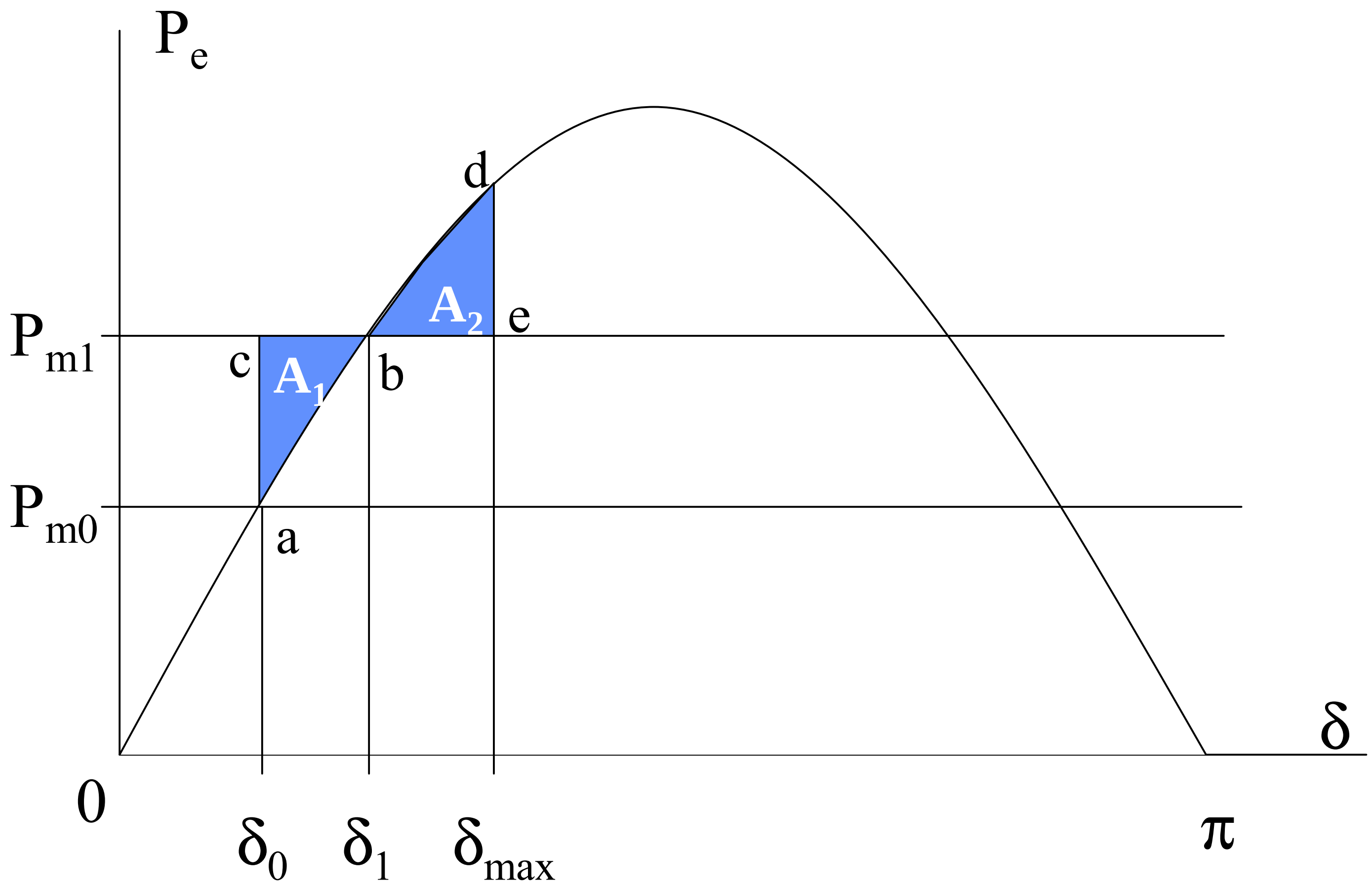
- with increase in the power angle, δ, the electrical power increases

$$P_e = P_{\max} \sin \delta$$

- when $\delta = \delta_1$, the electrical power equals the mechanical power, P_{m1}
- acceleration power is zero, but the rotor is running above synchronous speed, hence the power angle, δ, continues to increase
- now $P_{\mathrm{m1}} < P_{\mathrm{e}}$; the acceleration power is negative (deceleration), causing the rotor to decelerate to synchronous speed at $\delta = \delta_{\max}$
- an equal amount of energy must be given up by the rotating masses

$$U_{Potential} = \int_{\delta_0}^{\delta_1} (P_{m1} - P_e) d\delta - \int_{\delta_1}^{\delta_{\max}} (P_{m1} - P_e) d\delta = 0$$

Equal Area Criterion



Equal Area Criterion

- **The result is that the rotor swings to a maximum angle**
  - at which point the acceleration energy area and the deceleration energy area are equal

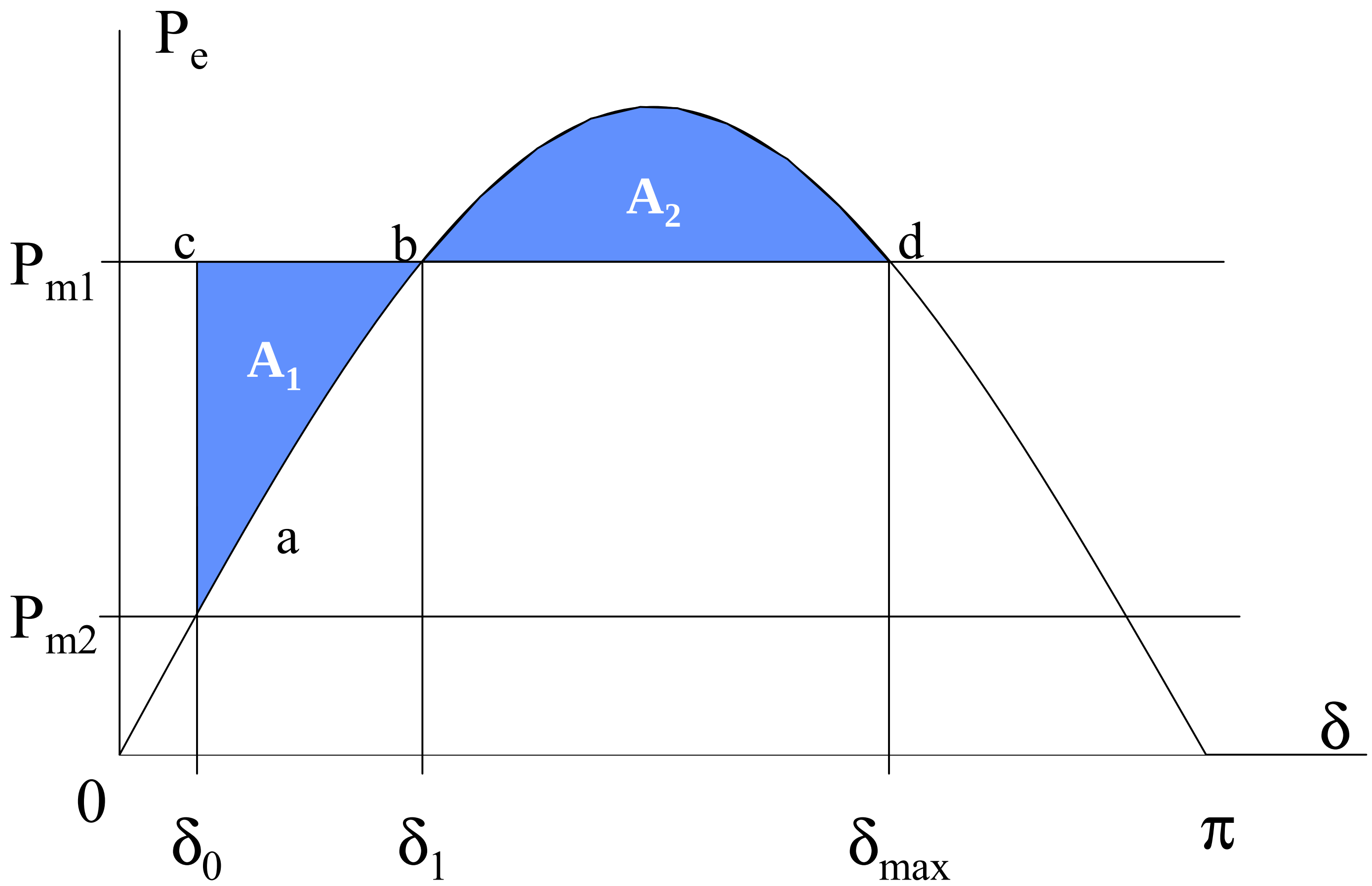
$$\int_{\delta_0}^{\delta_1} (P_{m1} - P_e)\, d\delta = \textbf{area}\, abc = \textbf{area}\, A_1$$

$$\int_{\delta_1}^{\delta_{\max}} (P_{m1} - P_e)\, d\delta = \textbf{area}\, bde = \textbf{area}\, A_2$$

$$\left|\textbf{area}\, A_1\right| = \left|\textbf{area}\, A_2\right|$$

  - this is known as the equal area criterion
  - the rotor angle will oscillate back and forth between δ and $\delta_{\max}$ at its natural frequency

Equal Area Criterion - ΔP mechanical

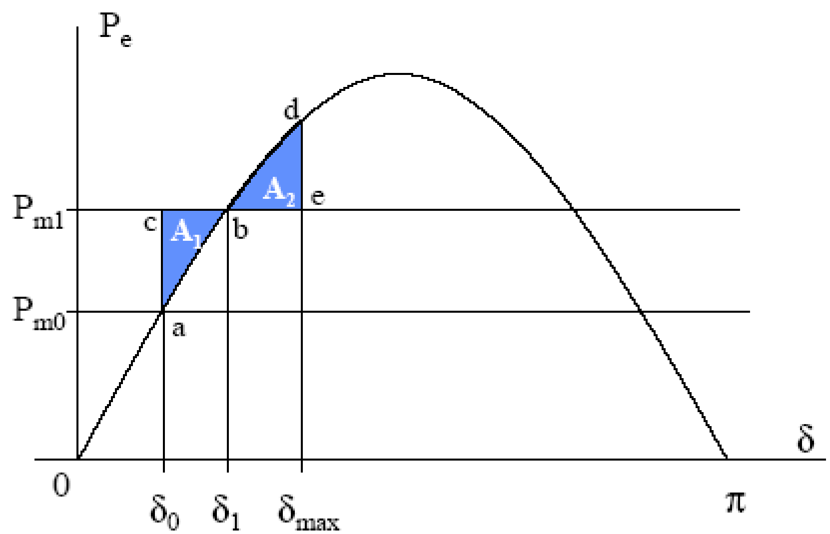


- The result is that the rotor swings to a maximum angle
  - at which point the acceleration energy area and the deceleration energy area are equal

$$\int_{\delta_0}^{\delta_1} (P_{m1} - P_e) d\delta = \textbf{area } abc = \textbf{area } A_1$$

$$\int_{\delta_1}^{\delta_{max}} (P_{m1} - P_e) d\delta = \textbf{area } bde = \textbf{area } A_2$$

$$|\textbf{area } A_1| = |\textbf{area } A_2|$$



  - this is known as the equal area criterion
  - the rotor angle will oscillate back and forth between δ and δ_{max} at its natural frequency

Equal Area Criterion - ΔP mechanical

$$P_{m1}(\delta_1 - \delta_0) - \int_{\delta_0}^{\delta_1} P_{\max} \sin\delta \, d\delta = \int_{\delta_1}^{\delta_{\max}} P_{\max} \sin\delta \, d\delta - P_{m1}(\delta_{\max} - \delta_1)$$

$$P_{m1}(\delta_{\max} - \delta_0) = P_{\max}(\cos\delta_0 - \cos\delta_{\max})$$

$$P_{m1} = P_{\max} \sin\delta_{\max}$$

$$(\delta_{\max} - \delta_0)\sin\delta_{\max} = \cos\delta_0 - \cos\delta_{\max}$$

$$\rightarrow P_{m1} = P_{\max} \sin\delta_1$$

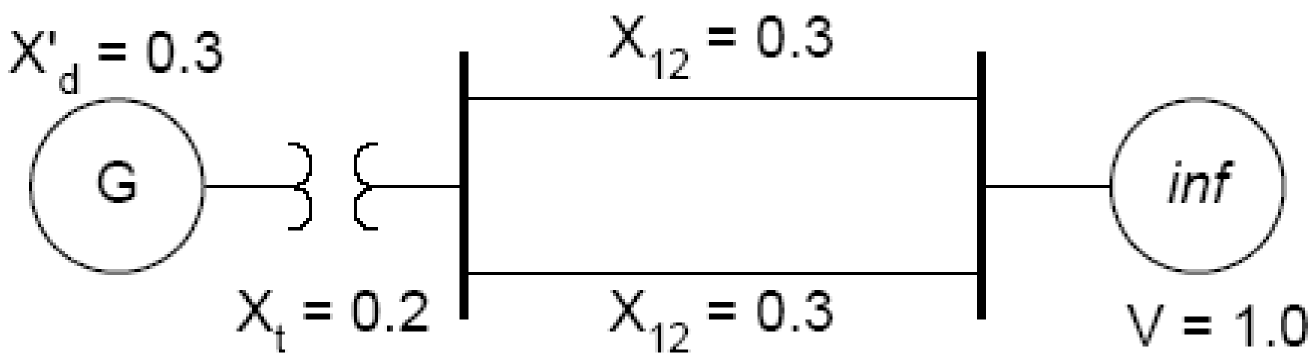
Function is nonlinear in δ_{max}
Solve using Newton-Raphson

Example

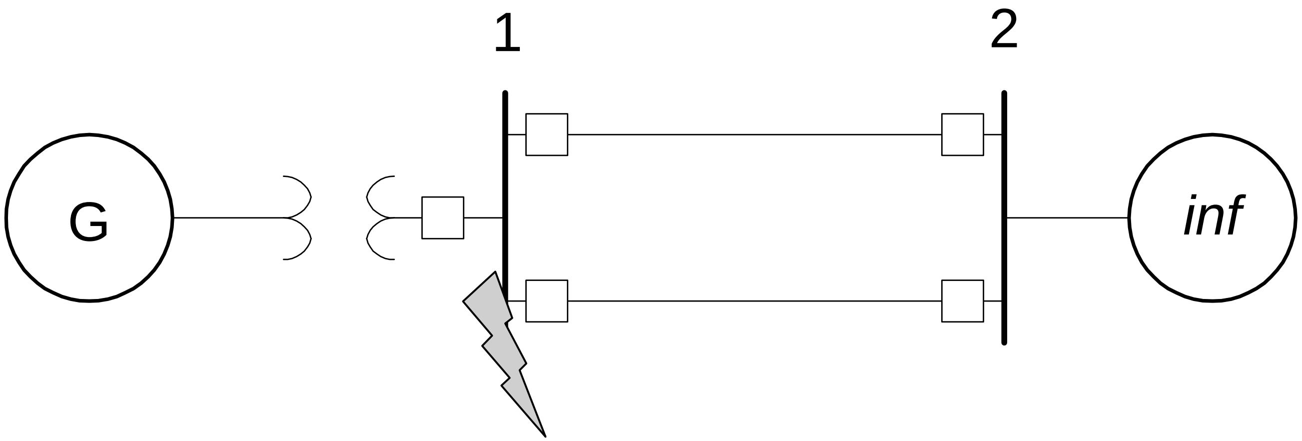
A 60 Hz synchronous generator having inertia constant H = 9.94 MJ/MVA and a transient reactance X'_d = 0.3 pu is connected to an infinite bus through the following network. The generator is delivering 0.6 *pu real power at 0.8 power* factor lagging to the infinite bus at a voltage of 1 *pu.*

(a) The maximum power input that can be applied without loss of synchronism.

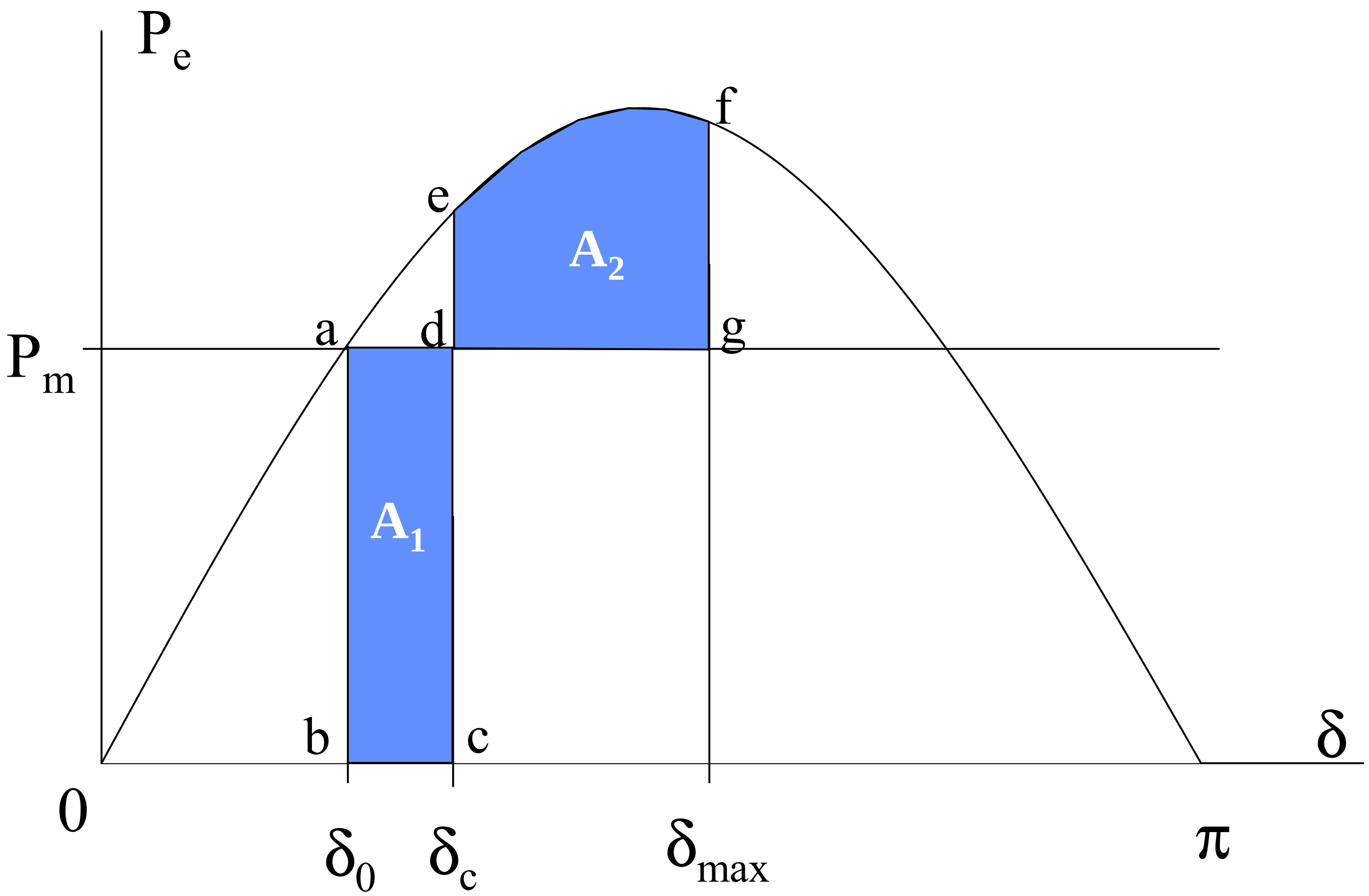
(b) Repeat (a) with zero initial power input. Assume the generator internal voltage remains constant at the value computed in (a).



3-Phase Fault



Equal Area Criterion - 3 phase fault



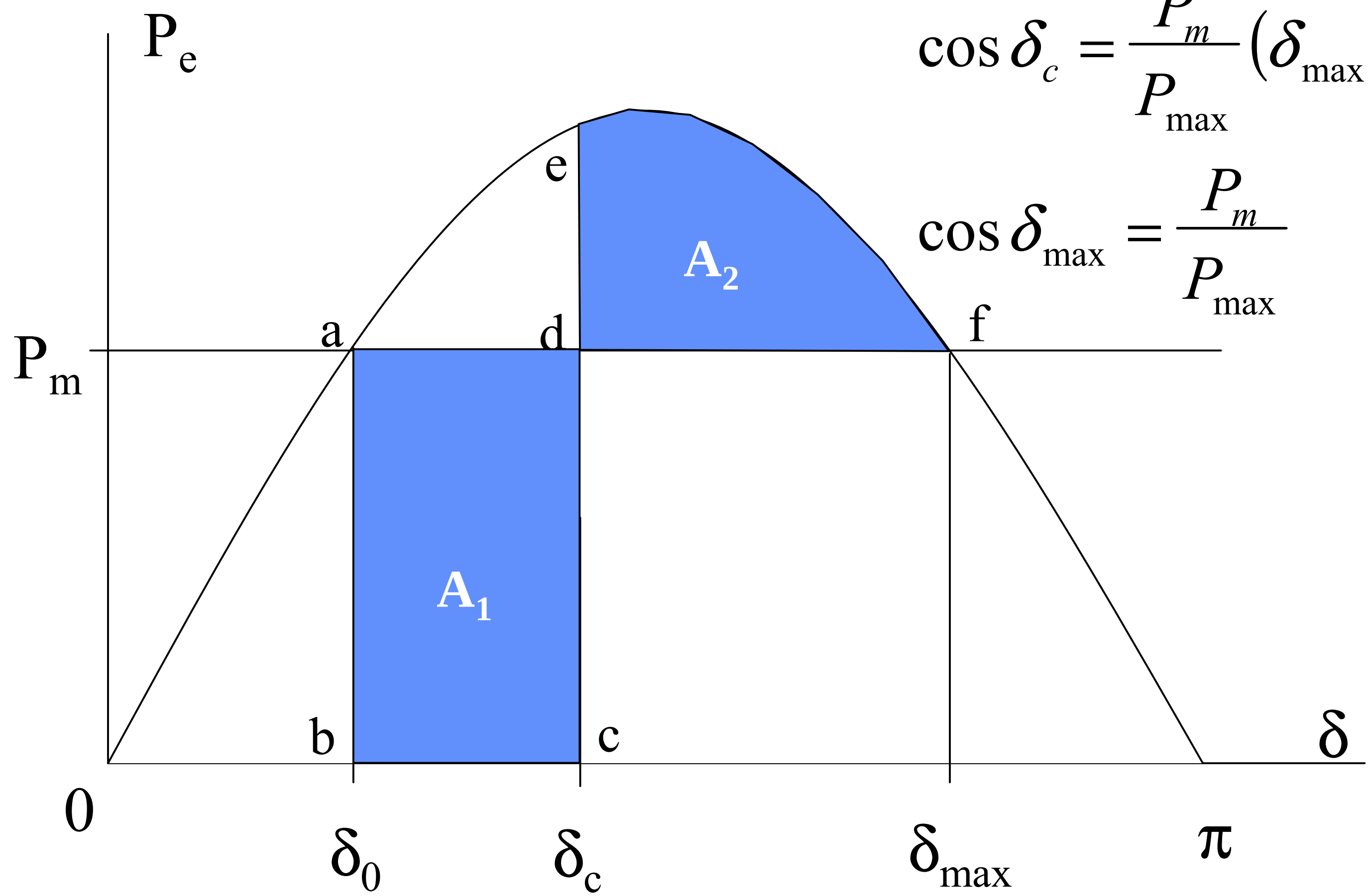
Equal Area Criterion - 3 phase fault

$$\int_{\delta_0}^{\delta_c} P_m \, d\delta = \int_{\delta_c}^{\delta_{\max}} \left(P_{\max} \sin\delta - P_m \right) d\delta$$

$$P_m\left(\delta_c - \delta_0\right) = P_{\max}\left(\cos\delta_c - \cos\delta_{\max}\right) - P_m\left(\delta_{\max} - \delta_c\right)$$

$$\cos\delta_c = \frac{P_m}{P_{\max}}\left(\delta_{\max} - \delta_c\right) + \cos\delta_{\max}$$

Critical Clearing Time



$$\cos\delta_c = \frac{P_m}{P_{\max}}(\delta_{\max} - \delta_c) + \cos\delta_{\max}$$

$$\cos\delta_{\max} = \frac{P_m}{P_{\max}}$$

Critical Clearing Time

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_e = P_m \neg \; P_e = 0$$

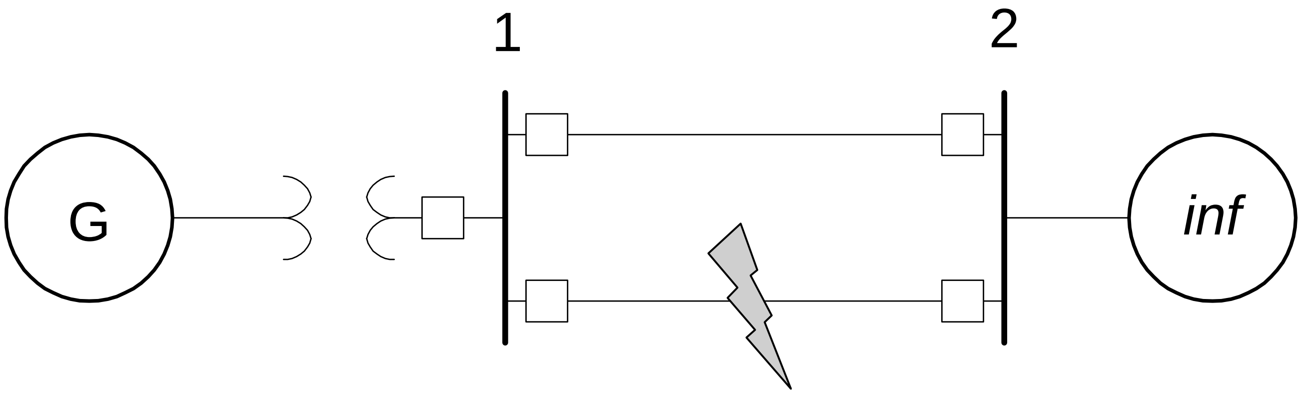
$$\frac{d^2\delta}{dt^2} = \frac{\pi f_0}{H} P_m$$

$$\frac{d\delta}{dt} = \frac{\pi f_0}{H} P_m \int_0^t dt = \frac{\pi f_0}{H} P_m t$$

$$\delta = \frac{\pi f_0}{2H} P_m t^2 + \delta_0$$

$$t_c = \sqrt{\frac{2H(\delta_c - \delta_0)}{\pi f_0 P_m}}$$

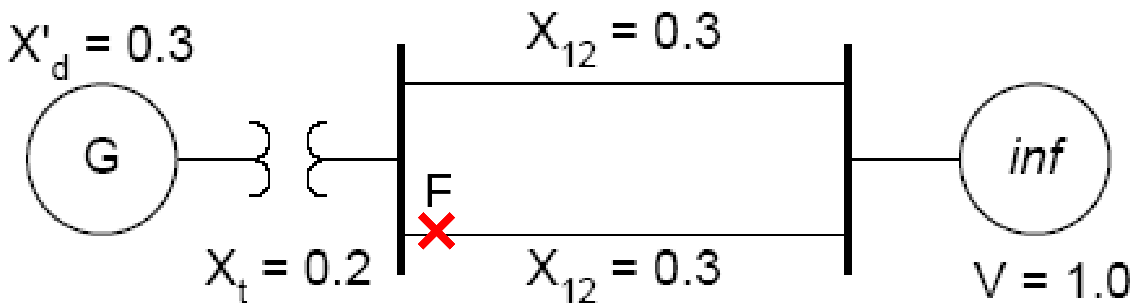
3-Phase Fault



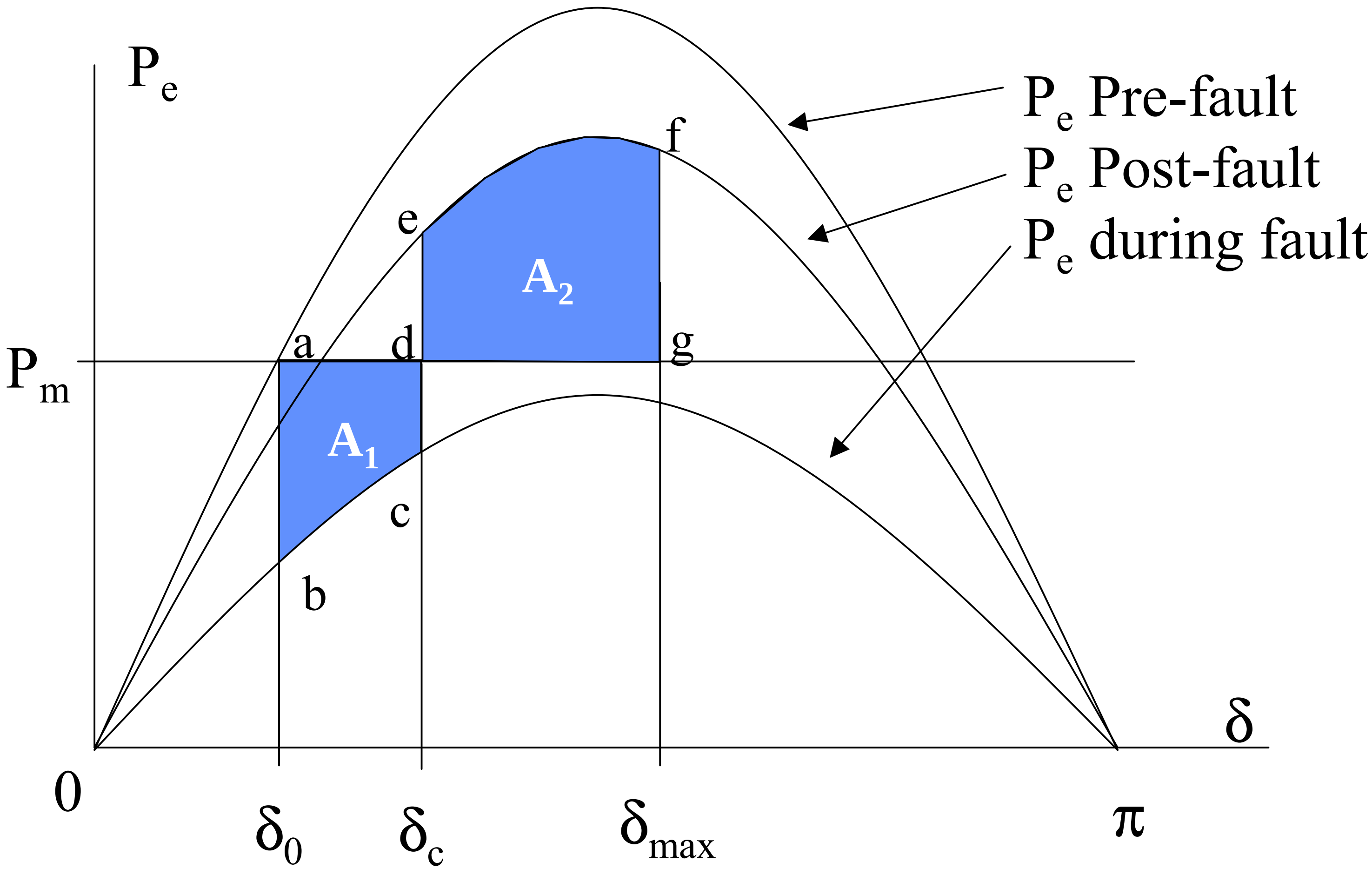
Example

A 60-Hz synchronous generator having inertia constant H = 5 MJ/MVA and a direct axis transient reactance X'_d = 0.3 pu is connected to an infinite bus through a purely reactive circuit as shown. Reactances are marked on the diagram on a common system base. The generator is delivering real power P = 0.8 pu and Q = 0.074 pu to the infinite bus at a voltage of V = 1 pu

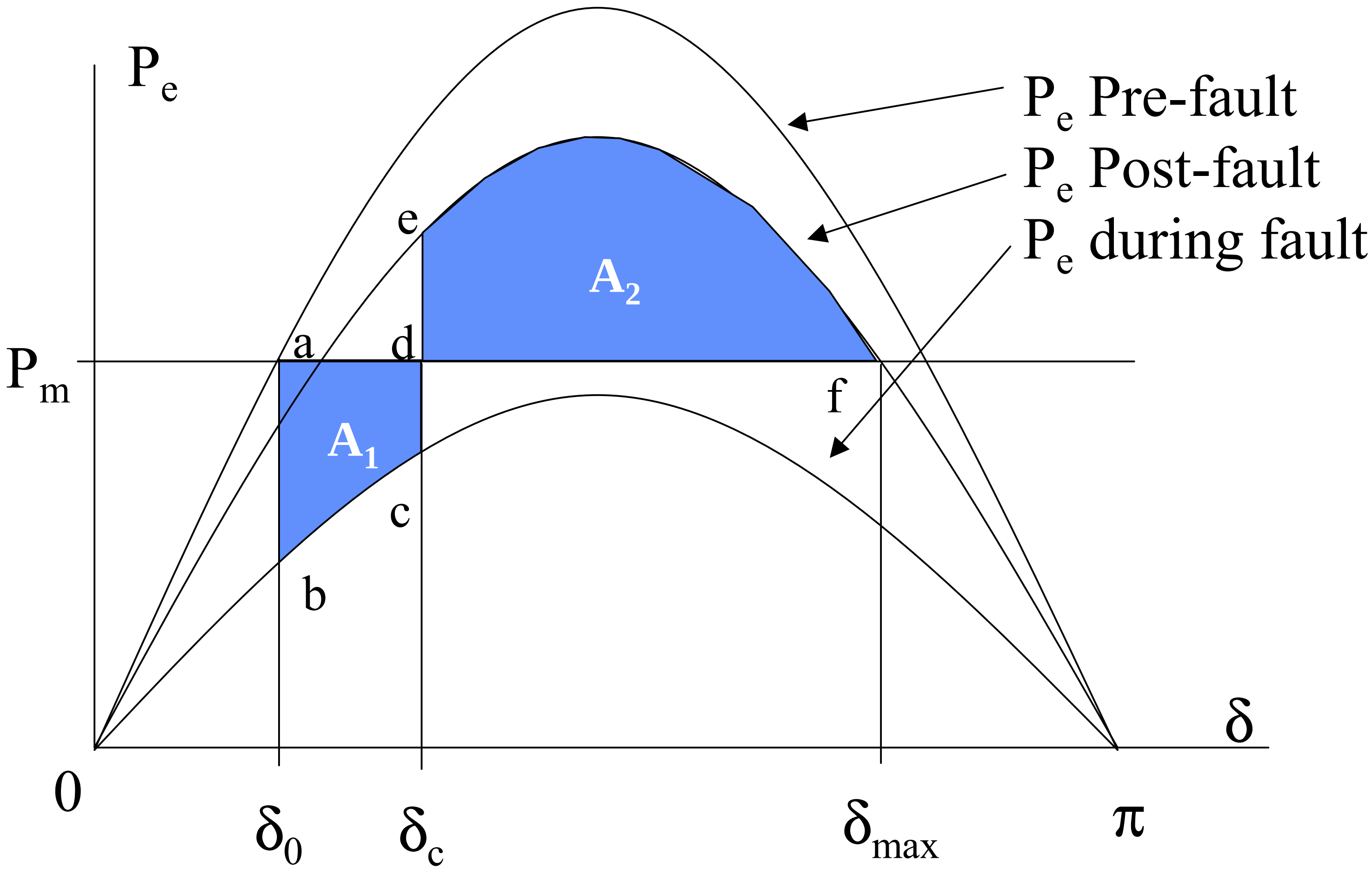
A temporary three-phase fault occurs at the sending end of the line at point *F*. When the fault is cleared, both lines are intact. Determine the critical clearing angle and the critical fault clearing time.



Equal Area Criterion



Critical Clearing Time



Critical Clearing Time

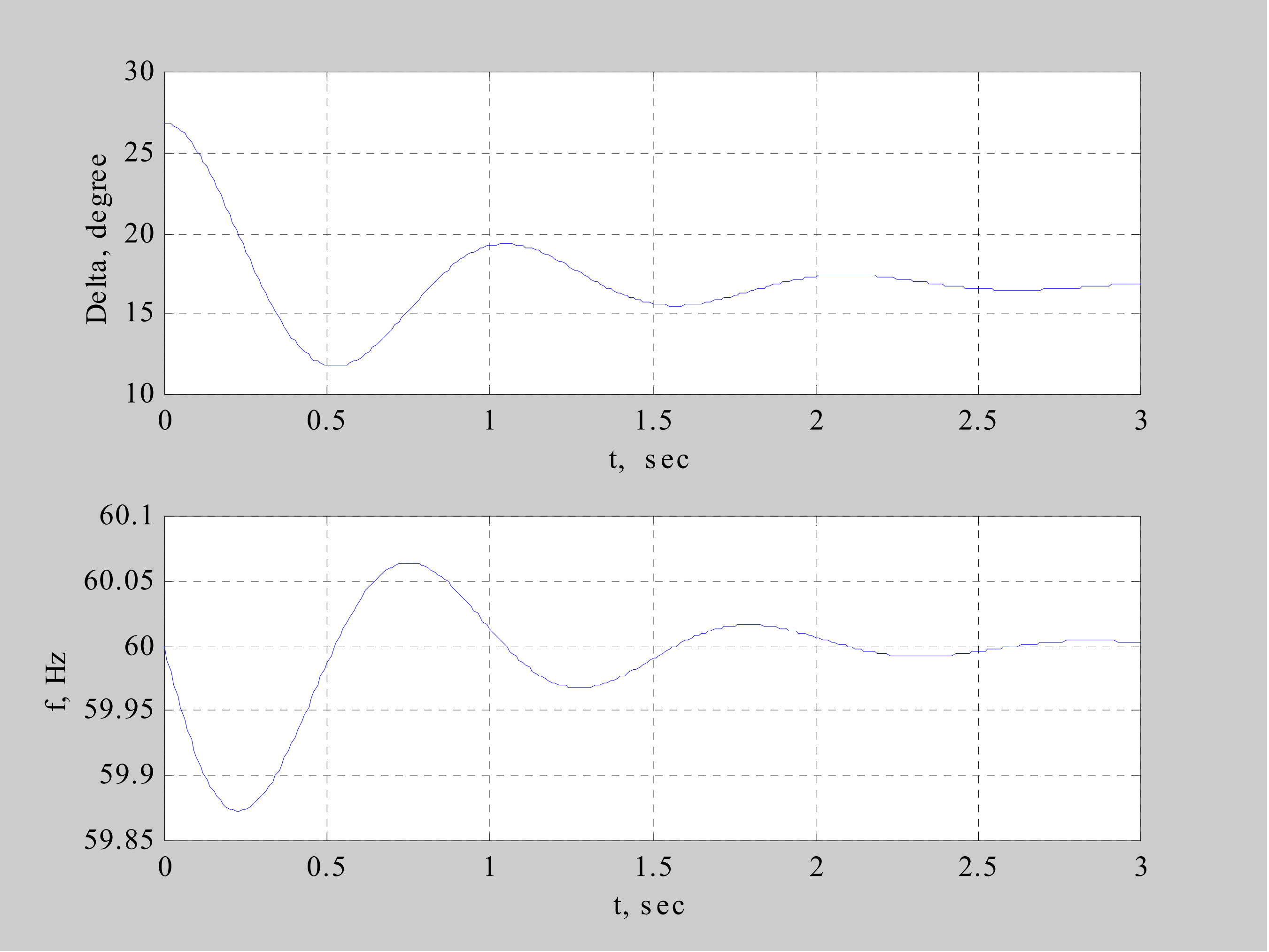
$$P_m\left(\delta_c-\delta_0\right)-\int_{\delta_0}^{\delta_c} P_{2\max}\sin\delta\, d\delta=\int_{\delta_c}^{\delta_{\max}} P_{3\max}\sin\delta\, d\delta-P_m\left(\delta_{\max}-\delta_c\right)$$

$$\cos\delta_c=\frac{P_m\left(\delta_{\max}-\delta_c\right)+P_{3\max}\cos\delta_{\max}-P_{2\max}\cos\delta_0}{P_{3\max}-P_{2\max}}$$

Steady State Stability

- **The ability of the power system to remain in synchronism when subject to small disturbances**
- **Stability is assured if the system returns to its original operating state (voltage magnitude and angle profile)**
- **The behavior can be determined with a linear system model**
- **Assumption:**
  - the automatic controls are not active
  - the power shift is not large
  - the voltage angles changes are small

Example



30
25
20
15
10
Delta, degree
0
0.5
1
1.5
2
2.5
3
t, sec
60.1
60.05
60
59.95
59.9
59.85
f, Hz
0
0.5
1
1.5
2
2.5
3
t, sec

Steady State Stability

- **Simplification of the swing equation**

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2}+\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2}=P_m-P_{max}\left[\sin\delta_0\cos\Delta\delta+\cos\delta_0\sin\Delta\delta\right]$$

Substitute the following approximations

$$\Delta\delta << \delta \qquad \cos\Delta\delta\approx 1 \qquad \sin\Delta\delta\approx\Delta\delta$$

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2}+\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2}=P_m-P_{max}\sin\delta_0-P_{max}\cos\delta_0\cdot\Delta\delta$$

Group steady state and transient terms

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2}-P_m+P_{max}\sin\delta_0=-\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2}-P_{max}\cos\delta_0\cdot\Delta\delta$$

Steady State Stability

- **Simplification of the swing equation**

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2} - P_m + P_{max}\sin\delta_0 = -\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} - P_{max}\cos\delta_0\cdot\Delta\delta$$

$$0 = \frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} + P_{max}\cos\delta_0\cdot\Delta\delta$$

Steady state term is equal to zero

$$\left.\frac{dP_e}{d\delta}\right|_{\delta_0} = \frac{d}{d\delta}P_{max}\sin\delta\Big|_{\delta_0} = P_{max}\cos\delta_0 = P_s$$

$$\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} + P_s\cdot\Delta\delta = 0$$

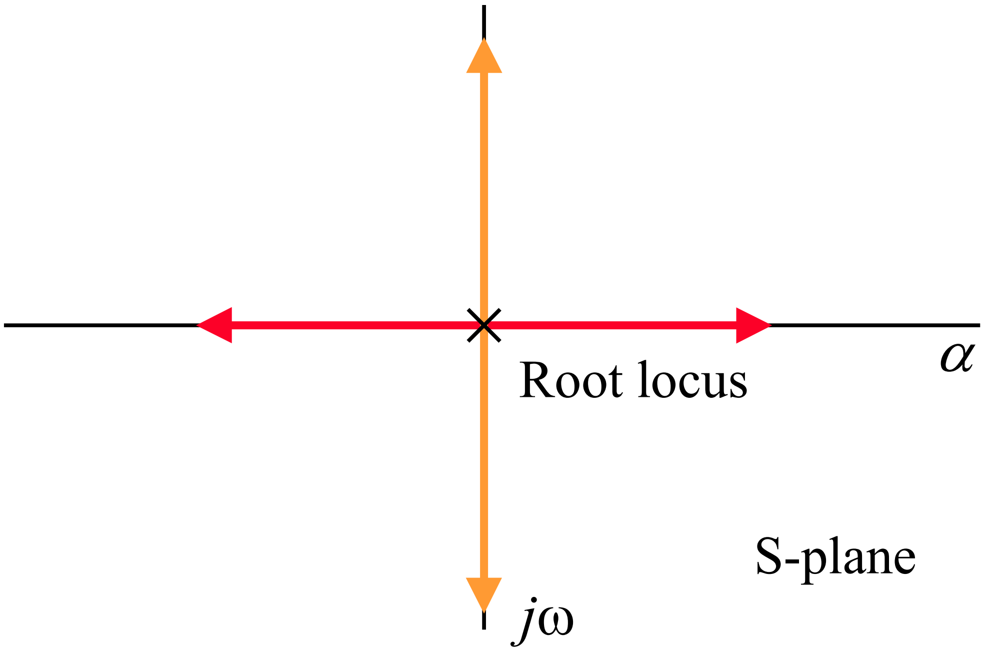
Second order equation.
The solution depends on the roots of the characteristic equation

Stability

- **Stability Assessment**
  - When P_s is negative, one root is in the right-half s-plane, and the response is exponentially increasing and stability is lost
  - When P_s is positive, both roots are on the $j\omega$ axis, and the motion is oscillatory and undamped, the natural frequency is:

$$s^2 = -\frac{\pi f_0}{H} P_S$$

$$\omega_n = \sqrt{\frac{\pi f_0}{H} P_S}$$



Damping Torque

$$P_D = D\frac{d\delta}{dt}$$ Damping force is due to air-gap interaction

$$\frac{H}{\pi\ f_0}\frac{d^2\Delta\delta}{dt^2} + D\frac{d\Delta\delta}{dt} + P_S\Delta\delta = 0$$

$$\frac{d^2\Delta\delta}{dt^2} + \frac{\pi\ f_0}{H}D\frac{d\Delta\delta}{dt} + \frac{\pi\ f_0}{H}P_S\Delta\delta = 0$$

$$\frac{d^2\Delta\delta}{dt^2} + 2\zeta\,\omega_n\frac{d\Delta\delta}{dt} + \omega_n^2\Delta\delta = 0$$

$$\zeta = \frac{D}{2}\sqrt{\frac{\pi\ f_0}{H\ P_S}}$$

Characteristic Equation

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$\zeta = \frac{D}{2}\sqrt{\frac{\pi\, f_0}{H\, P_S}} < 1$$ for normal operation conditions

$$s_1, s_2 = -\zeta\omega_n \pm j\,\omega_n\sqrt{1-\zeta^2}$$ complex roots

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$ the damped frequency of oscillation

Laplace Transform Analysis

$$x_1 = \Delta\delta, \quad x_2 = \frac{d\Delta\delta}{dt}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

$$\mathcal{L}\left\{\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}\right\} \to s\mathbf{X}(s) - \mathbf{x}(0) = \mathbf{A}\mathbf{X}(s)$$

$$\mathbf{X}(s) = \left(s\mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{x}(0)$$

$$\left(s\mathbf{I} - \mathbf{A}\right) = \begin{bmatrix} s & -1 \\ \omega_n^2 & s + 2\zeta\omega_n \end{bmatrix}$$

$$\mathbf{X}(s) = \frac{\begin{bmatrix} s + 2\zeta\omega_n & 1 \\ -\omega_n^2 & s \end{bmatrix}}{s^2 + 2\zeta\omega_n s + \omega_n^2}\mathbf{x}(0)$$

Laplace Transform Analysis

$$\Delta\delta(s) = \frac{\left(s + 2\zeta\omega_n\right)\Delta\delta_0}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\Delta\omega(s) = \frac{\omega_n^2 \Delta\delta_0}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\Delta\delta(t) = \frac{\Delta\delta_0}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_d t + \theta\right), \quad \theta = \cos^{-1}\zeta$$

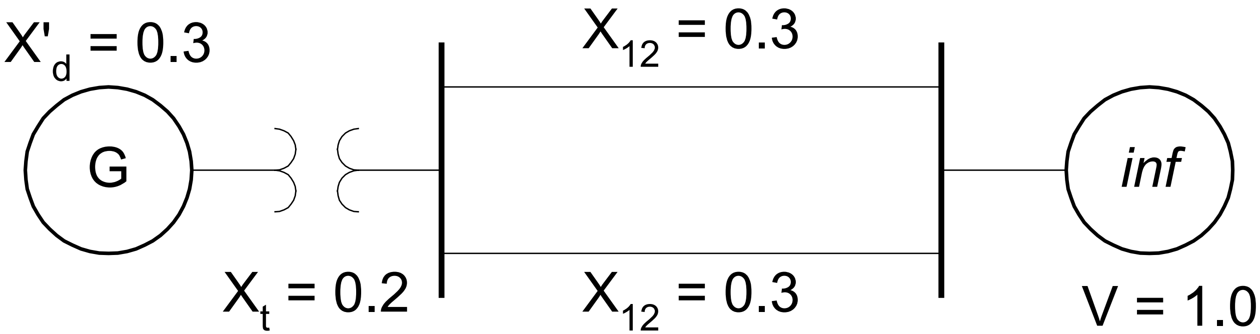
$$\Delta\omega(t) = -\frac{\omega_n \Delta\delta_0}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_d t\right)$$

$$\delta(t) = \delta_0 + \Delta\delta(t), \quad \omega(t) = \omega_0 + \Delta\omega(t)$$

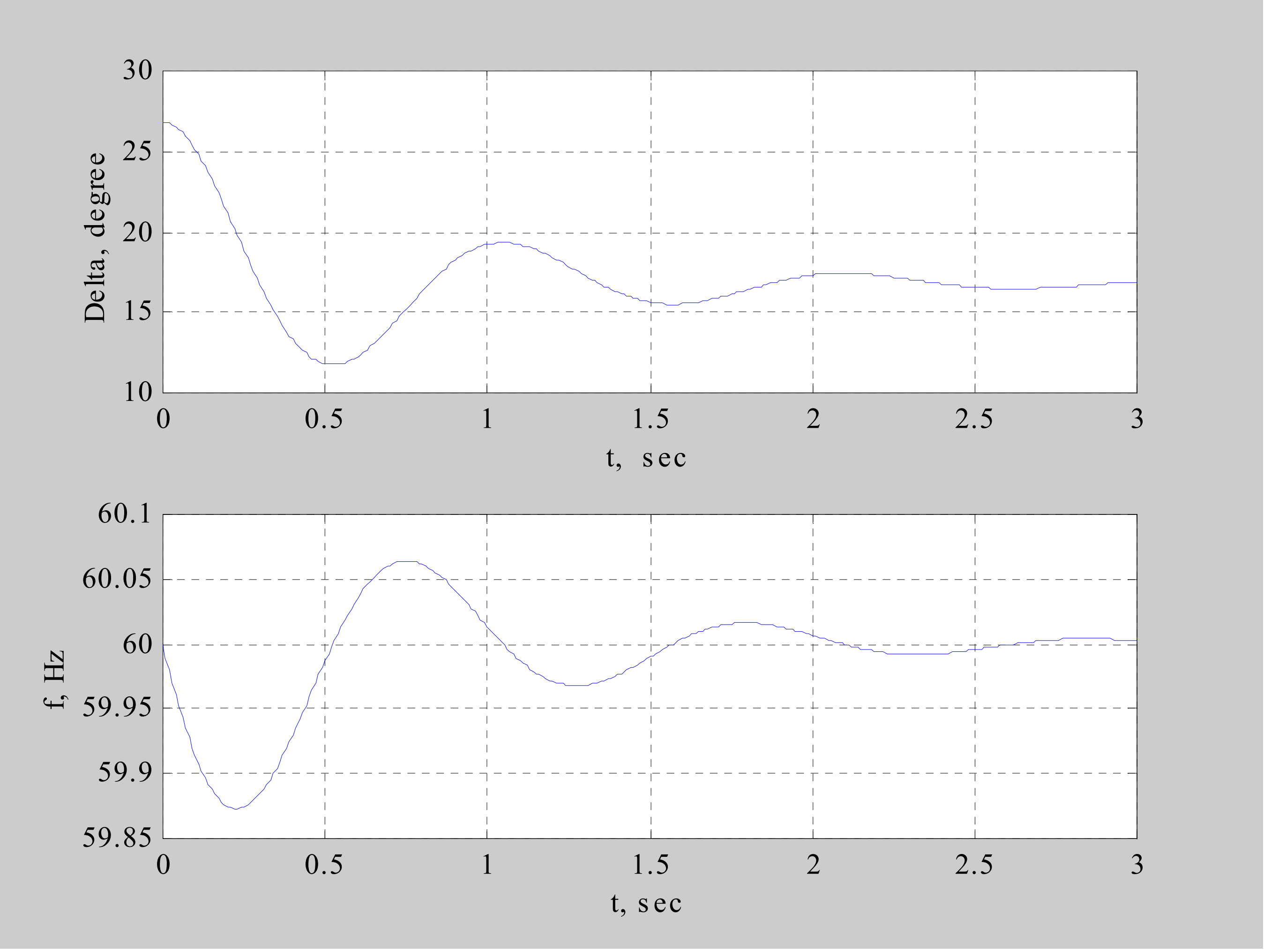
Example

- A 60 Hz synchronous generator having inertia constant H = 9.94 MJ/MVA and a transient reactance X'_d = 0.3 pu is connected to an infinite bus through the following network. The generator is delivering 0.6 *pu* real power at 0.8 power factor lagging to the infinite bus at a voltage of 1 *pu*. Assume the damping power coefficient is *D* = 0.138 *pu*. Consider a small disturbance of 10° or 0.1745 radians. Obtain equations of rotor angle and generator frequency motion.

Example



Example



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Delta, degree
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t, sec
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60.05
60
59.95
59.9
59.85
f, Hz
0
0.5
1
1.5
2
2.5
3
t, sec

Topic 3: Economic Dispatch

- Economic dispatch is used to determine the least cost means of using existing generating plants to meet electric demand

Power System Control Center

Economic Dispatch

- **In practice and in power flow analysis, there are many choices for setting the operating points of generators**
  - in the power flow analysis, generator buses are specified by P and |V|
  - generation capacity is more than load demand - generators can produce more than the customers can consume
    - there are many solution combinations for scheduling generation
  - in practice, power plants are not located at the same distance from the load centers
  - power plants use different types of fuel, which vary in cost from time to time
- **For interconnected systems, the objective is to find the real and reactive power scheduling so as to minimize some operating cost or cost function**

Optimization

- **General cost function:** $f\left(x_1, x_2, \cdots, x_n\right) = C$
- **Unconstrained parameter optimization, from calculus:**
  - the first derivative of f vanishes at a local extrema

    $$\frac{d}{dx} f\left(x\right) = 0$$

  - for f to be a local minimum, the second derivative must be positive at the point of the local extrema

    $$\frac{d^2}{dx^2} f\left(x\right) > 0$$

  - for a set of parameters, the gradient of f vanishes at a local extrema and to be a local minimum, the Hessian must be a positive definite matrix (i.e. positive eigenvalues)

    $$\frac{\partial f}{\partial x_i} = 0 \quad i = 1, \cdots, n \quad \text{or} \quad \nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \cdots, \frac{\partial f}{\partial x_n}\right) = 0$$

Example

- **Find the minimum of**

$$f\left(x_1, x_2, x_3\right) = x_1^{\,2} + 2x_2^{\,2} + 3x_3^{\,2} + x_1 x_2 + x_2 x_3 - 8x_1 - 16x_2 - 32x_3 + 110$$

  - evaluating the first derivatives to zero results in

$$\frac{\partial f}{\partial x_1} = 2x_1 + x_2 - 8 = 0$$
$$\frac{\partial f}{\partial x_2} = x_1 + 4x_2 + x_3 - 16 = 0 \quad or \quad \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 6 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 16 \\ 32 \end{bmatrix} \qquad \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$$
$$\frac{\partial f}{\partial x_3} = x_2 + 6x_3 - 32 = 0$$

Equality Constraints in Optimization

- **This type of problem arises when there are functional dependencies among the parameters to be found**
- **The problem**
  - minimize the cost function

    $$f\left(\hat{x}_1 \cdots \hat{x}_j \cdots \hat{x}_n\right)$$

  - subject to the equality constraints

    $$g_i\left(\hat{x}_1 \cdots \hat{x}_j \cdots \hat{x}_n\right) = 0 \quad i = 1, \cdots, k$$

- **Such problems may be solved by the *Lagrange muliplier* method**

Equality Constraints in Optimization

- **Lagrange Multiplier method**
  - introduce k-dimensional vector λ for the undetermined quantities

$$\mathrm{L} = f + \sum_{i=1}^{k} \lambda_i \, g_i$$

New cost function

  - The necessary conditions for finding the local minimum

$$\frac{\partial \mathrm{L}}{\partial x_i} = \frac{\partial f}{\partial x_i} + \sum_{i=1}^{k} \lambda_i \frac{g_i}{\partial x_i} = 0$$

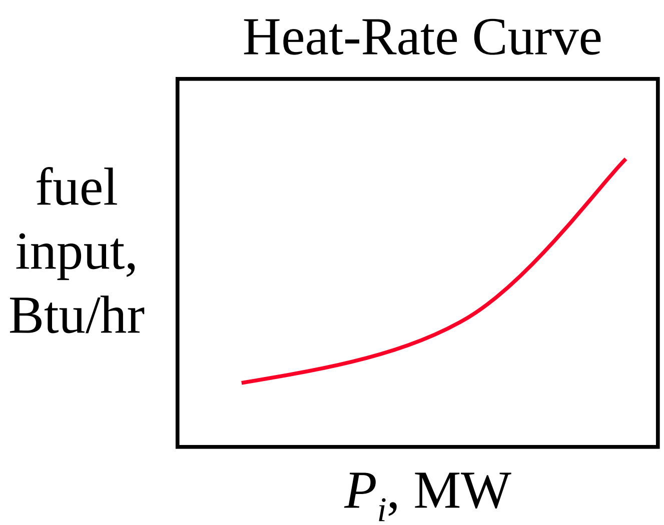
$$\frac{\partial \mathrm{L}}{\partial \lambda_i} = g_i = 0$$

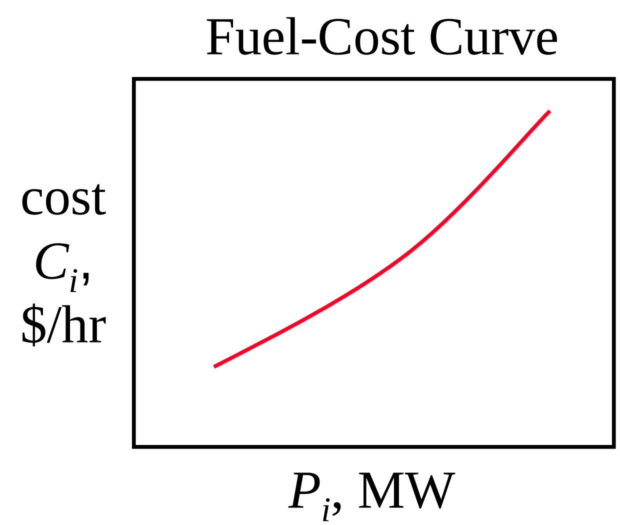
Operating Costs

- **Factors influencing the minimum cost of power generation**
  - operating efficiency of prime mover and generator
  - fuel costs
  - transmission losses
- **The most efficient generator in the system does not guarantee minimum costs**
  - may be located in an area with high fuel costs
  - may be located far from the load centers and transmission losses are high
- **The problem is to determine generation at different plants to minimize the total operating costs**

Operating Costs

- **Generator heat rate curves lead to the fuel cost curves**





  - The fuel cost is commonly express as a quadratic function

$$C_i = \alpha_i + \beta_i P_i + \gamma_i P_i^2$$

  - The derivative is known as the incremental fuel cost

$$dC_i \Big/ dP_i = \beta_i + 2\gamma_i P_i$$

Economic Dispatch

- **The simplest problem is when system losses and generator limits are neglected**
  - minimize the objective or cost function over all plants
  - a quadratic cost function is used for each plant

$$C_{total} = \sum_{i=1}^{n_{gen}} C_i = \sum_{i=1}^{n_{gen}} \alpha_i + \beta_i P_i + \gamma_i P_i^2$$

  - the total demand is equal to the sum of the generators' output; the equality constrant

$$\sum_{i=1}^{n_{gen}} P_i = P_{Demand}$$

Economic Dispatch

- **A typical approach using the Lagrange multipliers**

$$L = C_{total} + \lambda\left(P_{Demand} - \sum_{i=1}^{n_{gen}} P_i\right)$$

$$\frac{\partial L}{\partial P_i} = \frac{\partial C_{total}}{\partial P_i} + \lambda(0-1) = 0 \quad \rightarrow \quad \frac{\partial C_{total}}{\partial P_i} = \lambda$$

$$C_{total} = \sum_{i=1}^{n_{gen}} C_i \quad \rightarrow \quad \frac{\partial C_{total}}{\partial P_i} = \frac{dC_i}{dP_i} = \lambda \quad \forall i = 1, \ldots, n_g$$

$$\lambda = \frac{dC_i}{dP_i} = \beta_i + 2\gamma_i P_i$$

Economic Dispatch

- the second condition for optimal dispatch

$$\frac{dL}{d\lambda} = \left(P_{Demand} - \sum_{i=1}^{n_{gen}} P_i \right) = 0 \quad \rightarrow \quad \sum_{i=1}^{n_{gen}} P_i = P_{Demand}$$

- rearranging and combining the equations to solve for λ

$$P_i = \frac{\lambda - \beta_i}{2\gamma_i}$$

$$\sum_{i=1}^{n_{gen}} \frac{\lambda - \beta_i}{2\gamma_i} = P_{Demand} \qquad \lambda = \frac{P_{Demand} + \sum_{i=1}^{n_{gen}} \frac{\beta_i}{2\gamma_i}}{\sum_{i=1}^{n_{gen}} \frac{1}{2\gamma_i}}$$

Example

- **Neglecting system losses and generator limits, find the optimal dispatch and the total cost in $/hr for the three generators and the given load demand**

$$C_1 = 500 + 5.3P_1 + 0.004P_1^2 \; [\$ / MWhr]$$

$$C_2 = 400 + 5.5P_2 + 0.006P_2^2$$

$$C_3 = 200 + 5.8P_3 + 0.009P_3^2$$

$$P_{Demand} = 800MW$$

Example

$$\lambda = \frac{P_{Demand} + \sum_{i=1}^{n_{gen}} \frac{\beta_i}{2\gamma_i}}{\sum_{i=1}^{n_{gen}} \frac{1}{2\gamma_i}} = \frac{800 + \frac{5.3}{0.008} + \frac{5.5}{0.012} + \frac{5.8}{0.018}}{\frac{1}{0.008} + \frac{1}{0.012} + \frac{1}{0.018}} = \$8.5 / MWhr$$

$$P_i = \frac{\lambda - \beta_i}{2\gamma_i}$$

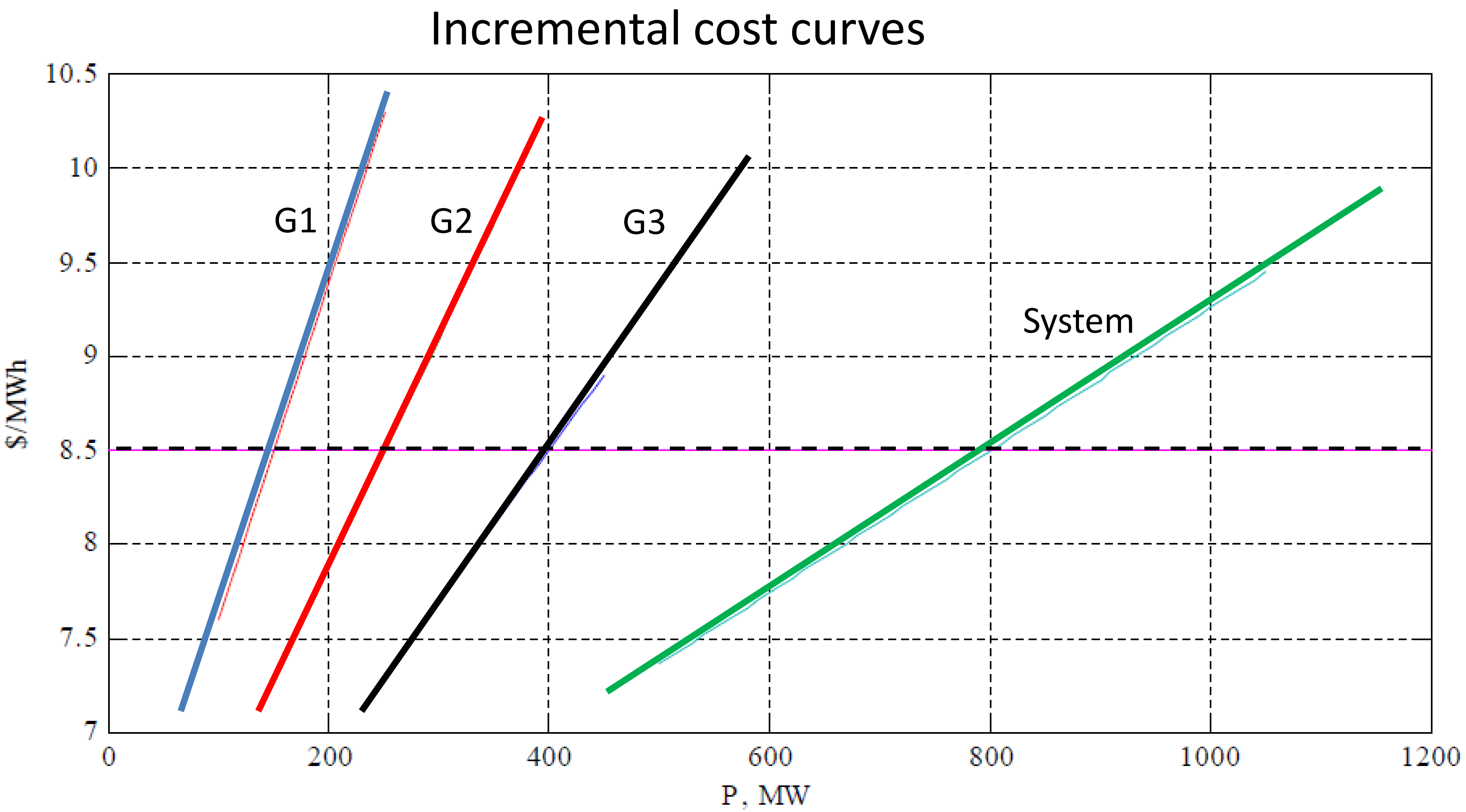
$$P_1 = \frac{8.5 - 5.3}{2(0.004)} = 400MW$$

$$P_2 = \frac{8.5 - 5.5_i}{2(0.006)} = 250MW$$

$$P_3 = \frac{8.5 - 5.8}{2(0.009)} = 150MW$$

$$P_{Demand} = 800 = 400 + 250 + 150$$

Example


Incremental cost curves
G1
G2
G3
System
$/MWh
P , MW
10.5
10
9.5
9
8.5
8
7.5
7
0
200
400
600
800
1000
1200

Discussion

- Key results for Economic Dispatch?
  - Incremental cost of all generating units is equal
  - This incremental cost is the Lagrangean multiplier, λ
  - 'λ' is called the 'System λ' and is the system-wide cost of generating electricity
    - This is the price charged to customers

Economic Dispatch with Generator Limits

- **The power output of any generator should not exceed its rating nor be below the value for stable boiler operation**
  - Generators have a minimum and maximum real power output limits
- **The problem is to find the real power generation for each plant such that cost are minimized, subject to:**
  - Meeting load demand - equality constraints
  - Constrained by the generator limits - inequality constraints
- **The Kuhn-Tucker conditions**

$$dC_i / dP_i = \lambda \quad \leftarrow \quad P_{i(\min)} < P_i < P_{i(\max)}$$

$$dC_i / dP_i \leq \lambda \quad \leftarrow \quad P_i = P_{i(\max)}$$

$$dC_i / dP_i \geq \lambda \quad \leftarrow \quad P_i = P_{i(\min)}$$

Example

- **Neglecting system losses, find the optimal dispatch and the total cost in $/hr for the three generators and the given load demand and generation limits**

$$C_1 = 500 + 5.3P_1 + 0.004P_1^2 \quad [\$/\text{MWhr}]$$

$$C_2 = 400 + 5.5P_2 + 0.006P_2^2$$

$$C_3 = 200 + 5.8P_3 + 0.009P_3^2$$

$$200 \le P_1 \le 450$$

$$150 \le P_2 \le 350$$

$$100 \le P_3 \le 225$$

$$P_{Demand} = 975 \text{ MW}$$

Example

$$\lambda = \frac{P_{Demand} + \sum_{i=1}^{n_{gen}} \frac{\beta_i}{2\gamma_i}}{\sum_{i=1}^{n_{gen}} \frac{1}{2\gamma_i}} = \frac{975 + \frac{5.3}{0.008} + \frac{5.5}{0.012} + \frac{5.8}{0.018}}{\frac{1}{0.008} + \frac{1}{0.012} + \frac{1}{0.018}} = \$9.163/\text{MWh}$$

$$P_i = \frac{\lambda - \beta_i}{2\gamma_i}$$

$$P_1 = \frac{9.16 - 5.3}{2(0.004)} = 483 \text{ MW}$$

$$P_2 = \frac{9.16 - 5.5_i}{2(0.006)} = 305 \text{ MW}$$

$$P_3 = \frac{9.16 - 5.8}{2(0.009)} = 187 \text{ MW}$$

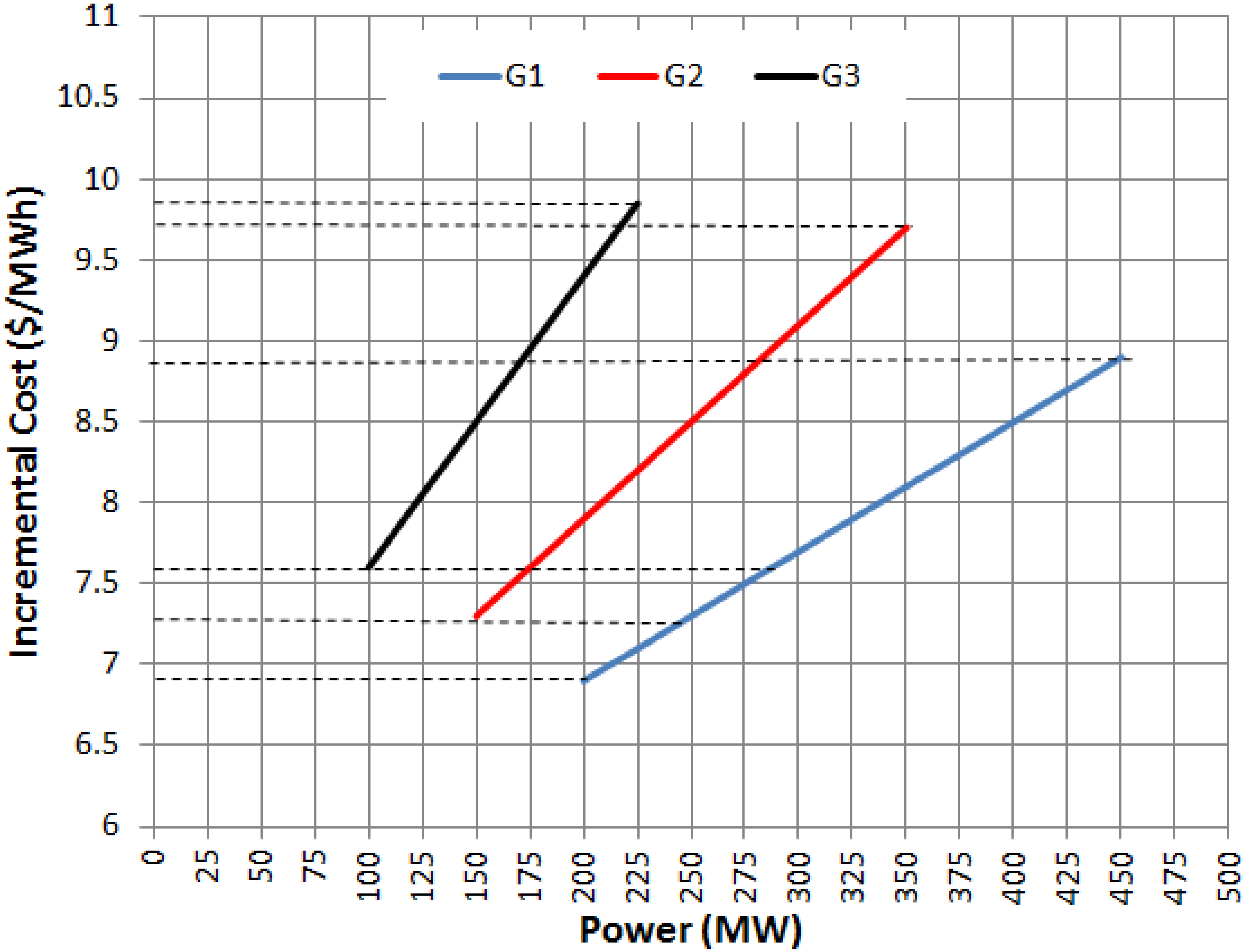
$$P_{Demand} = 975 = 450 + 315 + 210$$

Upper limit violated:
→ P1= 450 MW

→ solve the dispatch problem with two generators:
P2 + P3 = 525 MW

→ λ = \$9.4/MWh
→ P2 = 315 MW
→ P3 = 210 MW

Example



G1
G2
G3
Incremental Cost ($/MWh)
11
10.5
10
9.5
9
8.5
8
7.5
7
6.5
6
0
25
50
75
100
125
150
175
200
225
250
275
300
325
350
375
400
425
450
475
500
Power (MW)

Economic Dispatch including Losses

- **For large interconnected system where power is transmitted over long distances with low load density areas**
  - transmission line losses are a major factor
  - losses affect the optimum dispatch of generation
- **One common practice for including the effect of transmission losses is to express the total transmission loss as a quadratic function of the generator power outputs**
  - simplest form: $$P_L = \sum_{i=1}^{n_{gen}} \sum_{j=1}^{n_{gen}} P_i B_{ij} P_j$$
  - Kron's loss formula: $$P_L = \sum_{i=1}^{n_{gen}} \sum_{j=1}^{n_{gen}} P_i B_{ij} P_j + \sum_{j=1}^{n_{gen}} B_{0j} P_j + B_{00}$$

Economic Dispatch including Losses

- B_{ij} **are called the loss coefficients**
  - they are assumed to be constant
  - reasonable accuracy is expected when actual operating conditions are close to the base case conditions used to compute the coefficients
- **The economic dispatch problem is to minimize the overall generation cost, C, which is a function of plant output**
- **Constraints:**
  - the generation equals the total load demand plus transmission losses
  - each plant output is within the upper and lower generation limits - inequality constraints

Economic Dispatch including Losses

$$f: \quad C_{total} = \sum_{i=1}^{n_{gen}} C_i = \sum_{i=1}^{n_{gen}} \alpha_i + \beta_i P_i + \gamma_i P_i^2$$

$$g: \quad \sum_{i=1}^{n_{gen}} P_i = P_{demand} + P_{losses}$$

$$u: \quad P_{i(\min)} \le P_i \le P_{i(\max)} \quad i = 1, \cdots, n_{gen}$$

The resulting optimization equation

$$L = C_{total} + \lambda \left(P_{demand} + P_{losses} - \sum_{i=1}^{n_{gen}} P_i \right) + \sum_{i=1}^{n_{gen}} \mu_{i(\max)} \left(P_{i(\max)} - P_i \right)$$

$$+ \sum_{i=1}^{n_{gen}} \mu_{i(\min)} \left(P_i - P_{i(\min)} \right)$$

$$P_i < P_{i(\max)}: \quad \mu_{i(\max)} = 0 \qquad\qquad P_i > P_{i(\min)}: \quad \mu_{i(\min)} = 0$$

Economic Dispatch including Losses

- **When generator limits are not violated:**

$$\frac{\partial L}{\partial P_i} = 0 = \frac{\partial C_{total}}{\partial P_i} + \lambda\left(0 + \frac{\partial P_L}{\partial P_i} - 1\right)$$

$$\frac{\partial C_{total}}{\partial P_i} = \frac{\partial}{\partial P_i}\left(C_1 + C_2 + \cdots + C_{n_{gen}}\right) = \frac{dC_i}{dP_i}$$

$$\therefore \quad \lambda = \frac{dC_i}{dP_i} + \lambda \frac{\partial P_L}{\partial P_i} = \left(\frac{1}{1 - \partial P_L / \partial P_i}\right)\frac{dC_i}{dP_i} = L_i \frac{dC_i}{dP_i}$$

$$\frac{\partial L}{\partial \lambda} = 0 = P_D + P_L - \sum_{i=1}^{n_{gen}} P_i \qquad \therefore \sum_{i=1}^{n_{gen}} P_i = P_D + P_L$$

The Penalty Factor

- **The incremental transmission loss equation becomes the penalty factor**

$$L_i = \frac{1}{1 - \dfrac{\partial P_L}{\partial P_i}}$$

  - The effect of transmission losses introduces a penalty factor that depends on the location of the plant
  - The minimum cost is obtained when the incremental cost of each plant multiplied by its penalty factor is the same for all plants

Example

- **Find the optimal dispatch and the total cost in $/hr**
  - fuel costs and plant output limits

$$C_1 = 200 + 7.0P_1 + 0.008P_1^2 \ [\$/\mathrm{hr}] \quad 10 \le P_1 \le 85 \ \mathrm{MW}$$

$$C_2 = 180 + 6.3P_2 + 0.009P_2^2 \quad 10 \le P_2 \le 80$$

$$C_3 = 140 + 6.8P_3 + 0.007P_3^2 \quad 10 \le P_3 \le 70$$

  - real power loss and total load demand

$$P_{loss} = 0.000218\, P_1^2 + 0.000228\, P_2^2 + 0.000179\, P_3^2$$

$$P_{Demand} = 150 \ \mathrm{MW}$$

Example

$$\left(\frac{1}{1-.000436P_1}\right)(7+.016P_1)=\left(\frac{1}{1-.000456P_2}\right)(6.3+.018P_2)$$

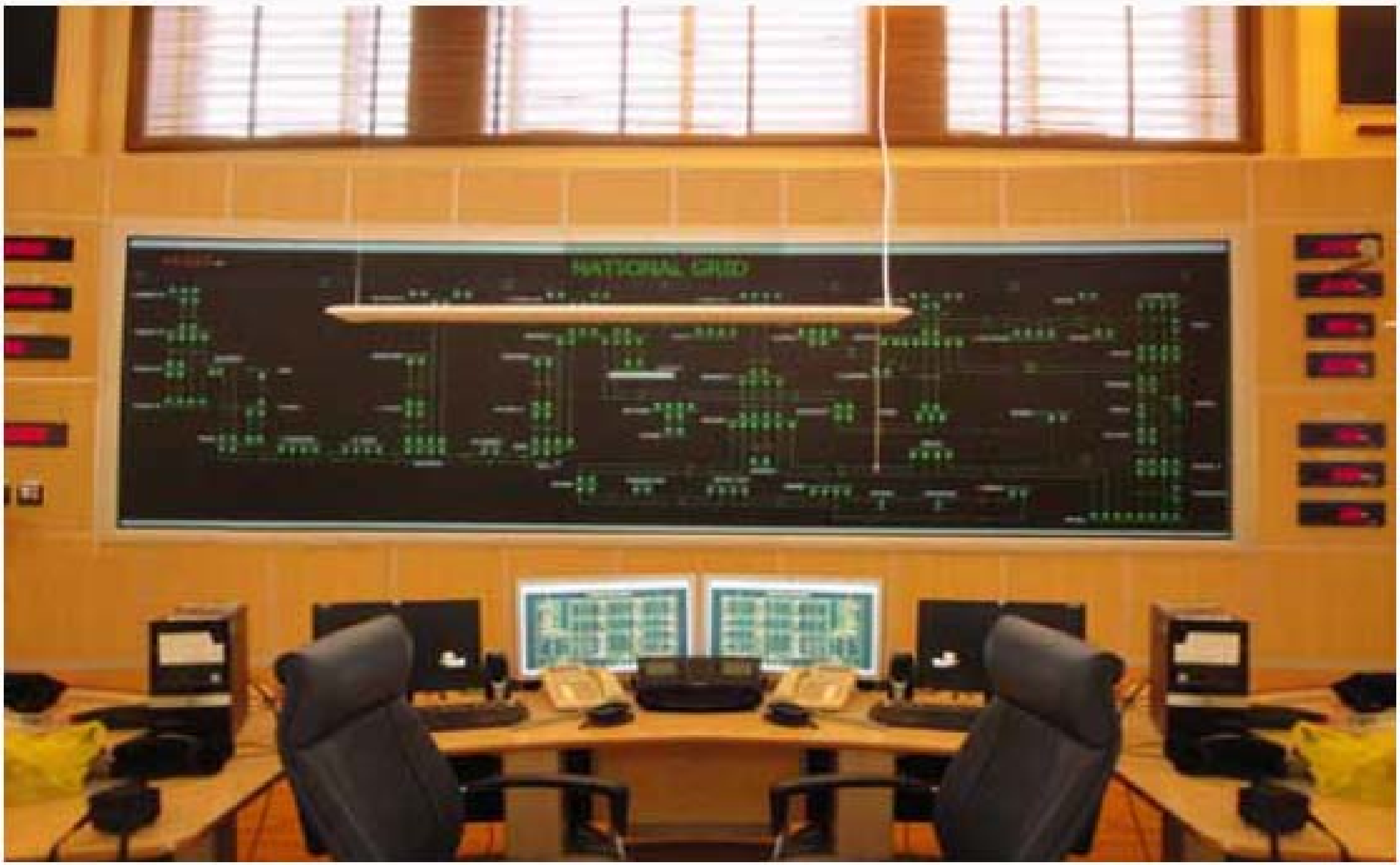
$$=\left(\frac{1}{1-.000358P_3}\right)(6.8+.014P_3),$$

$$P_1+P_2+P_3-150=P_{loss}$$

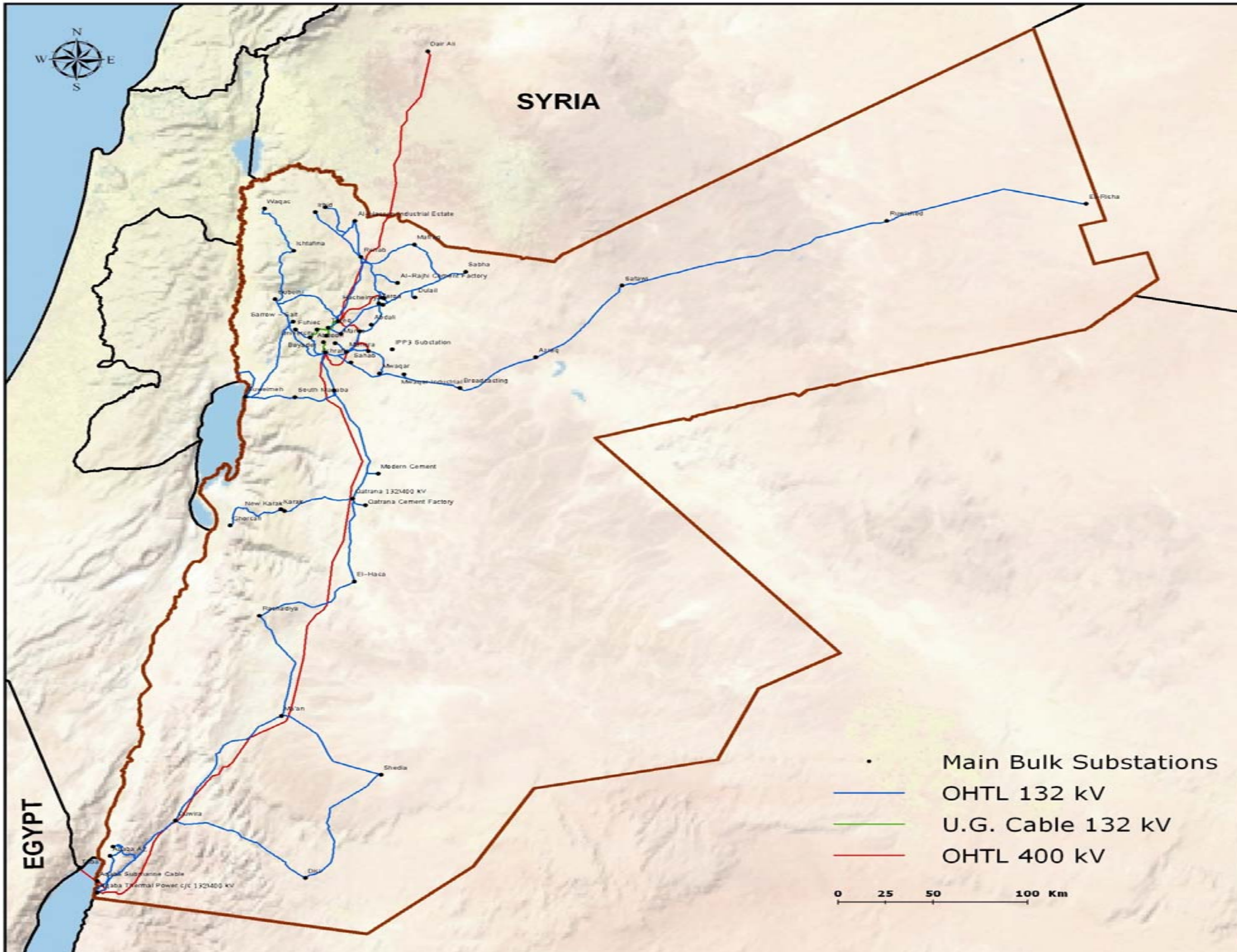
Results (obtained numerically):

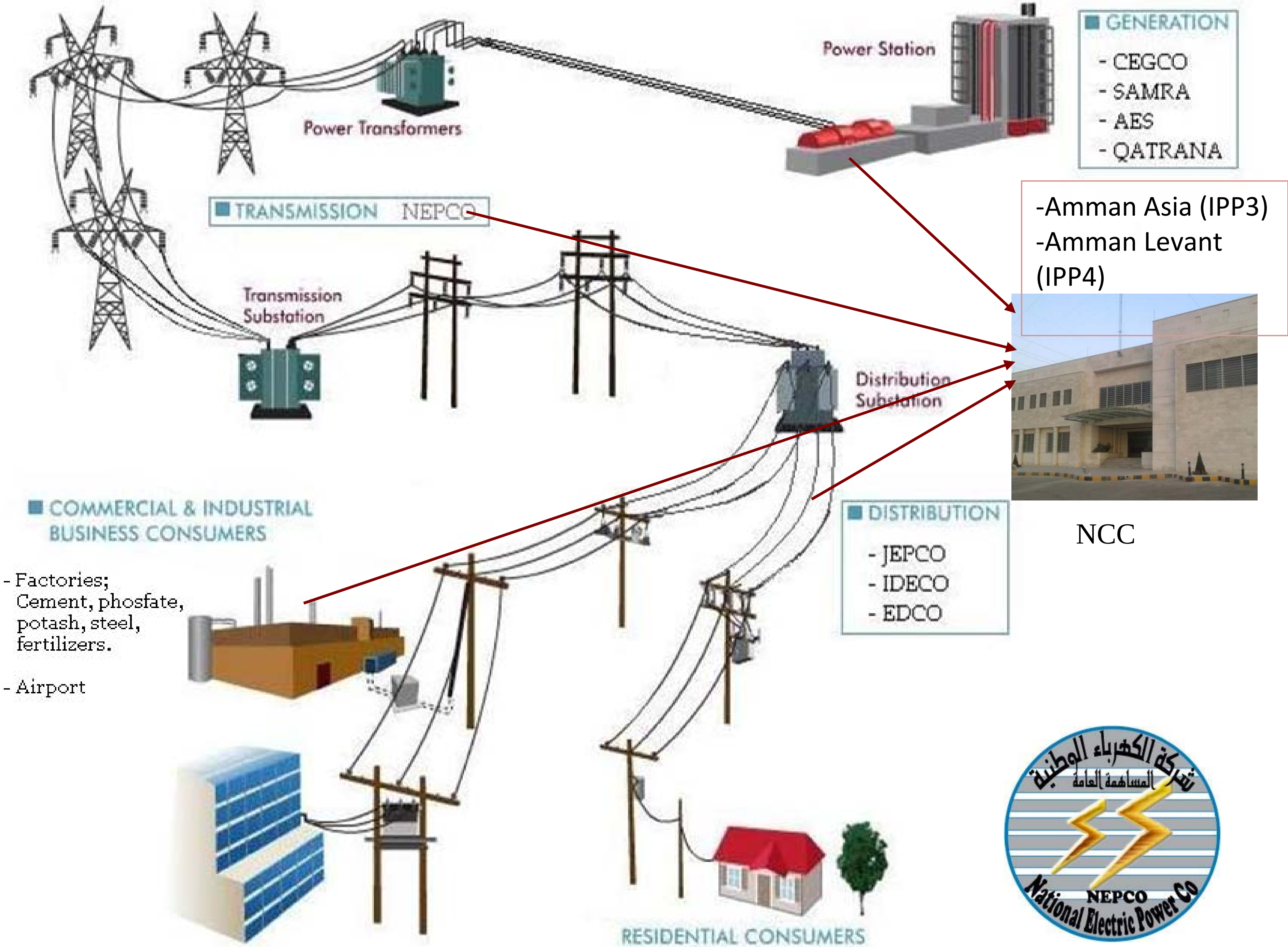
- P_1 = 35.1 MW
- P_2 = 64.1 MW
- P_3 = 52.5 MW
- P_{loss} = 1.7 MW
- P_{demand} = 150 MW

National Control Ceter



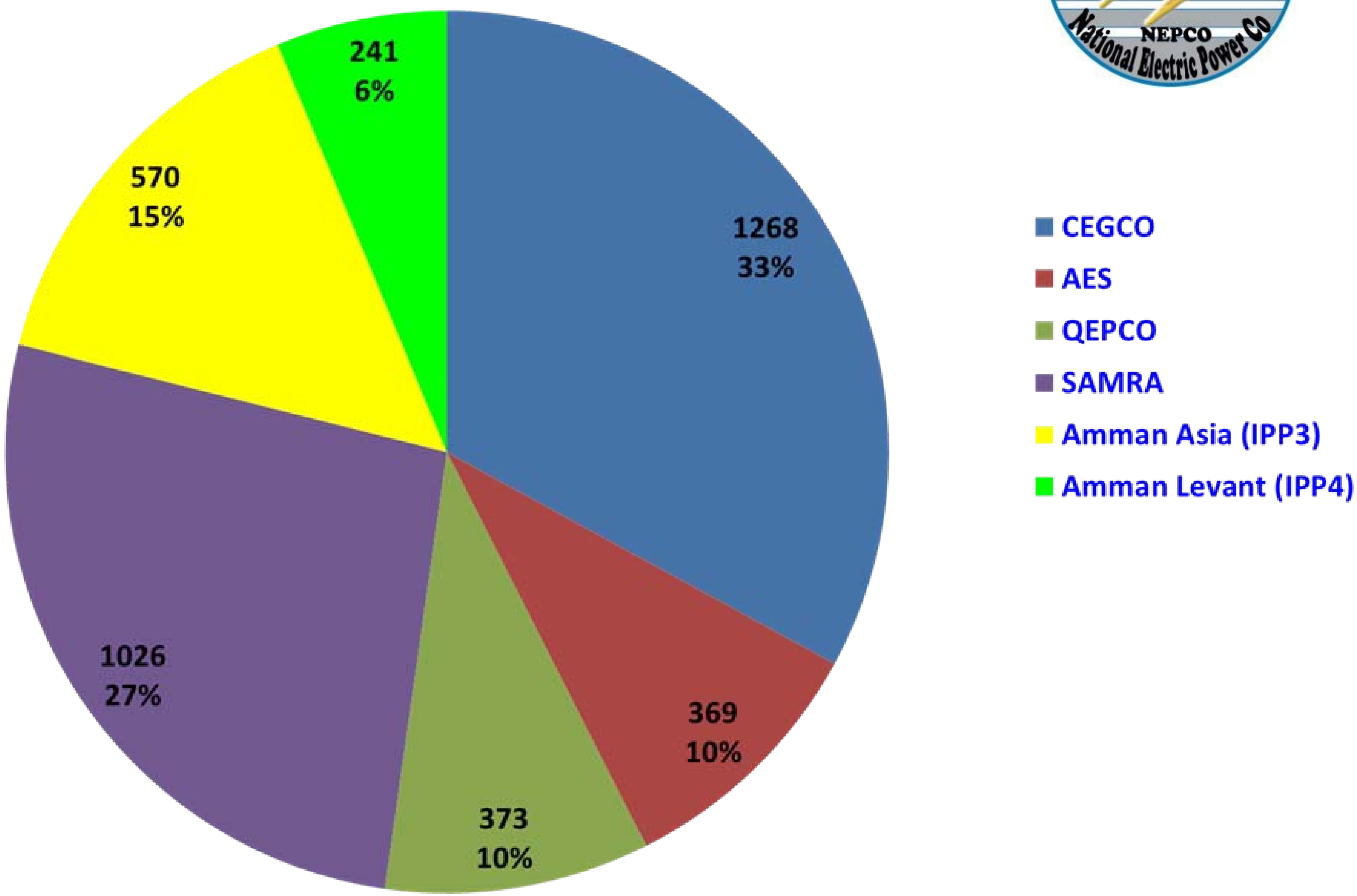
National Transmission Grid



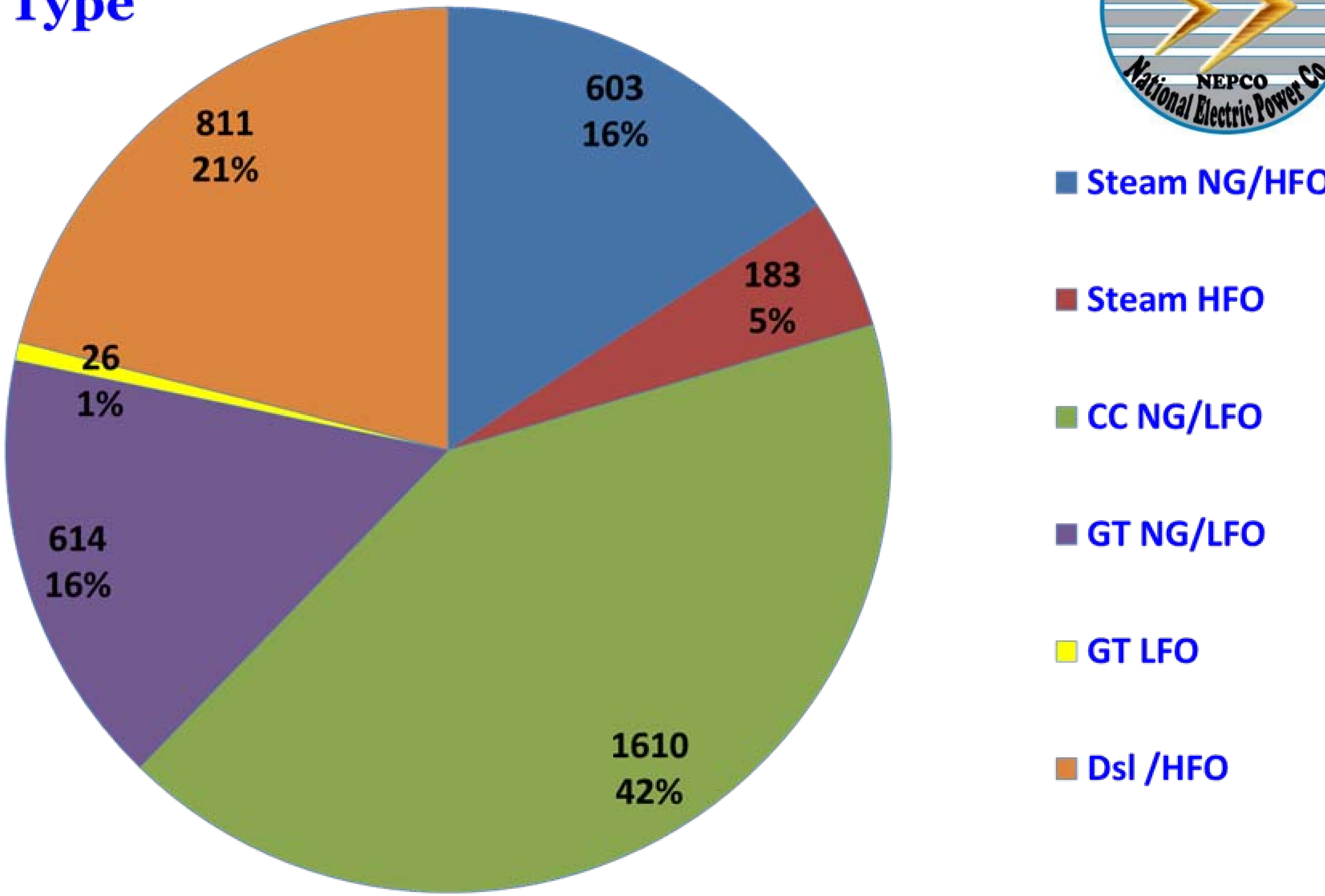

GENERATION
- CEGCO
- SAMRA
- AES
- QATRANA
Power Station
Power Transformers
-Amman Asia (IPP3)
-Amman Levant (IPP4)
TRANSMISSION NEPCO
Transmission Substation
Distribution Substation
NCC
COMMERCIAL & INDUSTRIAL BUSINESS CONSUMERS
- Factories; Cement, phosfate, potash, steel, fertilizers.
- Airport
DISTRIBUTION
- JEPCO
- IDECO
- EDCO
RESIDENTIAL CONSUMERS
شركة الكهرباء الوطنية
المساهمة العامة
NEPCO
National Electric Power Co

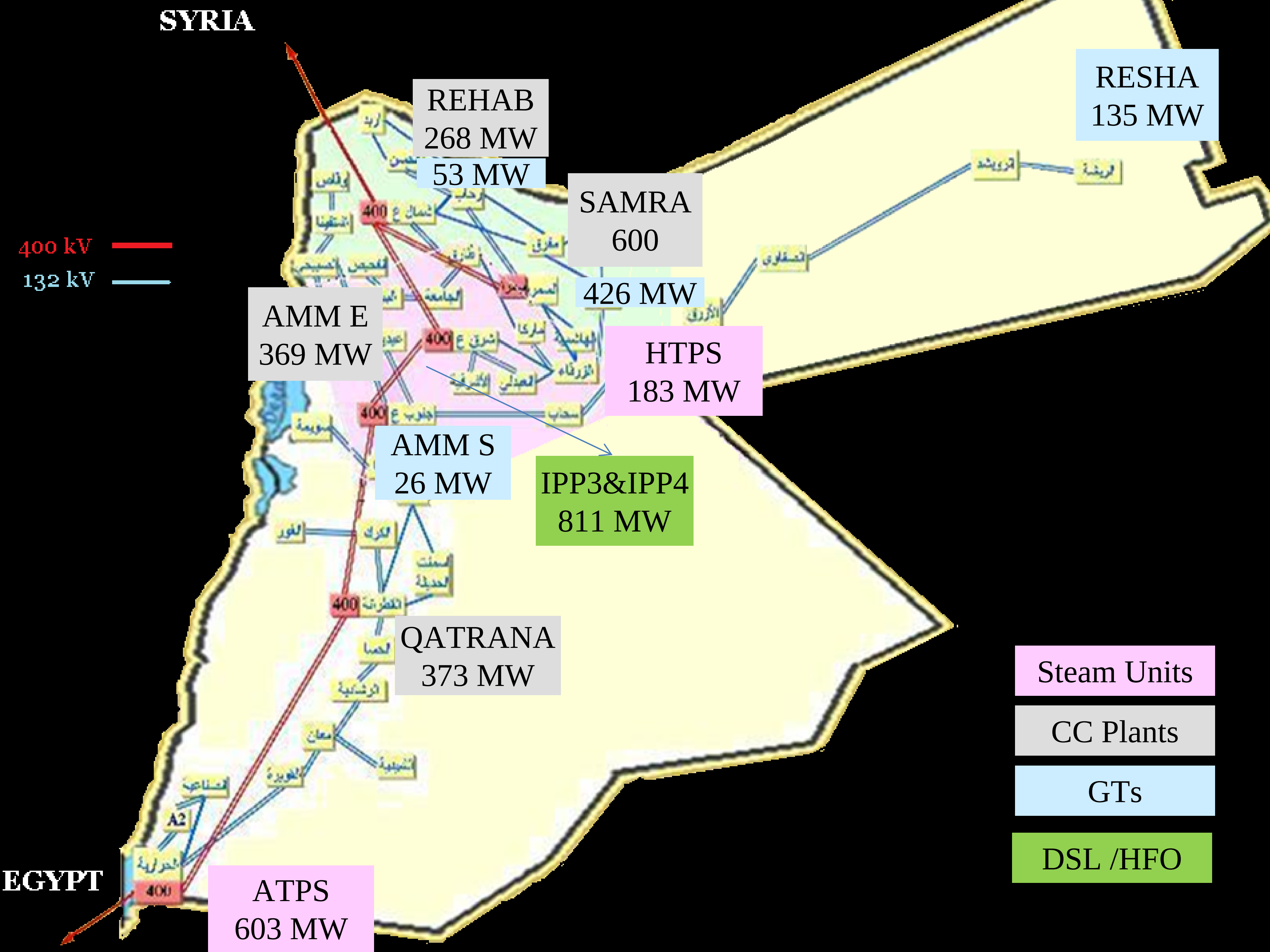
Generation

By Company

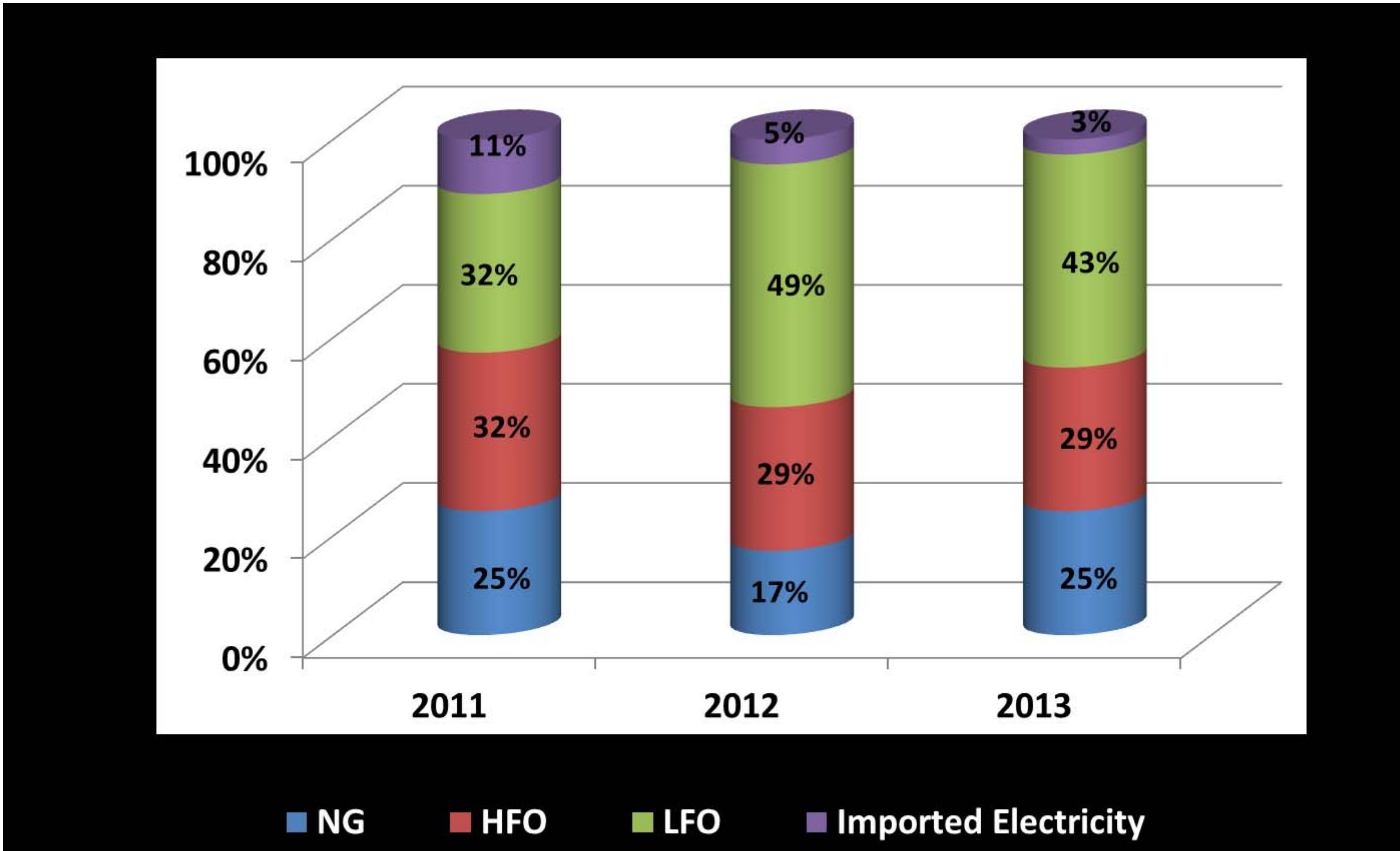


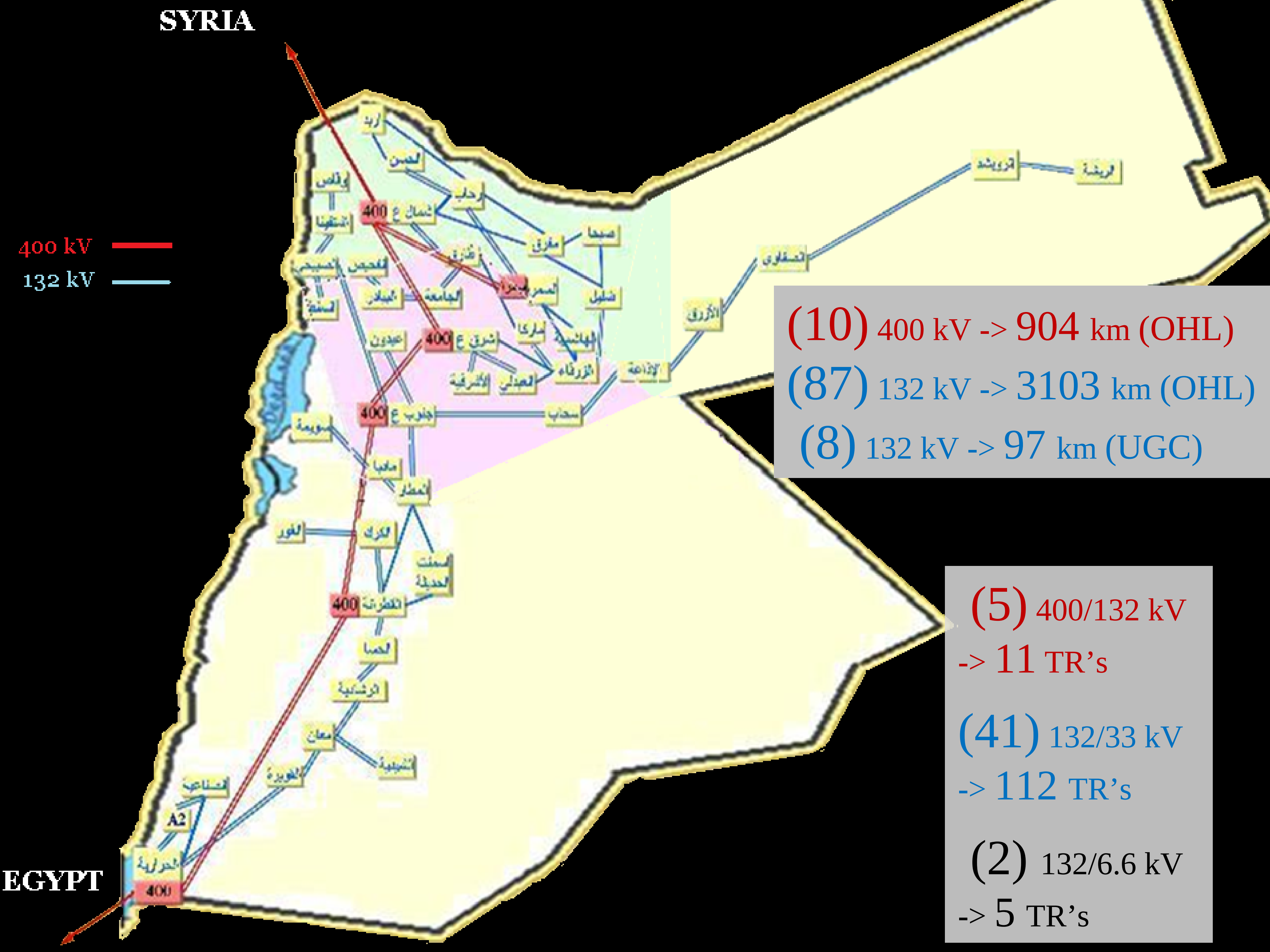
Generation
By Type




SYRIA
EGYPT
400 kV
132 kV
REHAB
268 MW
53 MW
SAMRA
600
426 MW
RESHA
135 MW
AMM E
369 MW
HTPS
183 MW
AMM S
26 MW
IPP3&IPP4
811 MW
QATRANA
373 MW
ATPS
603 MW
A2
Steam Units
CC Plants
GTs
DSL /HFO

Energy generated by fuel type:





SYRIA
400 kV
132 kV
EGYPT
A2
(10) 400 kV -> 904 km (OHL)
(87) 132 kV -> 3103 km (OHL)
(8) 132 kV -> 97 km (UGC)
(5) 400/132 kV
-> 11 TR's
(41) 132/33 kV
-> 112 TR's
(2) 132/6.6 kV
-> 5 TR's

Economic Dispatch

- Operators load the available generating units to maintain the equation:

  Generation = Load + Losses

- Based on “Daily Dispatch Schedule” which is an hourly based schedule prepared ahead of time, showing the forecasted system loads and suggested generation loading to meet such loads.